

Relic neutrinos at accelerator experiments

Jack Shergold

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Durham
University

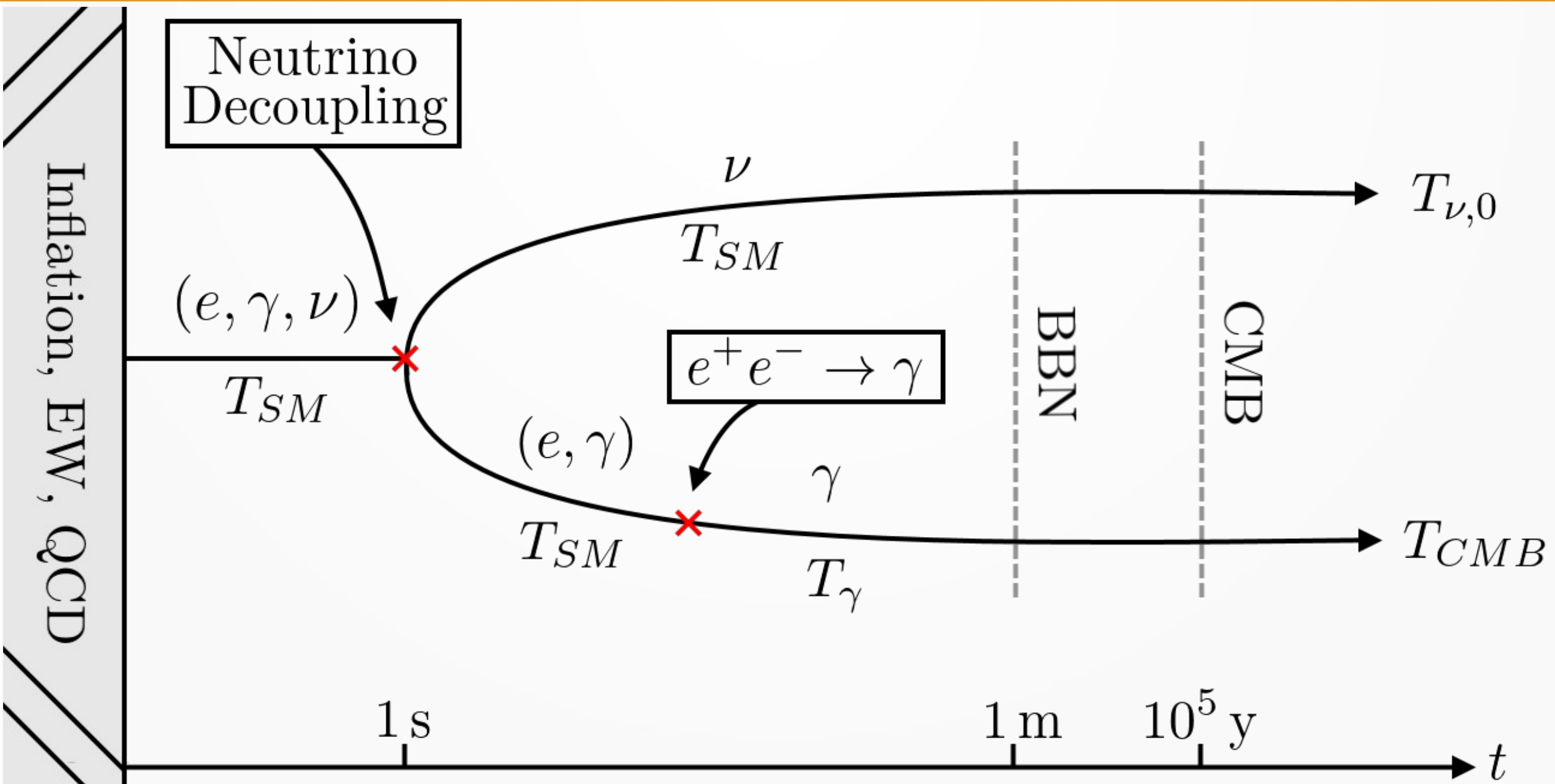
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- What is the $C\nu B$?
- PTOLEMY (-on-a-beam)
- Resonant neutrino capture
- Experimental challenges

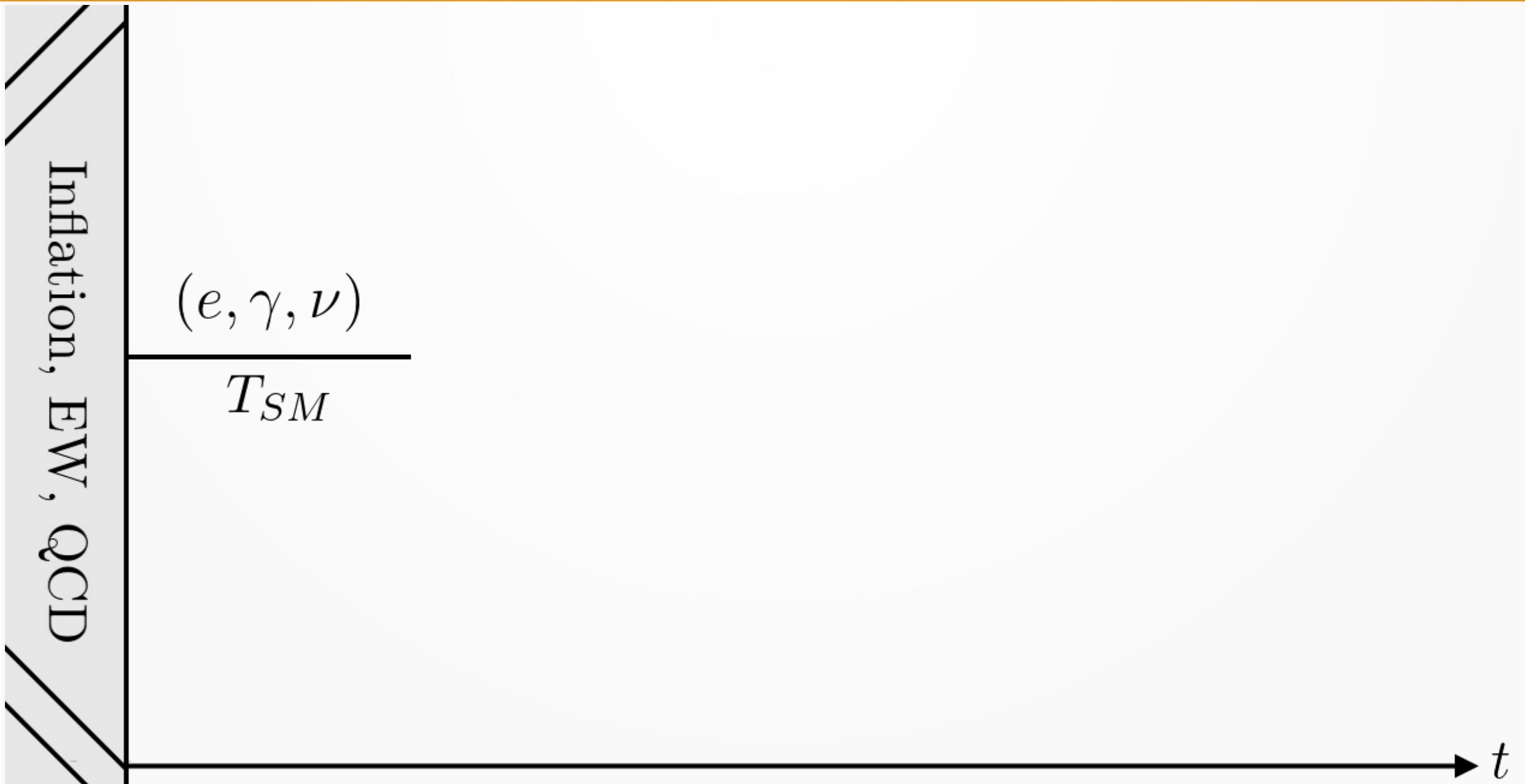
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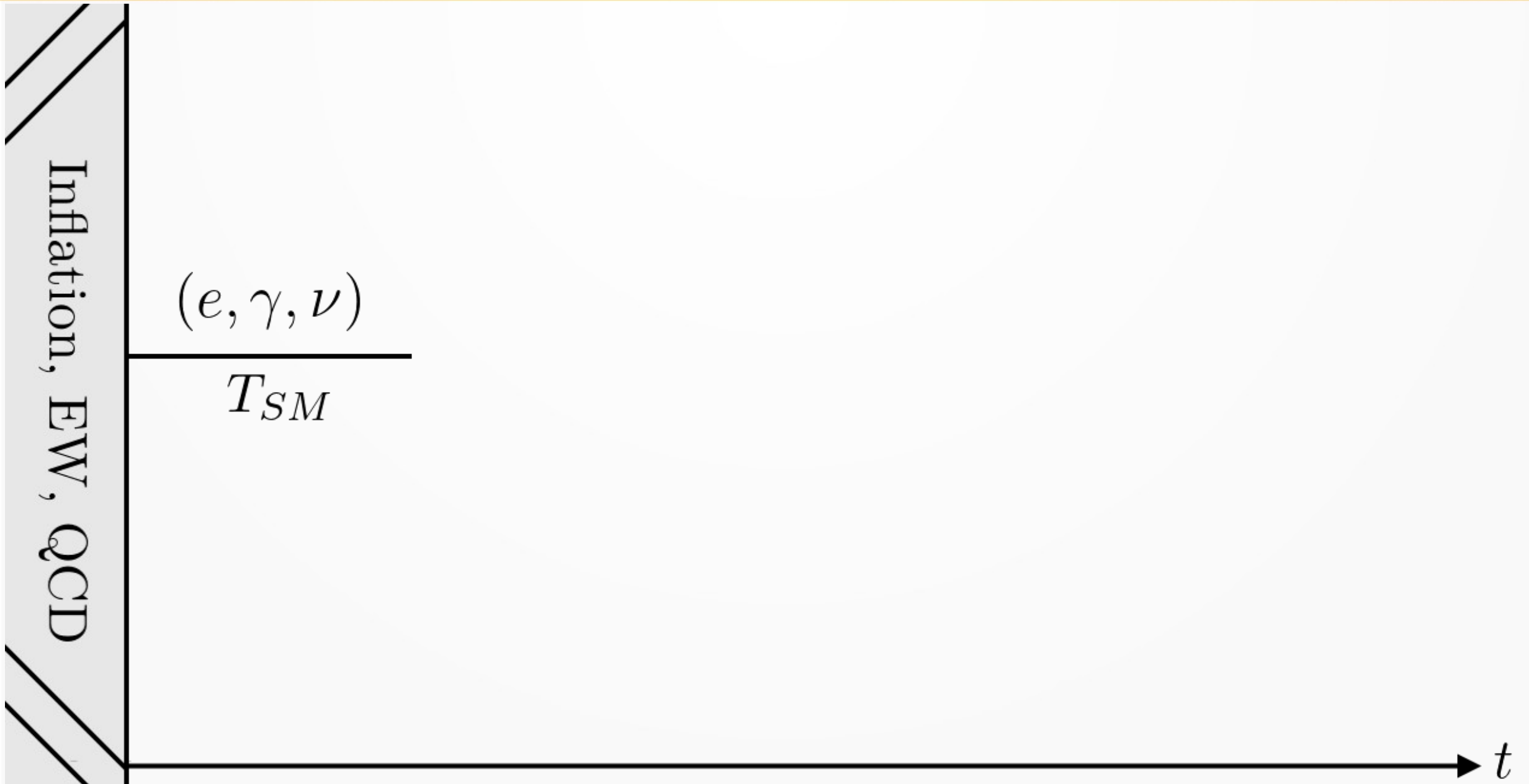
$$e^+ + e^- \leftrightarrow \gamma$$

- Neutrinos and electrons are kept in equilibrium through weak interactions:

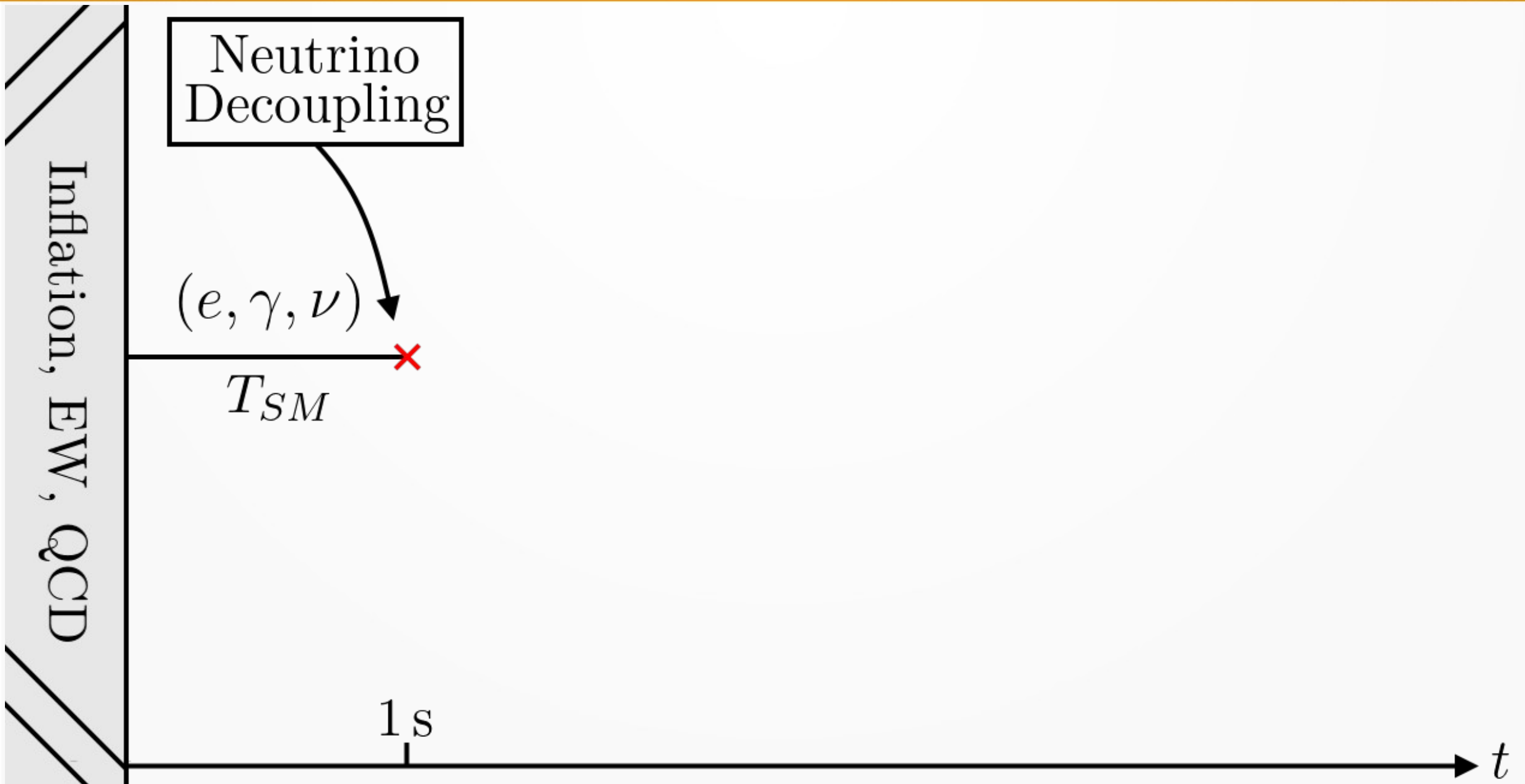
$$\nu + e \rightarrow \nu + e$$

$$\nu + \bar{\nu} \leftrightarrow e^+ + e^-$$

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$$\rho \propto \sum_{e,\gamma,\nu} \int \frac{p_i d^3 p_i}{e^{\frac{p_i}{T_i}} \pm 1} \propto T_{SM}^4$$

