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Citation: Proc. Mtgs. Acoust. **38**, 045015 (2019); doi: 10.1121/2.0001095
View online: https://doi.org/10.1121/2.0001095
View Table of Contents: https://asa.scitation.org/toc/pma/38/1
Published by the Acoustical Society of America

Volume 38 http://acousticalsociety.org/

2019 International Congress on Ultrasonics

Bruges, Belgium

3-6 September 2019

Physical Acoustics: PUA (2/2) Presentation 5

The nonlinear dynamics of flexural ultrasonic transducers

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Dynamic nonlinearity can manifest as changes in characteristic properties of a vibrating system in response to variations in excitation. This study investigates the nonlinearity in the vibration response of the flexural ultrasonic transducer. This device is typically employed for industrial measurement, but little is known about the influence of changes in excitation on its dynamics. In general, the resonance frequency of an ultrasonic device is known to shift as excitation amplitude is increased, displaying either hardening nonlinear behaviour, where resonance frequency increases, or softening associated with resonance frequency decrease. In typical operation, the vibration response of the flexural ultrasonic transducer has been found to be weakly nonlinear. Different physical mechanisms can cause nonlinearity, including structural configuration, the physical responses of components such as the transducer membrane, and thermomechanical properties inherent in piezoelectric materials. The nonlinear behaviour of flexural ultrasonic transducers is shown in the context of typical operation in practical application, through laser Doppler vibrometry and supported by fundamental mathematics. The results demonstrate the existence of nonlinear behaviour for different types of flexural ultrasonic transducer for modest changes in excitation amplitude, and show that the influence of dynamic nonlinearity should be considered in the practical application of flexural ultrasonic transducers.

Published by the Acoustical Society of America