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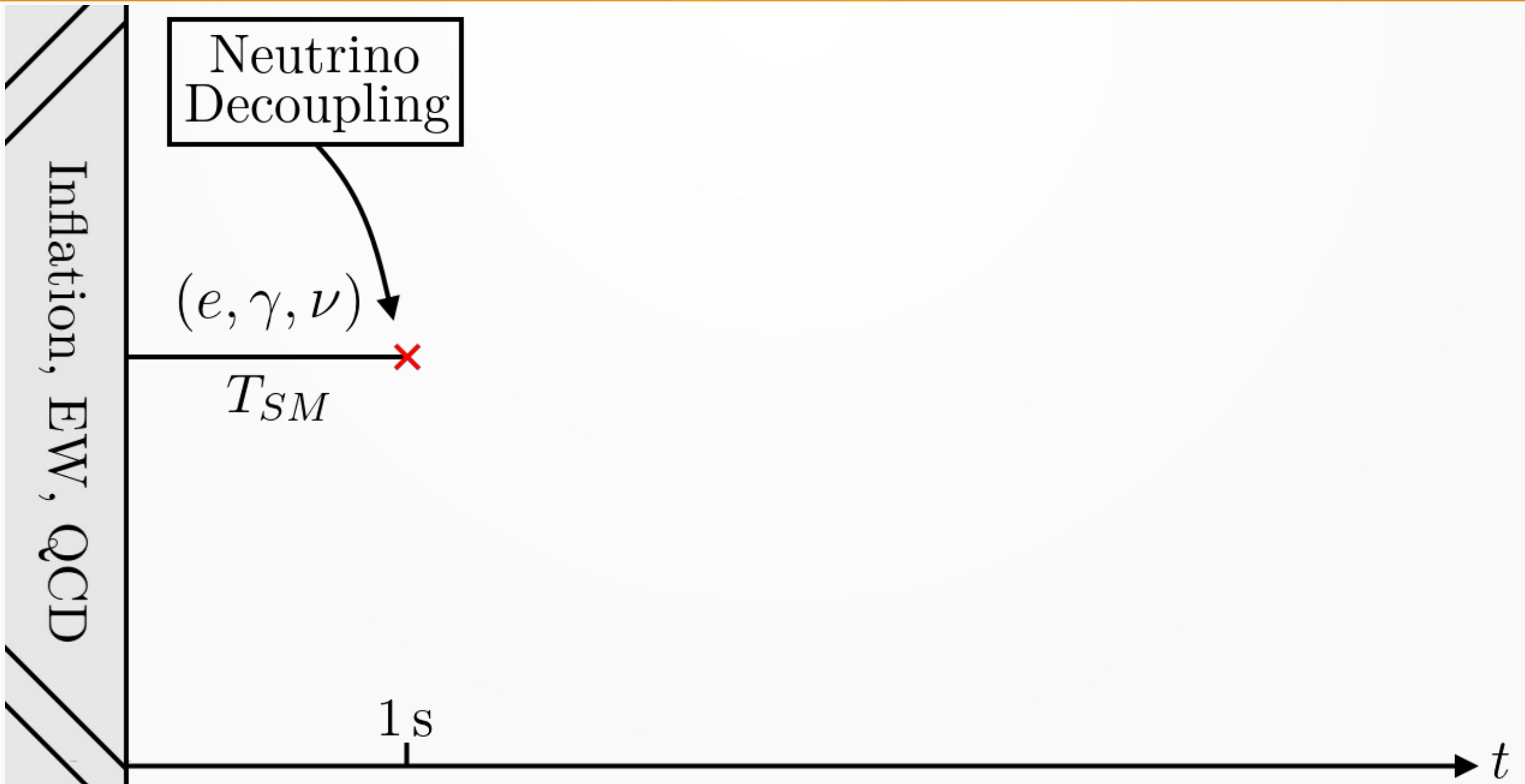
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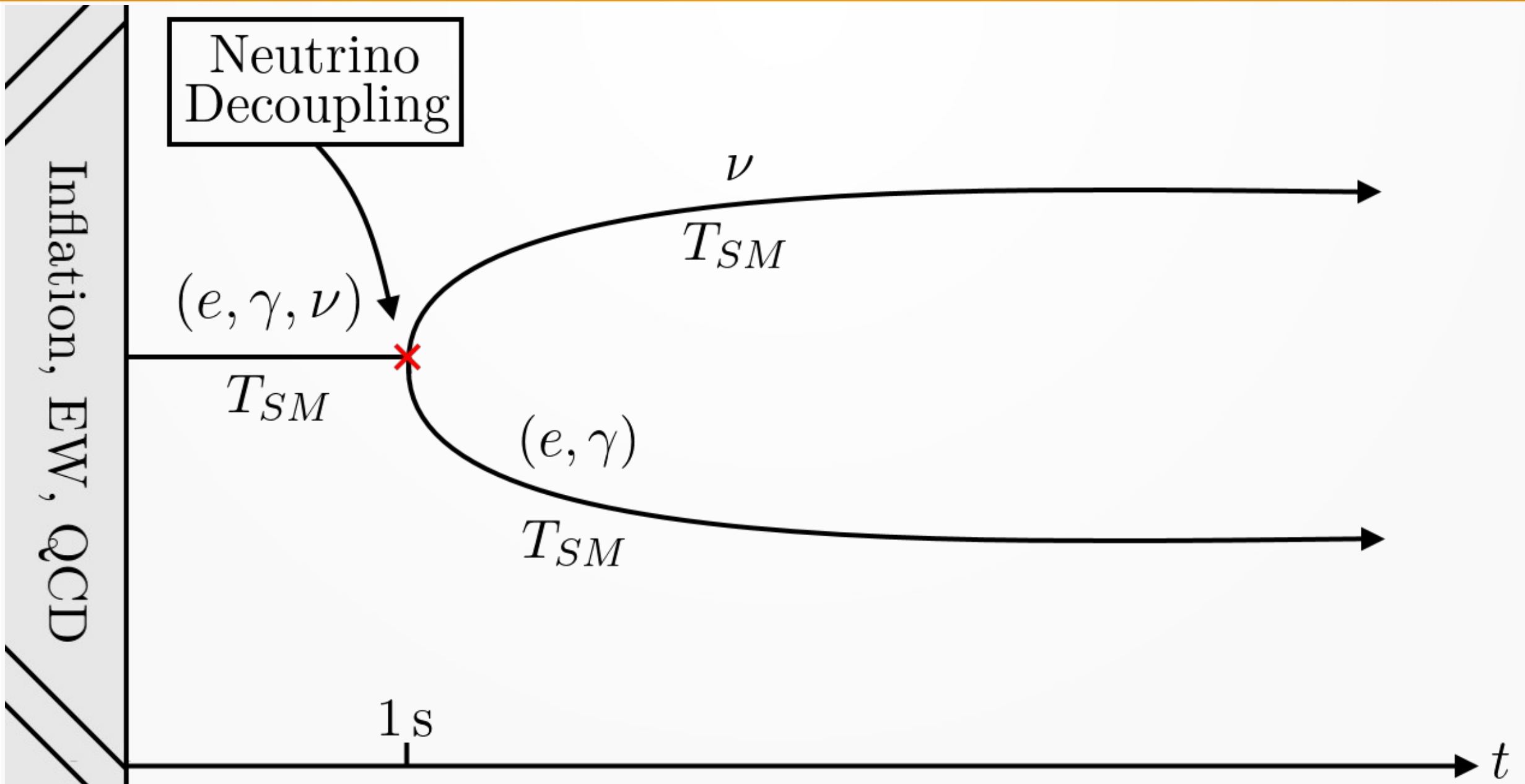
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$$t_{\text{dec}} = \frac{1}{2H} \sim 1 \text{ s}$$

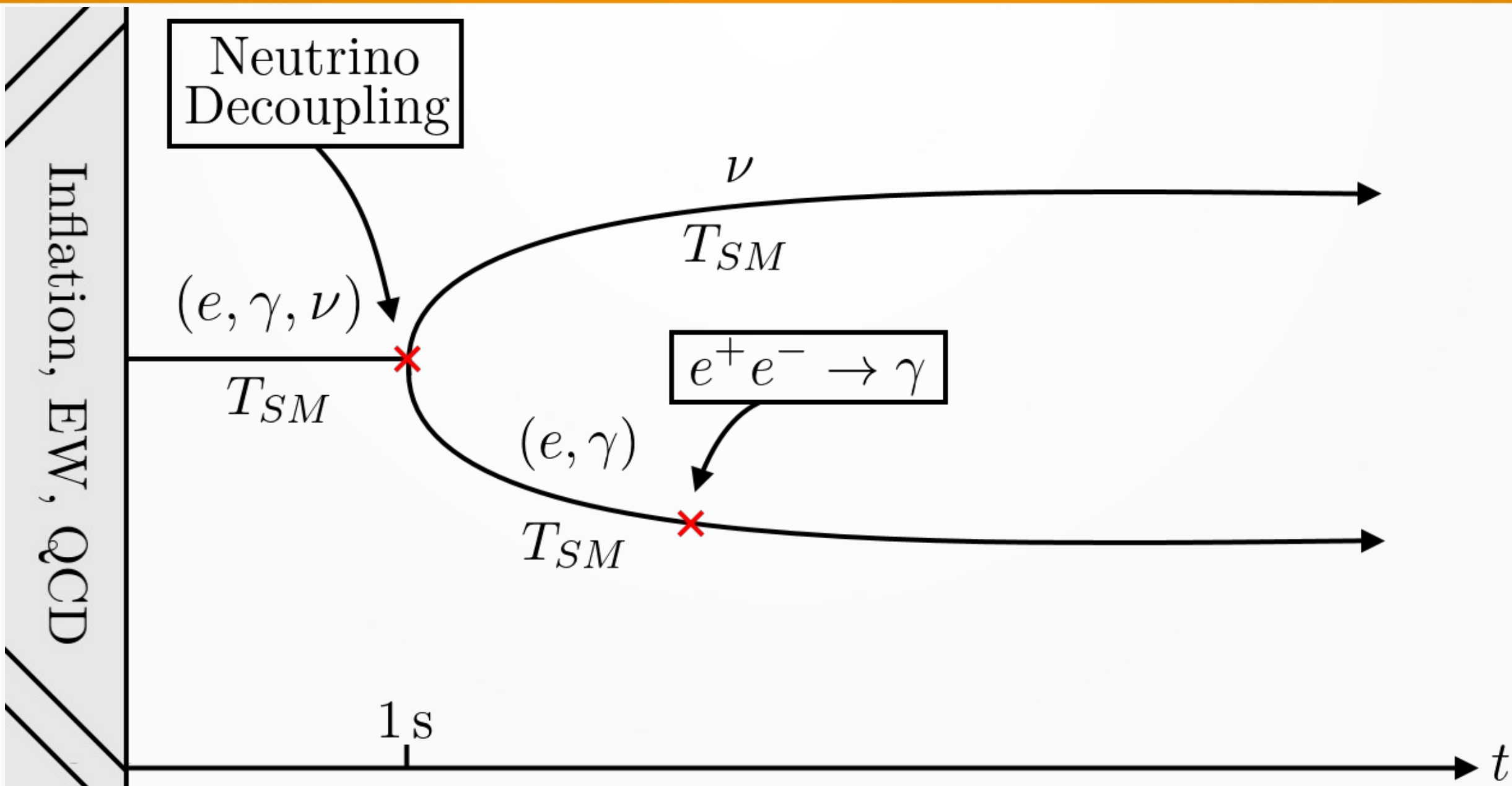
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- This process changes the photon temperature!

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$$g_s^*(T_{SM}) T_{SM}^3 = g_s^*(T_\gamma) T_\gamma^3$$

- In general:

$$g_s^*(T) = \sum_{\text{bosons}} g_i + \frac{7}{8} \sum_{\text{fermions}} g_i$$

What is the CνB?

- Before annihilation:

$$g_s^*(T_{SM}) = \underbrace{2}_{\gamma} + \frac{7}{8} \left(\underbrace{2 \times 2}_e \right)$$

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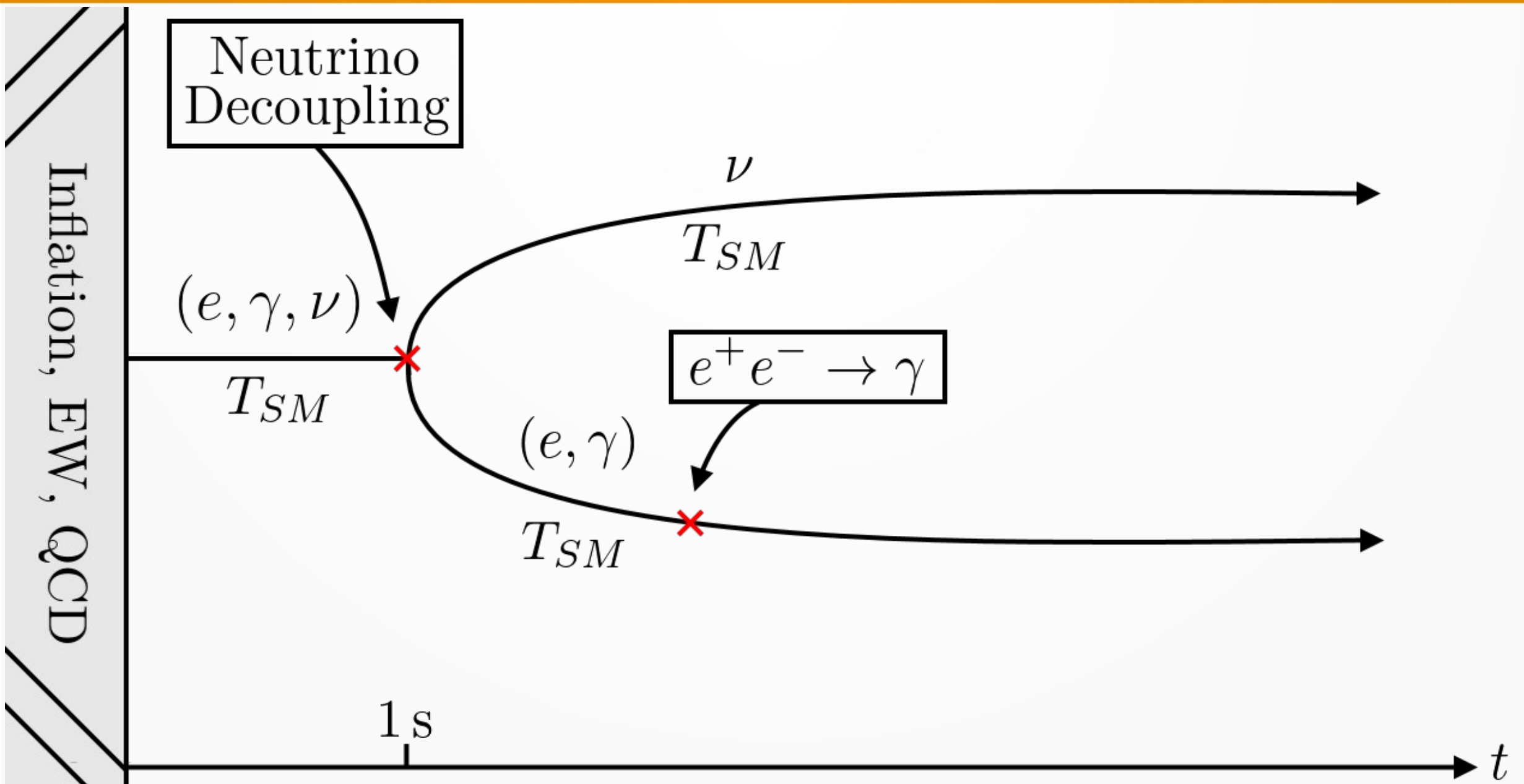
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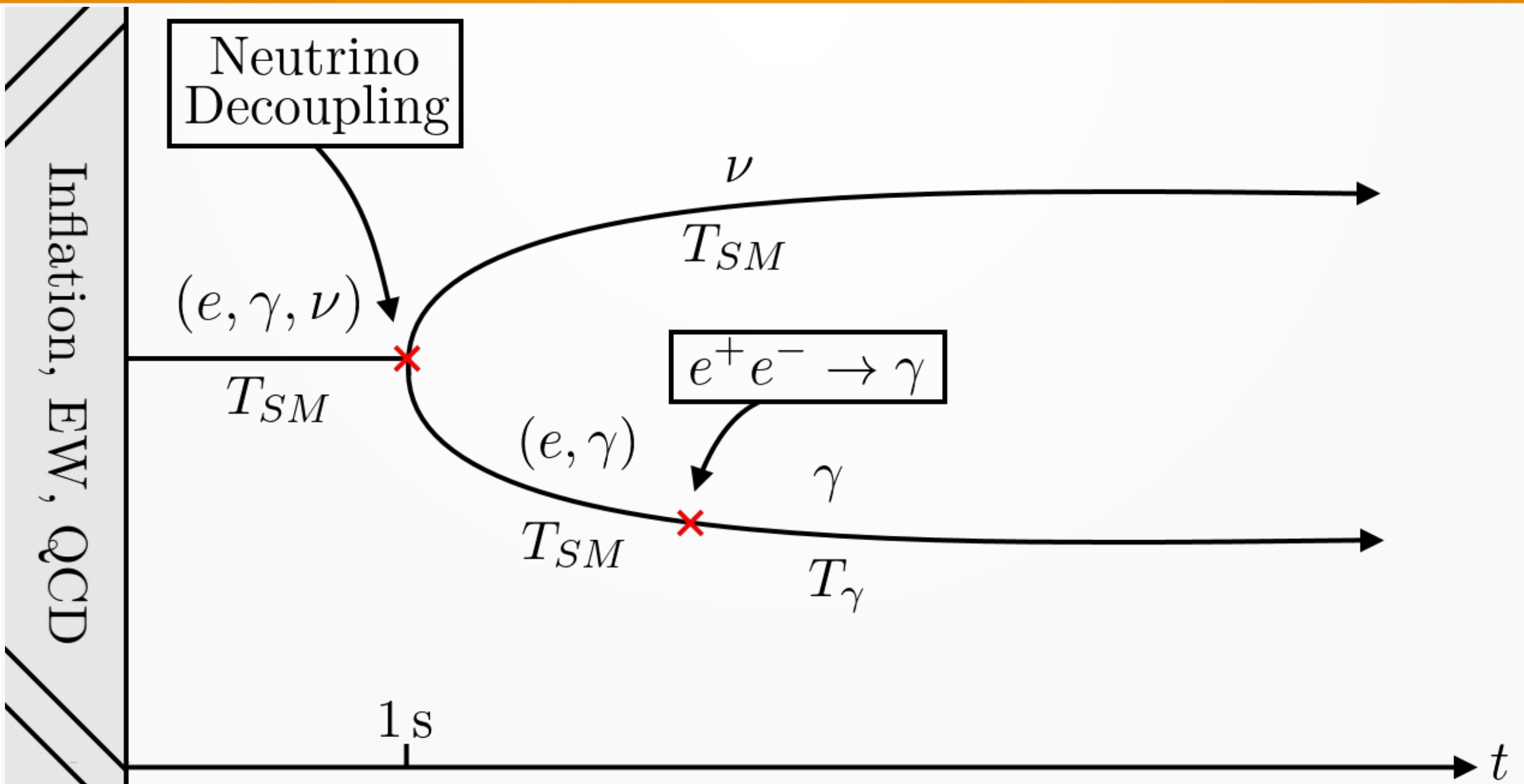
- Recalling that the neutrinos are still at T_{SM} :

$$T_\nu = \left(\frac{4}{11} \right)^{\frac{1}{3}} T_\gamma$$

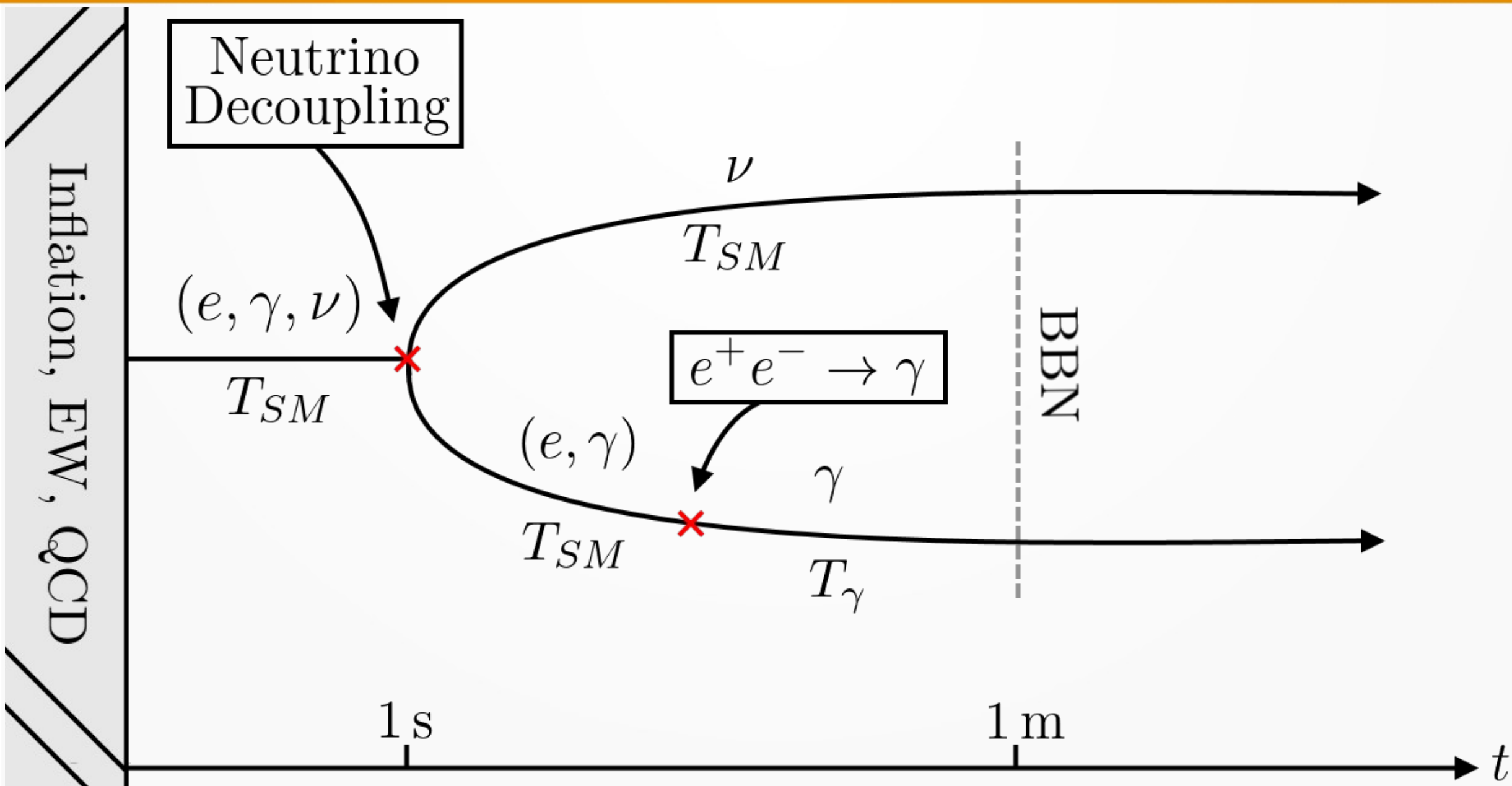
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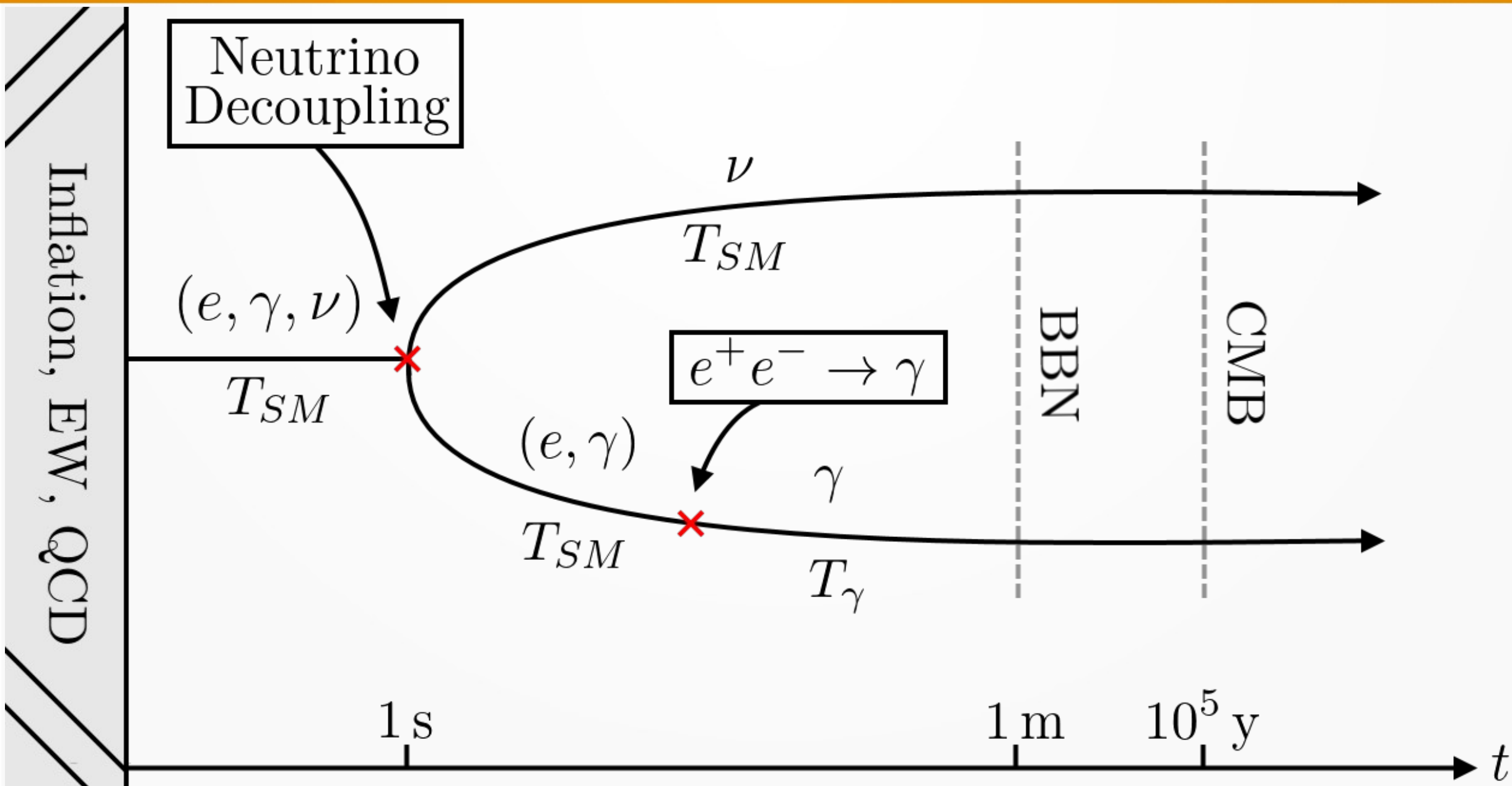
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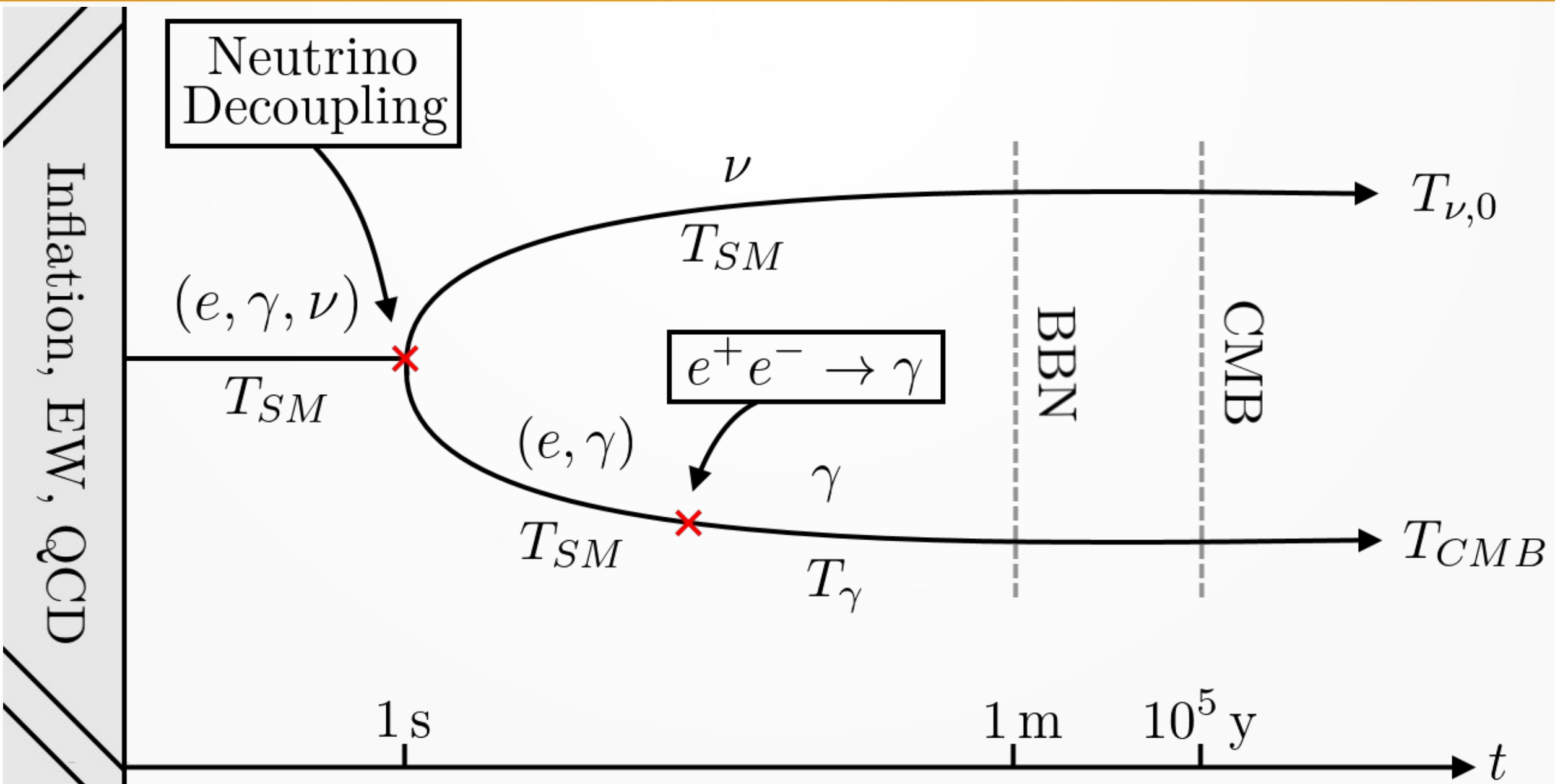
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The CνB today

- Redshifted to temperature:

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_{CMB}$$

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- At least two neutrinos states are non-relativistic!
- Exist today as freely propagating mass eigenstates

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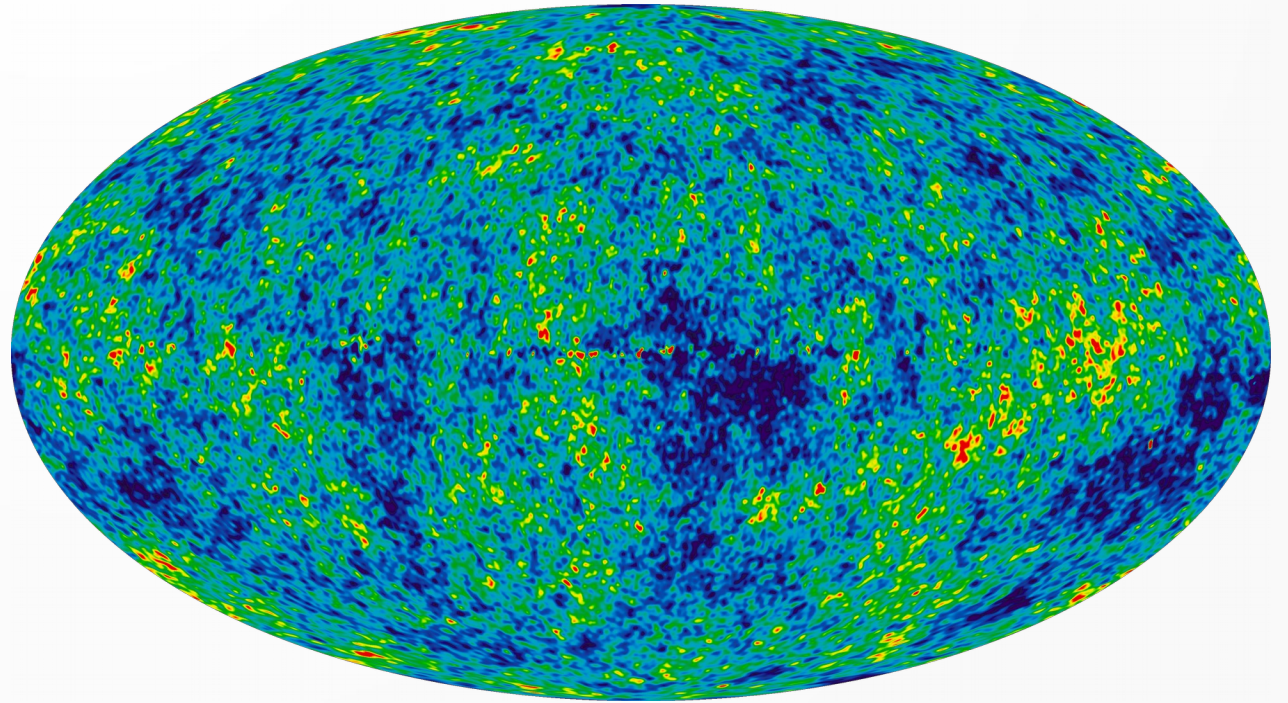
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- This may lead to CDM profile, overdensities, helicity mixing etc.

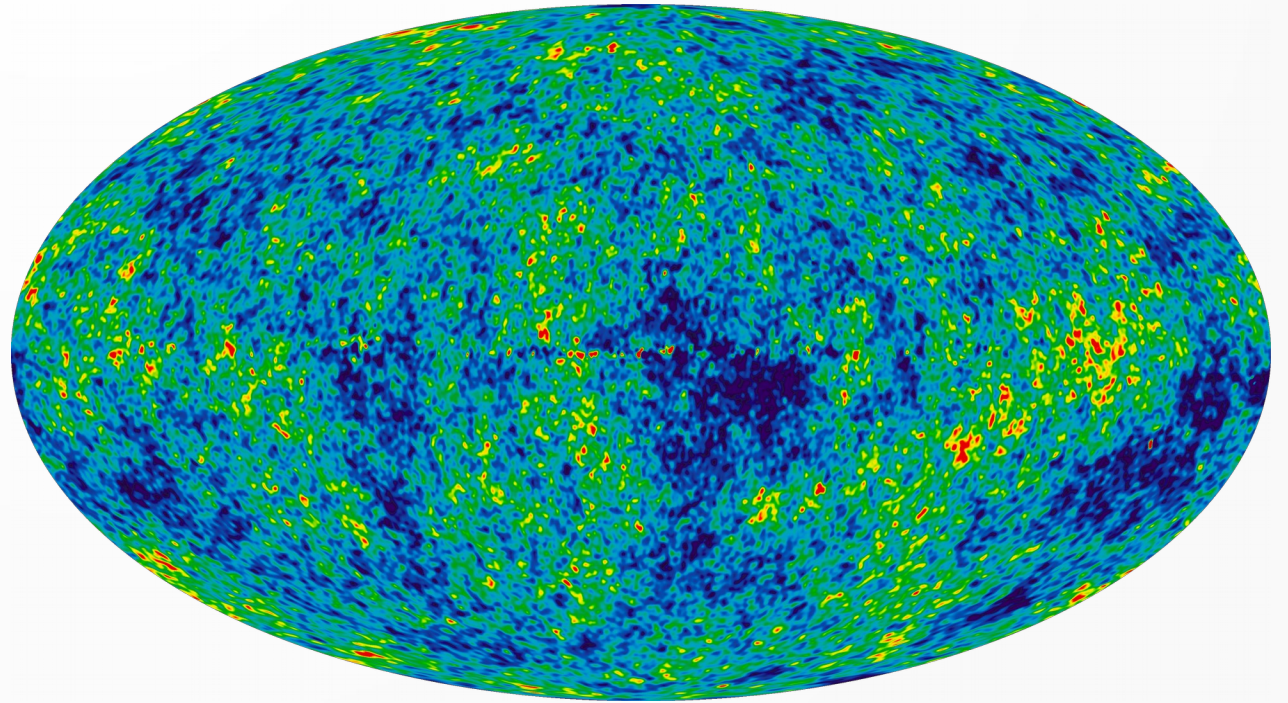
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- The CMB is the furthest we can currently look back through time



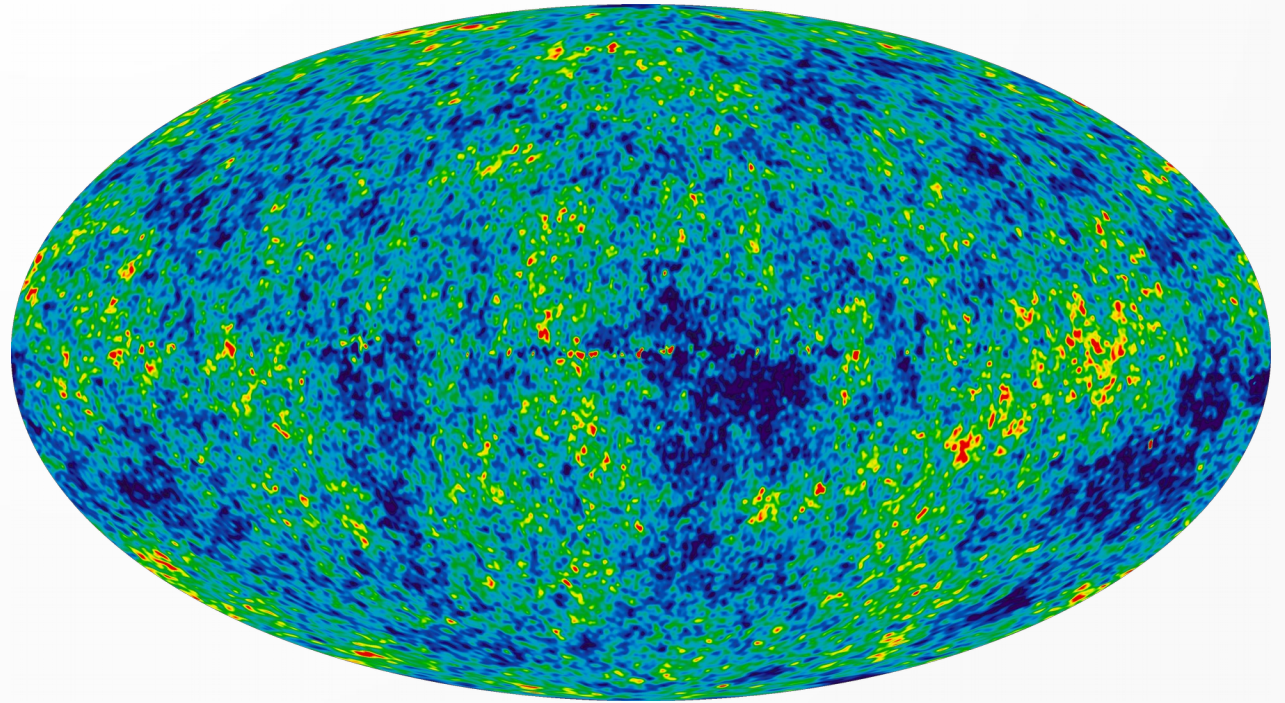
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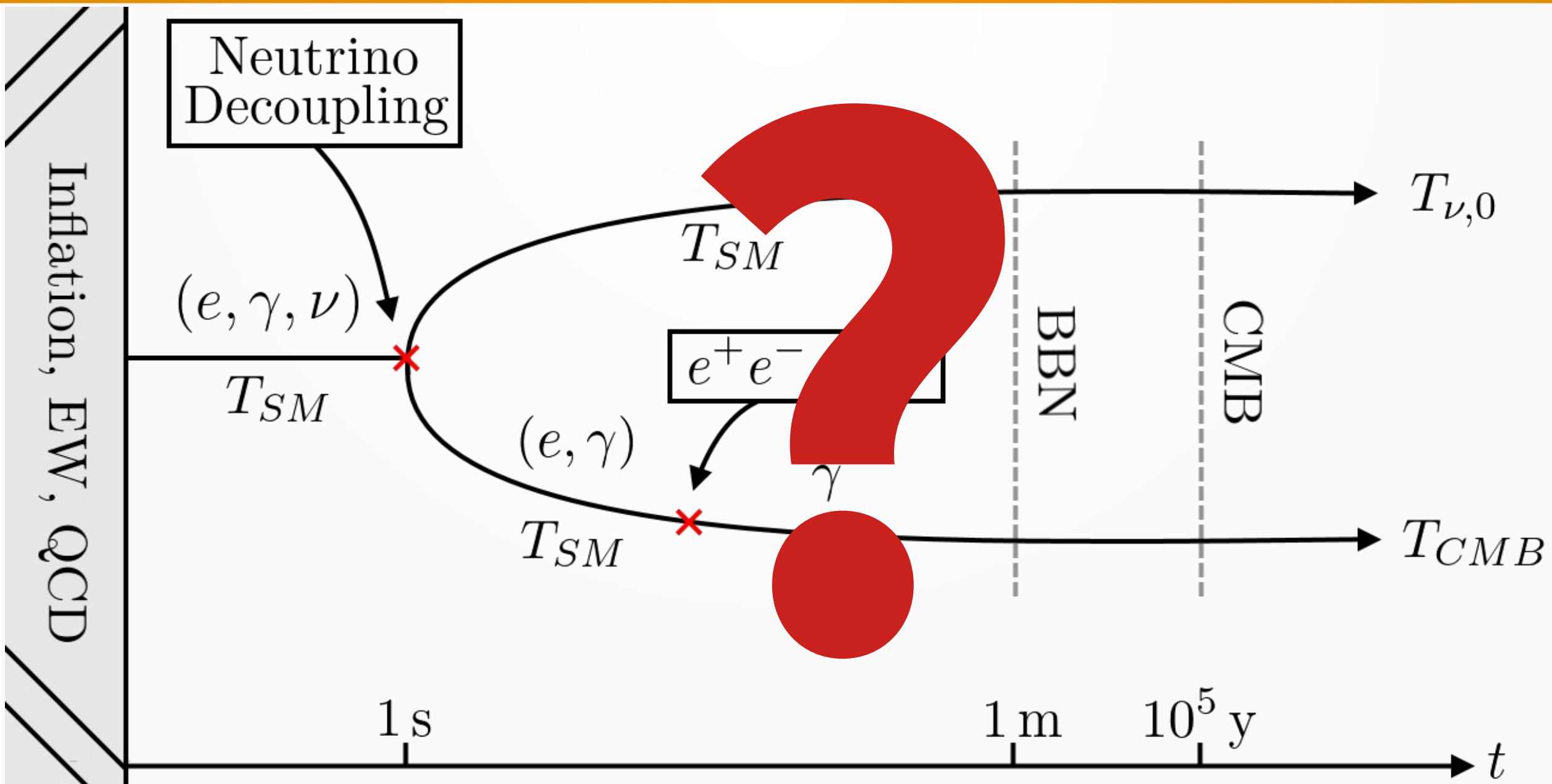


Why detect the CνB?

- The CMB is the furthest we can currently look back through time
- A rare source of non-relativistic neutrinos!
- Perhaps they're not there at all



Why detect the CνB?



So...why haven't we detected them yet?

- Neutrinos are notoriously hard to look for...

$$\sigma_\nu \sim G_F^2 E_\nu^2 \sim 5 \cdot 10^{-50} \left(\frac{E_\nu}{1 \text{ keV}} \right)^2 \text{ cm}^2$$

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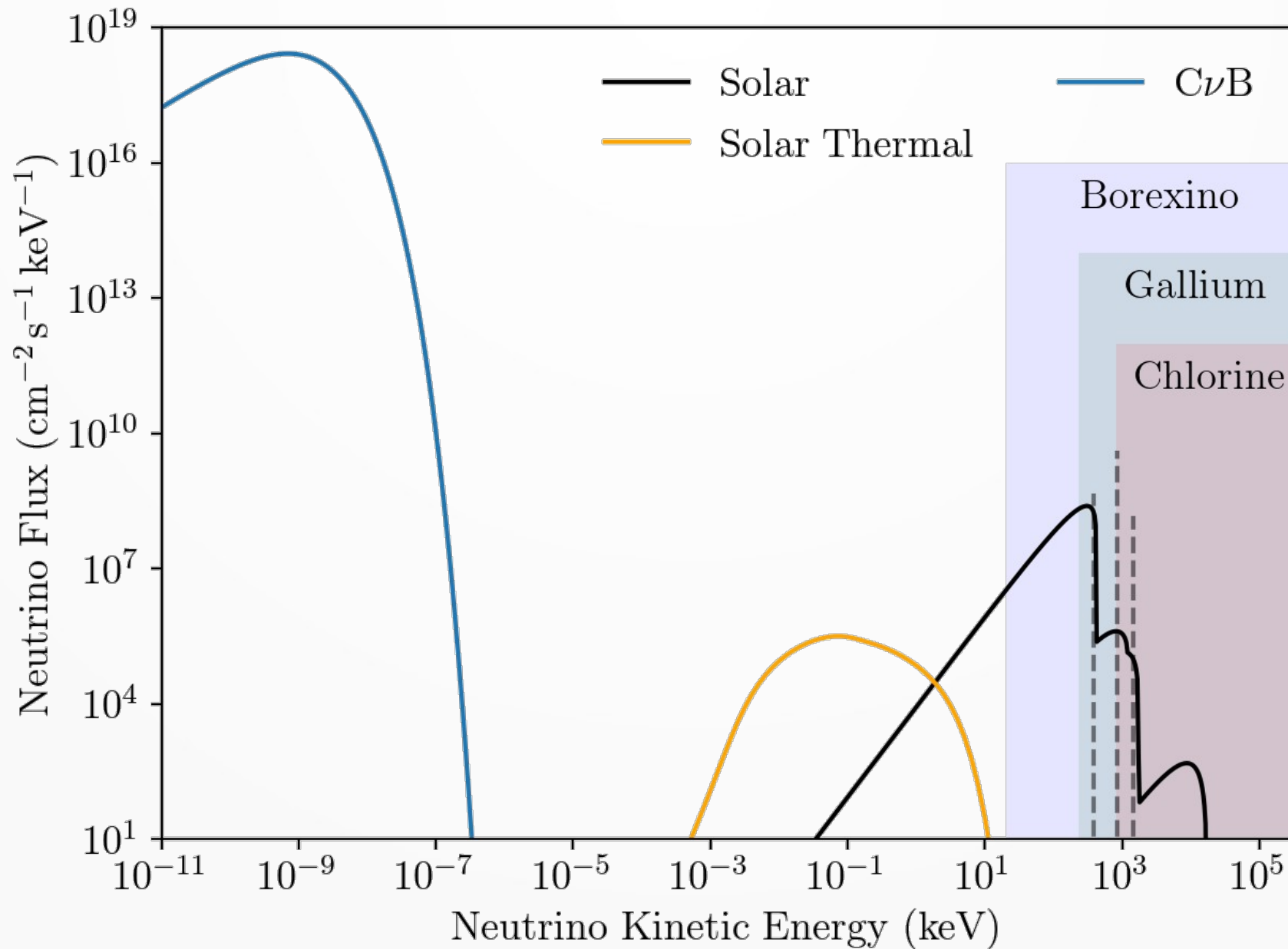
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- Existing neutrino detection experiments have thresholds:

$$\bar{\nu}_e + p + (1.8 \text{ MeV}) \rightarrow e^- + n$$

But...there is hope!



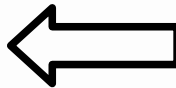
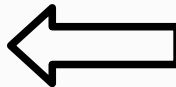
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- Threshold:
 - Remove it completely!
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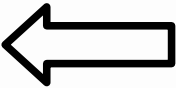
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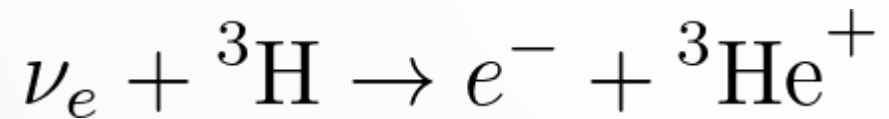
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The PTOLEMY experiment

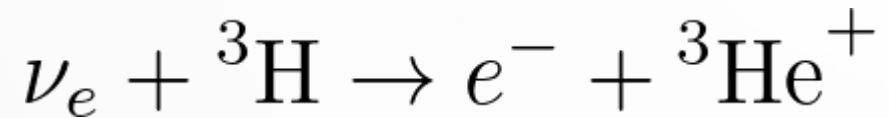
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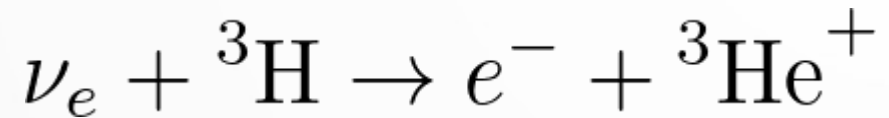
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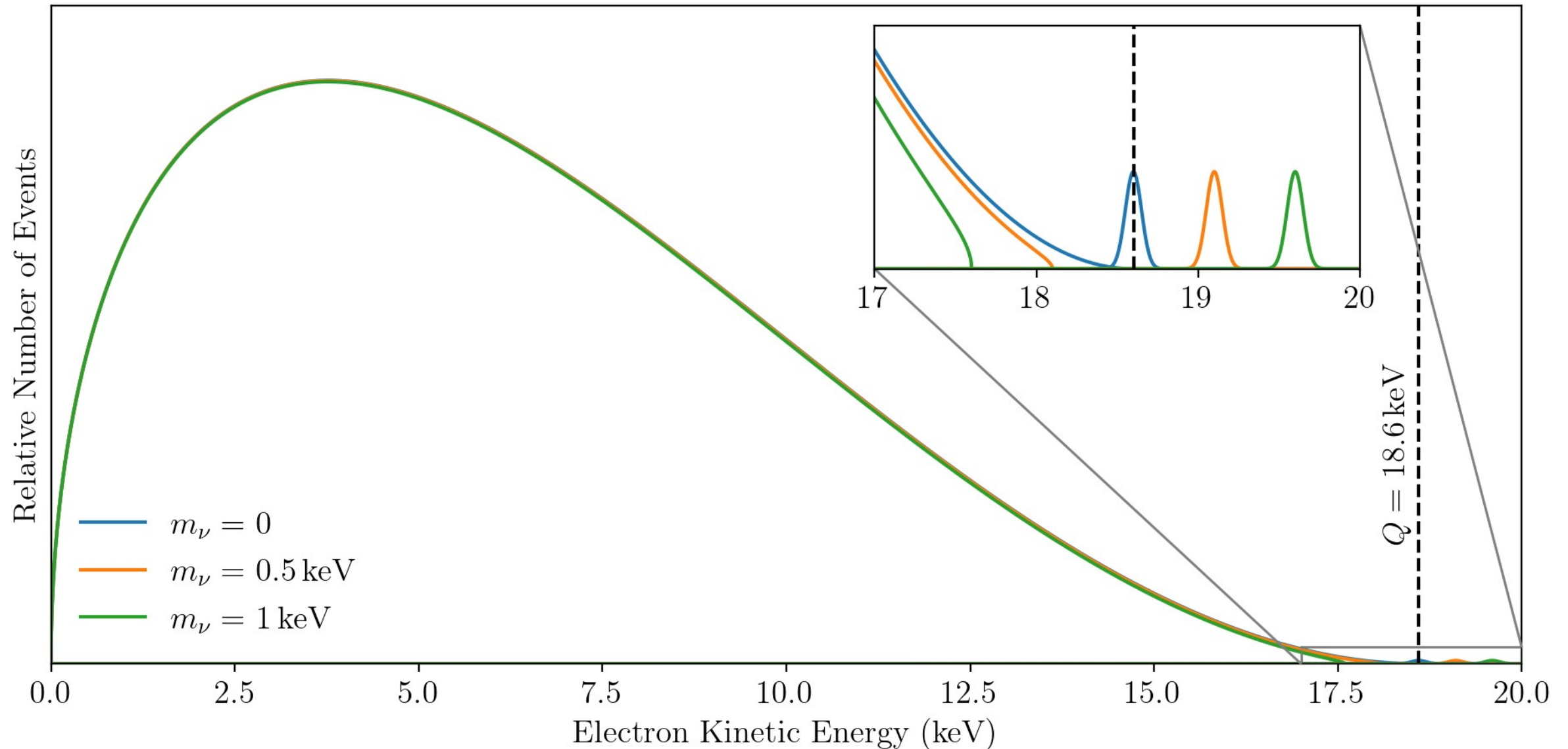


- This process has no threshold
- Tritium already well understood from neutrino mass experiments



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- This event rate is doubled for Majorana neutrinos

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- cf. KATRIN, uses $\sim 300\mu\text{g}$ of tritium [3]

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- We can increase our neutrino energy by using a beam

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- Relevant beam rest frame quantities:

$$\tilde{E}_\nu \simeq \frac{m_\nu}{M} E \quad \tilde{\phi} = \gamma \phi \quad \tilde{t} = \frac{t}{\gamma} \quad \tilde{R} = \gamma R$$

Setup

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$$\langle \sigma \tilde{\beta}_\nu \rangle \propto G_F^2 \tilde{E}_e \tilde{p}_e$$

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- Quadratic enhancement begins when:

$$\tilde{E}_\nu > 2m_e \implies E \gtrsim 3 \text{ PeV}$$

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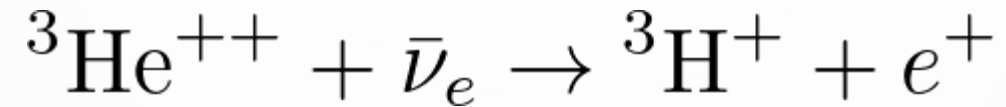
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- ...but, large energy presents an opportunity!

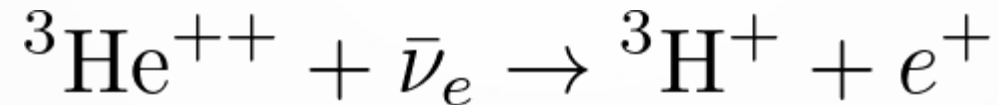
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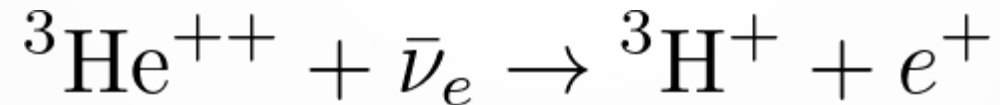
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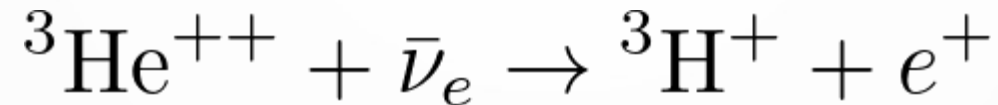
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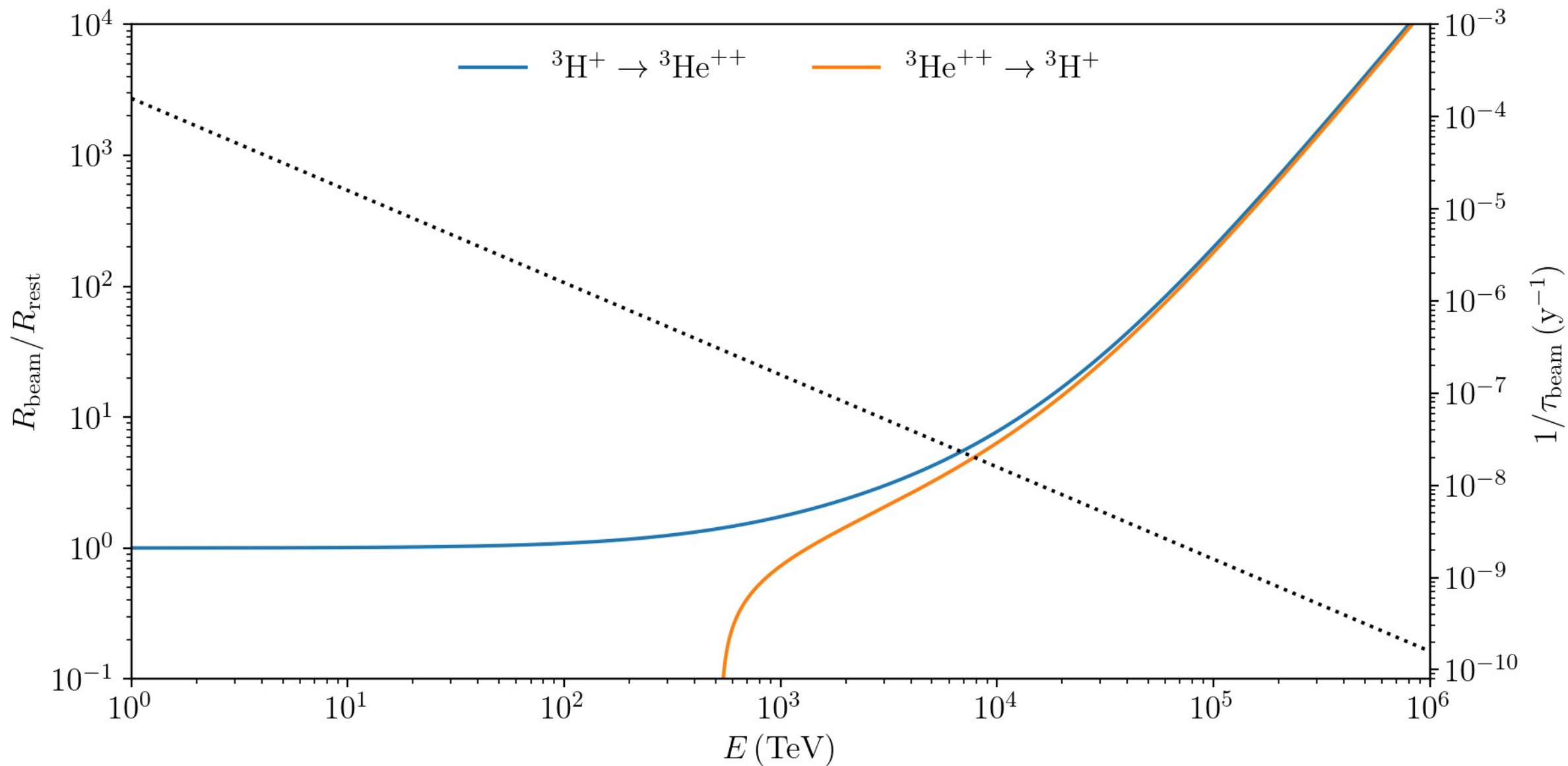
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- This process has a ‘unique’ signal
- Signal is now unstable

“Inverse PTOLEMY-on-a-beam”



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- But we have learnt some lessons!

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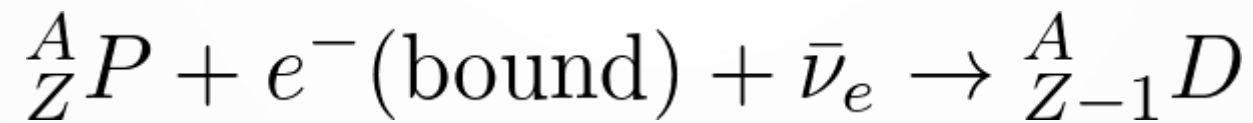
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- Tiny cross sections \rightarrow use a resonance!
- Tunable beam energy naturally invites resonances
- e.g. Z-resonance: $\nu + \bar{\nu}_{\text{C}\nu\text{B}} \rightarrow Z \rightarrow ?$
- Vastly larger cross section: $\sigma \propto \frac{1}{M_Z^2} \propto G_F$

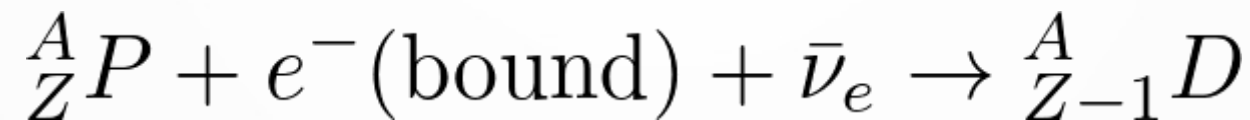
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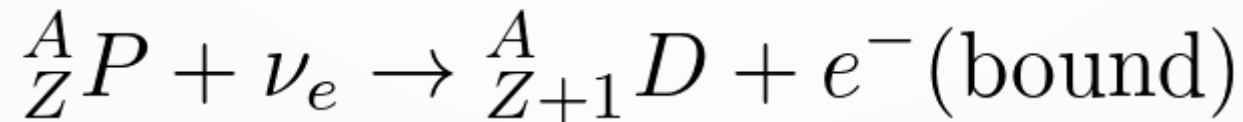


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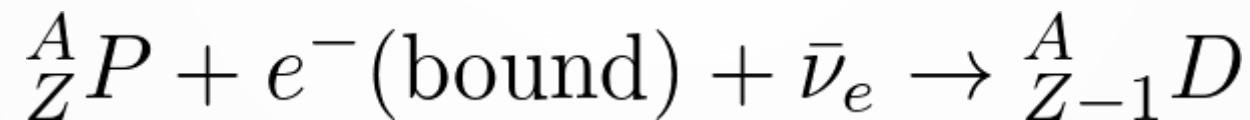


- Resonant bound beta decay (RB β):

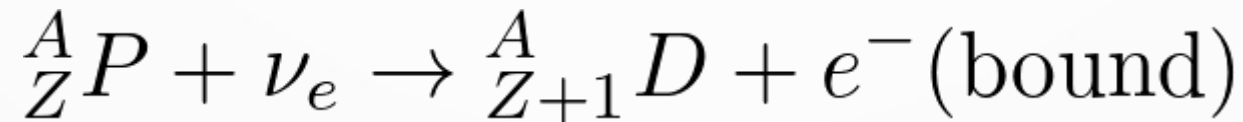


Resonant neutrino capture

- Resonant electron capture (REC):



- Resonant bound beta decay (RB β):



- Parent ionised down to one s-shell electron (REC) or completely ionised (RB β)

Resonant neutrino capture

- Cross section for resonant neutrino capture [4]:

$$\sigma \propto \frac{1}{\tilde{E}_\nu^2} \left[\frac{\Gamma^2/4}{(\tilde{E}_\nu - Q)^2 - \Gamma^2/4} \right] \text{Br}(D \rightarrow P)$$

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- Peak cross section is independent of G_F !

[4] R. G. C. Oldeman, M. Meloni and B. Saïtta, Eur. Phys. J. C **65**, 81-87 (2010)

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$$\sigma_{\text{peak}} = 2.5 \cdot 10^{-15} \left(\frac{1 \text{ keV}}{Q} \right)^2 \text{Br}(D \rightarrow P) \text{ cm}^2$$

- Peak cross section is independent of G_F !

Resonant neutrino capture

- Capture rate per target given by:

$$\frac{R}{N_T} = \int_Q^\infty d\tilde{E}_\nu \sigma(\tilde{E}_\nu) \frac{d\phi}{d\tilde{E}_\nu}$$

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- For narrow resonances:

$$\left[\frac{\Gamma^2/4}{(\tilde{E}_\nu - Q)^2 - \Gamma^2/4} \right] \rightarrow \pi\Gamma \delta(\tilde{E}_\nu - Q)$$

Resonant neutrino capture

- Capture rate per target given by:

$$\frac{R}{N_T} = \frac{\pi}{2} \sigma_{\text{peak}} \Gamma \left. \frac{d\phi}{d\tilde{E}_\nu} \right|_{\tilde{E}_\nu=Q}$$

Resonant neutrino capture

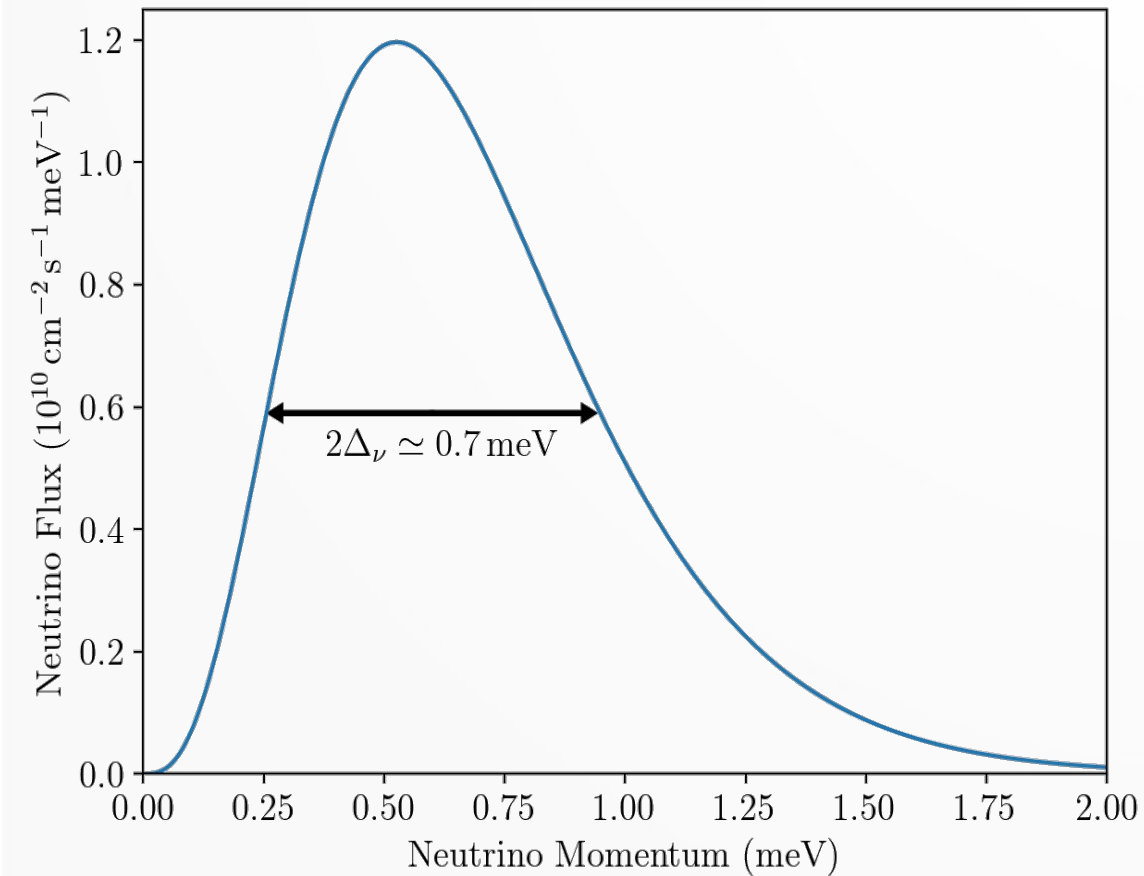
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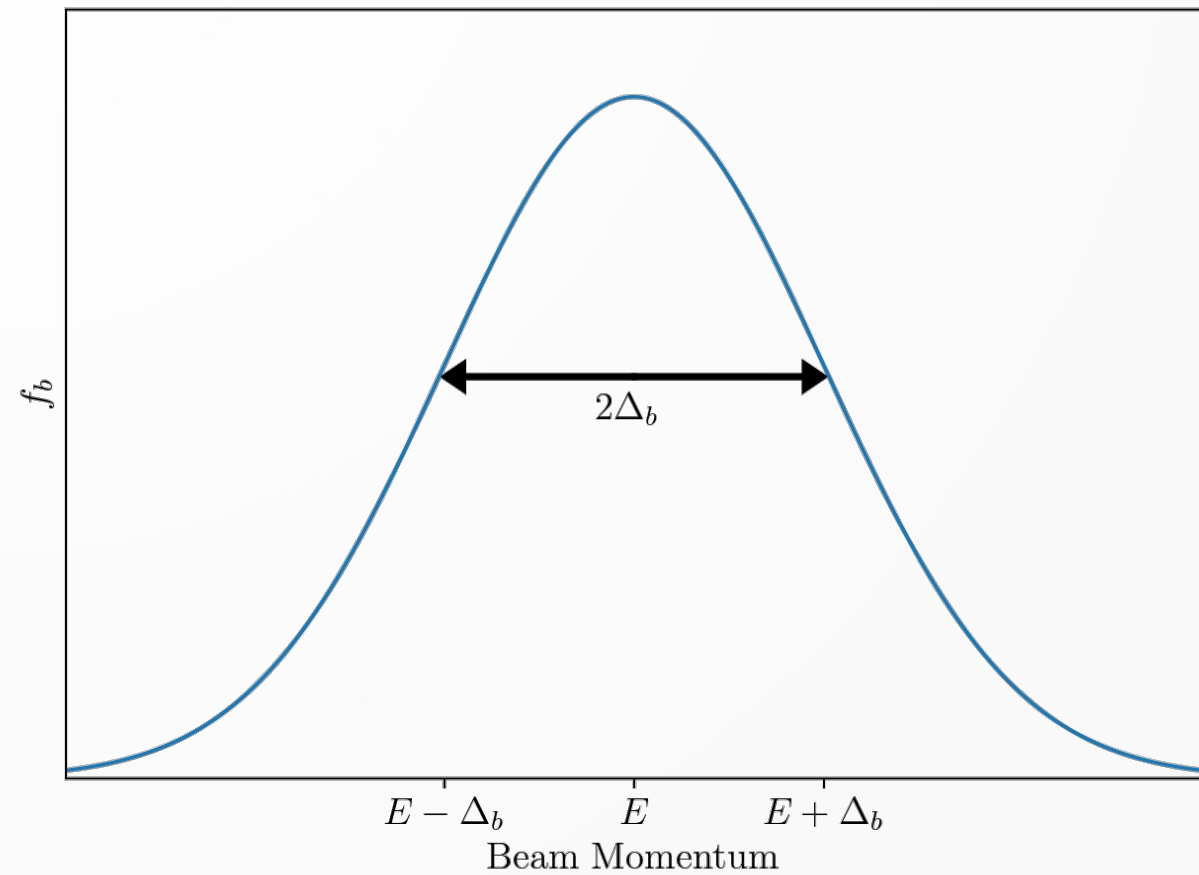
- Assuming Gaussian distribution:

$$\frac{R}{N_T} \propto \frac{\Gamma}{Q^2} \frac{\phi}{\tilde{\Delta}_E} \text{Br}(D \rightarrow P)$$

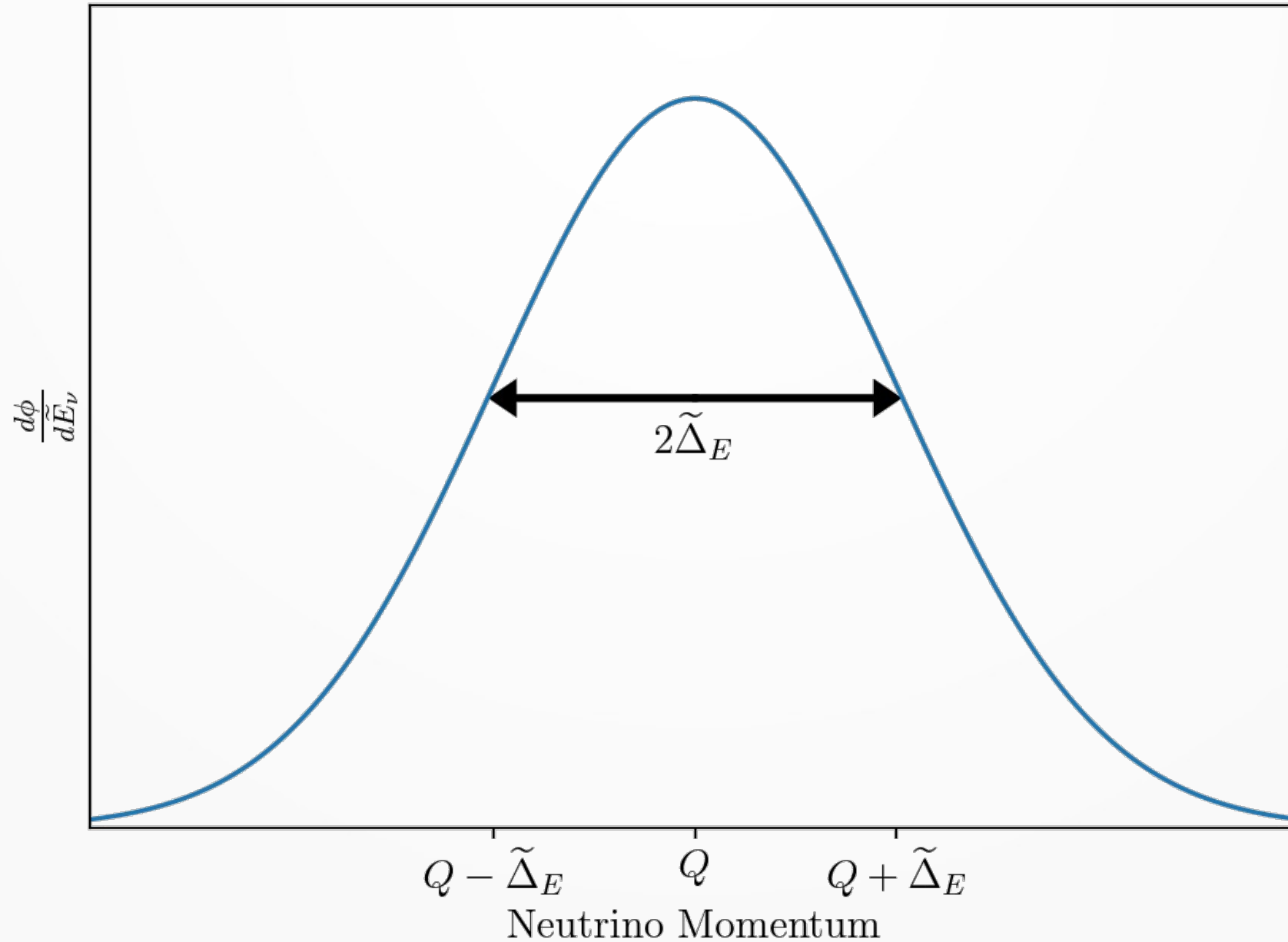
Accounting for the finite width of the $C\nu B$



+



Accounting for the finite width of the $C\nu B$



Accounting for the finite width of the $C\nu B$

- Treating widths of distributions as uncertainty:

$$\tilde{\Delta}_E = \sqrt{\left(\Delta_\nu \frac{\partial \tilde{E}_\nu}{\partial p_\nu}\right)^2 + \left(\Delta_b \frac{\partial \tilde{E}_\nu}{\partial p}\right)^2}$$

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- For non-relativistic neutrinos, relativistic beam:

$$\tilde{\Delta}_E = Q \sqrt{\delta_\nu^2 + \delta_b^2}$$

Accounting for the finite width of the $C\nu B$

- Total capture rate per target:

$$\frac{R}{N_T} \propto \frac{\Gamma}{Q^3} \frac{\phi}{\sqrt{\delta_\nu^2 + \delta_b^2}} \text{Br}(D \rightarrow P)$$

Accounting for the finite width of the $C\nu B$

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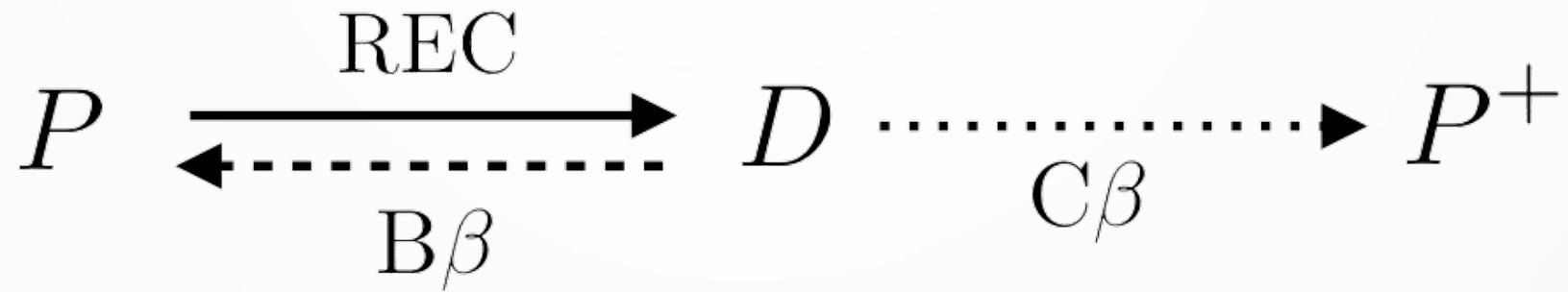
$$\frac{R}{N_T} \propto \frac{\Gamma}{Q^3} \frac{\phi}{\sqrt{\delta_\nu^2 + \delta_b^2}} \text{Br}(D \rightarrow P)$$

- More convenient to introduce *quality factor*:

$$R_\tau = \frac{\gamma}{\Gamma} \frac{R}{N_T} = 1.7 \cdot 10^{-17} \frac{\text{Br}(D \rightarrow P)}{\sqrt{\delta_\nu^2 + \delta_b^2}} \left[\frac{0.1 \text{ eV}}{m_\nu} \right] \left[\frac{1 \text{ keV}}{Q} \right]^2$$

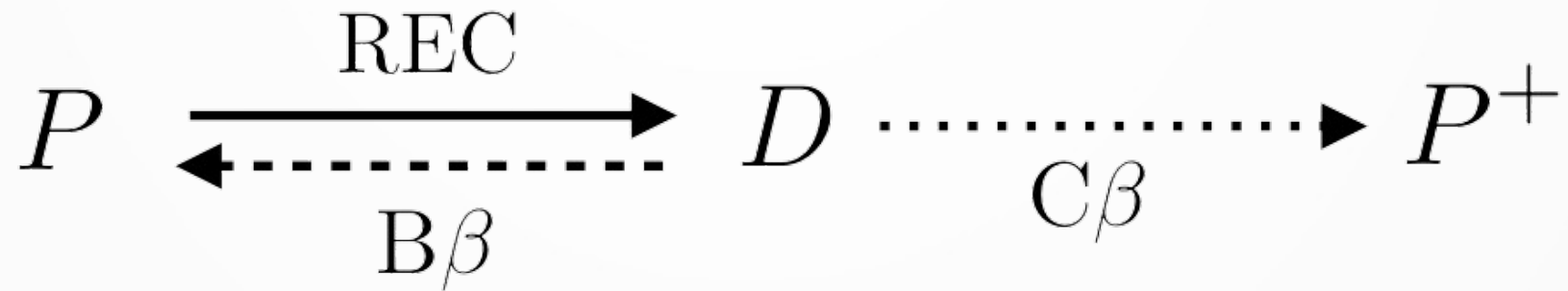
2-state systems

- Resonant electron capture:

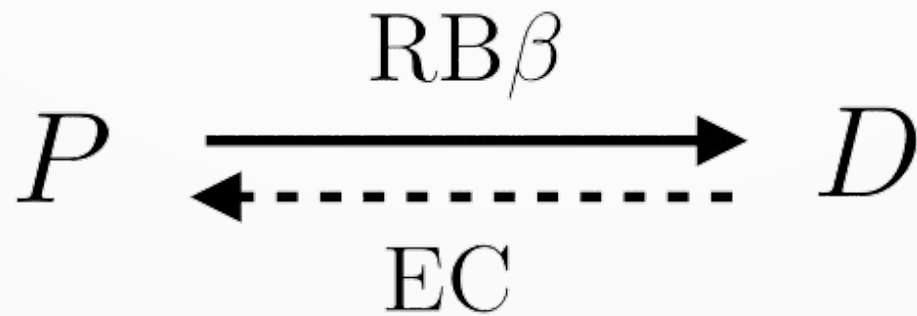


2-state systems

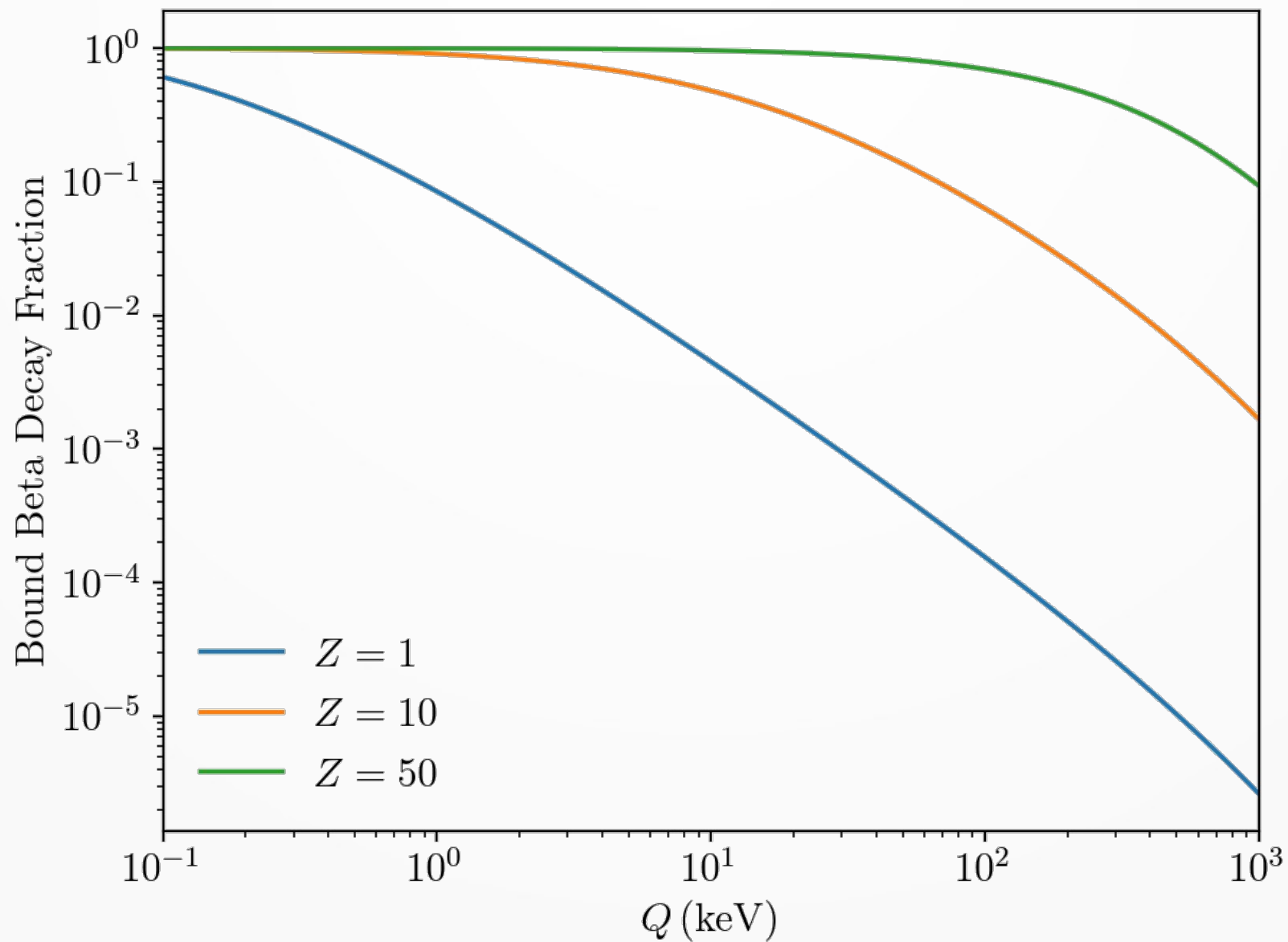
- Resonant electron capture:



- Resonant bound beta decay:



2-state systems



2-state systems

- Number of states on the beam:

$$\frac{dN_P}{d\tilde{t}} = -\gamma \frac{R}{N_T} N_P(\tilde{t}) + \frac{\text{Br}(D \rightarrow P)}{\tau_D} N_D(\tilde{t})$$

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- Working in terms of dimensionless variables:

$$x = \frac{t}{\gamma \tau_D} = \frac{m_\nu}{Q} \frac{t}{\tau_D} \quad R_\tau = \frac{\gamma}{\Gamma} \frac{R}{N_T} = \gamma \tau_D \frac{R}{N_T}$$

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- Number of daughter states reaches an equilibrium value!

2-state systems

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2-state systems

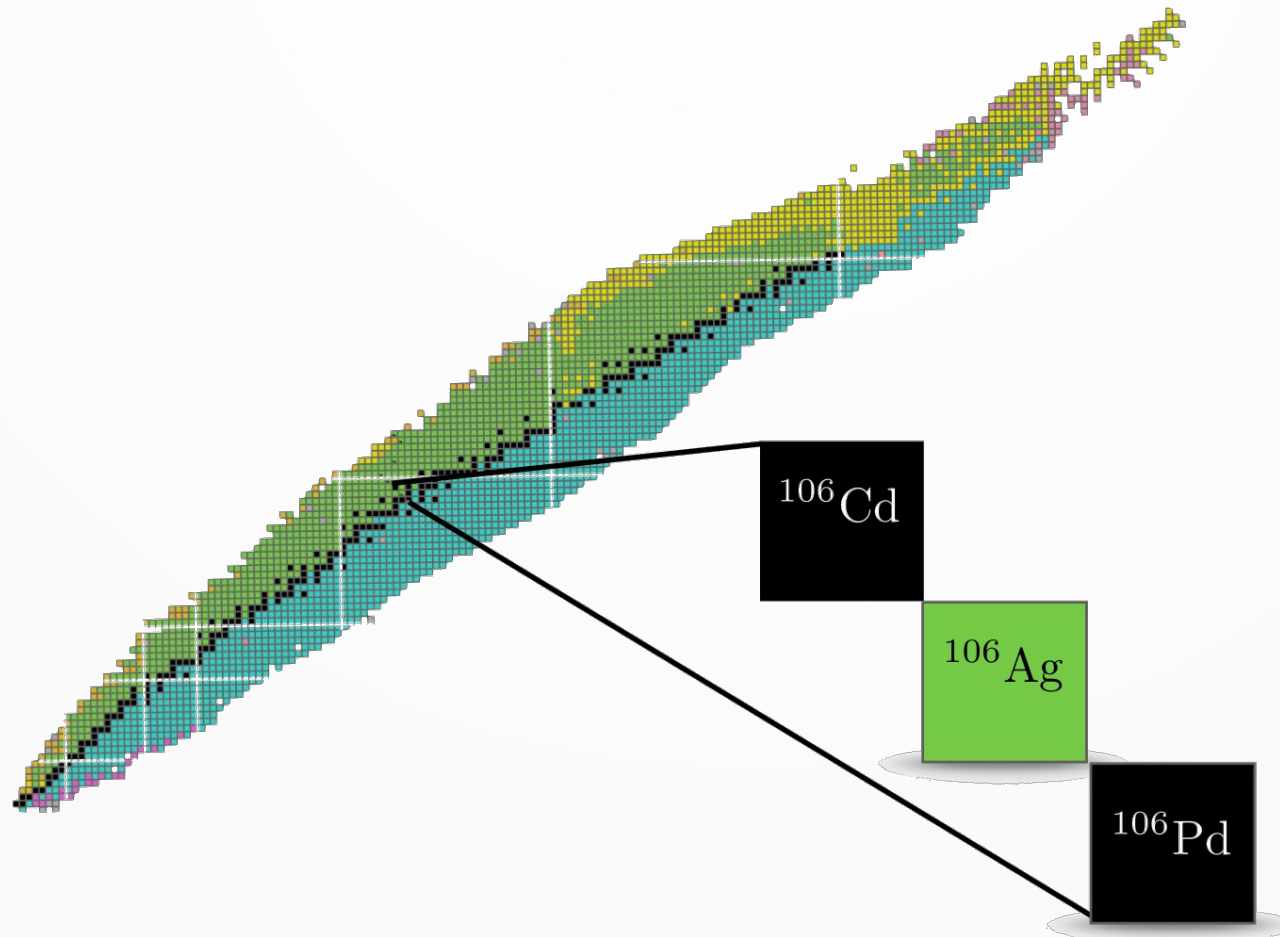
- Number of states on the beam:

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- 2-state systems are limited to converting small fraction of the beam
- Can we do better?

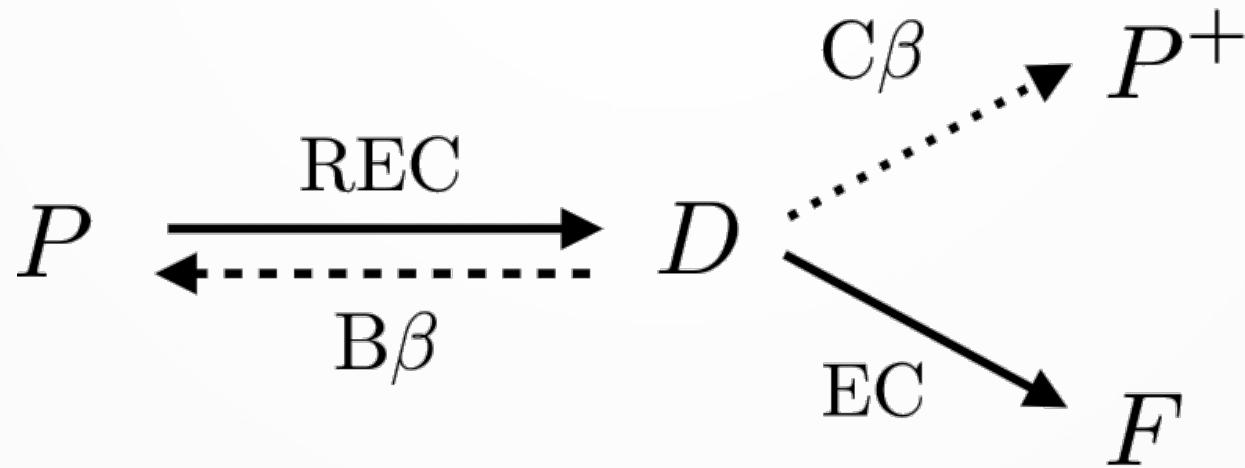
3-state systems

- Introduce a third, stable signal state:



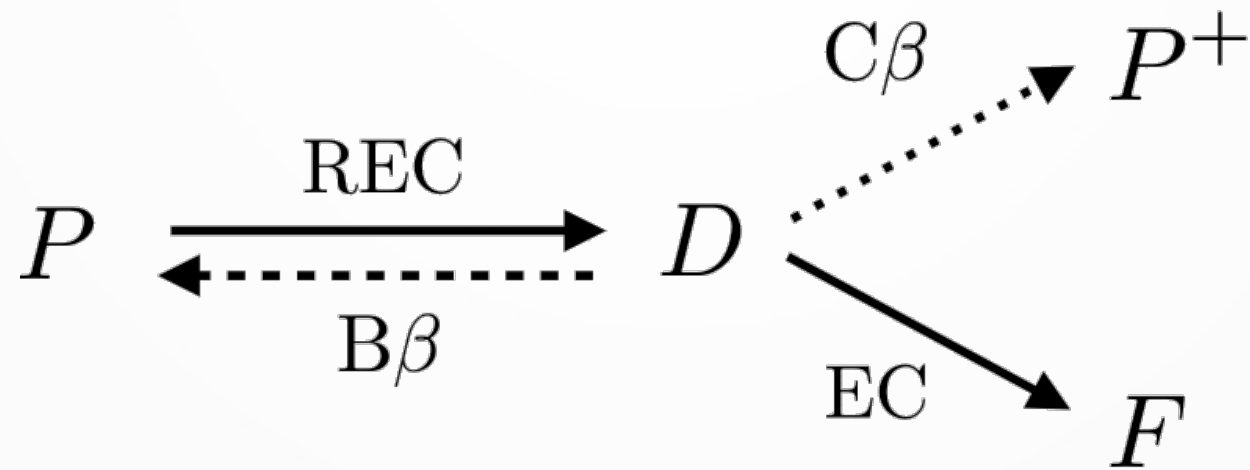
3-state systems

- 3-state resonant electron capture:

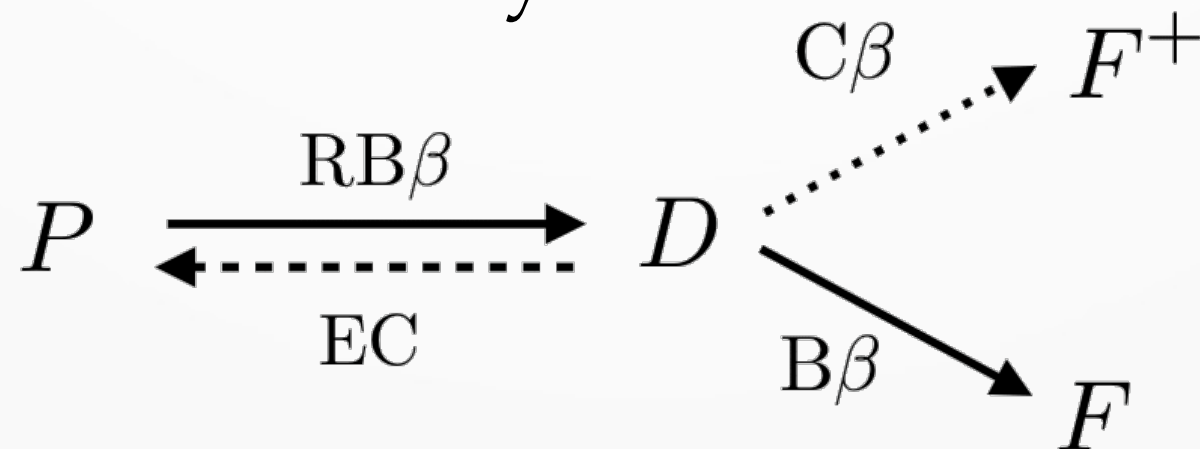


3-state systems

- 3-state resonant electron capture:



- 3-state bound beta decay:



3-state systems

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$$\frac{dN_F}{dx} = \text{Br}(D \rightarrow F) N_D(x)$$

- Number of final (F) states increases monotonically!

3-state systems

- Now able to convert a significant fraction of the beam:

$$\lim_{x \rightarrow \infty} N_F(x) = \frac{N_0 \text{Br}(D \rightarrow F)}{1 - \text{Br}(D \rightarrow P)} \gg N_0 R_\tau$$

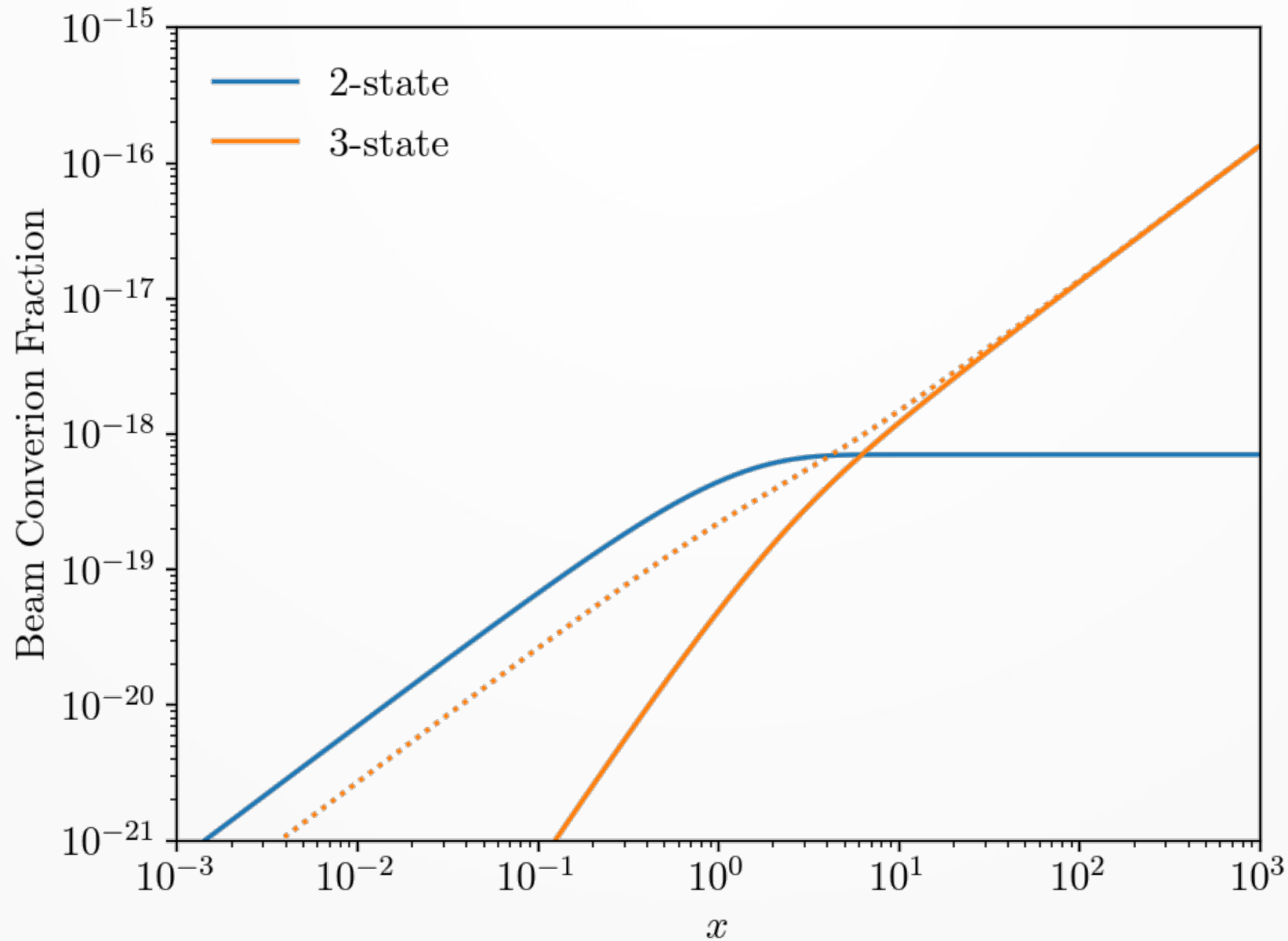
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- We now have a *stable, clean signal* with a large cross section!

3-state systems



Real world examples

- 2-state system:



Real world examples

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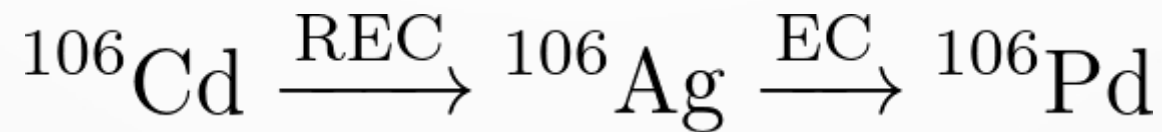
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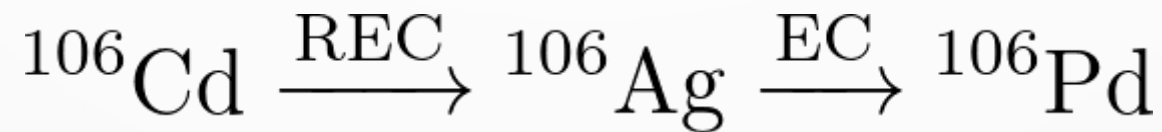


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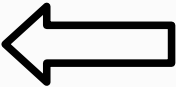
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- 3-state system:



$$\frac{N_D + N_F}{N_0} \simeq 10^{-23} \quad \frac{E}{A} \simeq 2 \text{ PeV}$$

Contents

- What is the $C\nu B$?
- PTOLEMY (-on-a-beam)
- Resonant neutrino capture
- Experimental challenges 

Experimental challenges

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- Large number of targets required → reduce threshold, purpose built experiment

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- Seek processes with small threshold
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 - Possibility to convert huge fraction of the beam

Summary

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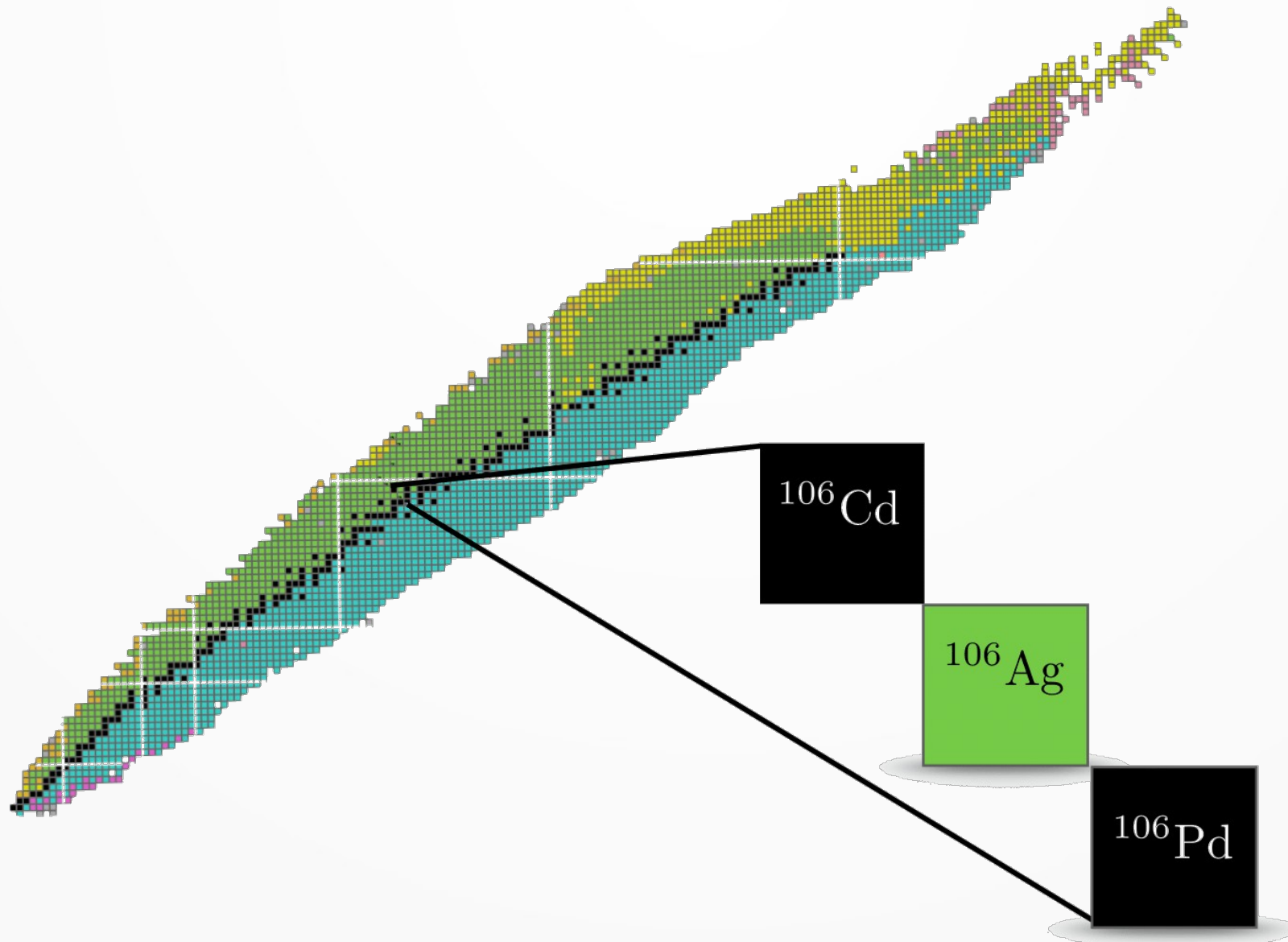
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- Great deal of parameter space left to be explored

Summary



Thank you!
Questions?

Neutrino mass uncertainty

- Assuming wrong neutrino mass \rightarrow incorrectly centred beam energy

$$\delta_m = \frac{m_{\nu,\text{true}} - m_{\nu,\text{pred}}}{m_{\nu,\text{true}}}$$

$$R_{\tau,\text{eff}} = R_{\tau} (1 - \delta_m)^2 e^{-\frac{\delta_m^2}{2(\delta_{\nu}^2 + \delta_b^2)}}$$

- Only capturing neutrinos from tail end of spectrum
- Partially rectifiable by appropriate choice of δ_b

Neutrino mass uncertainty

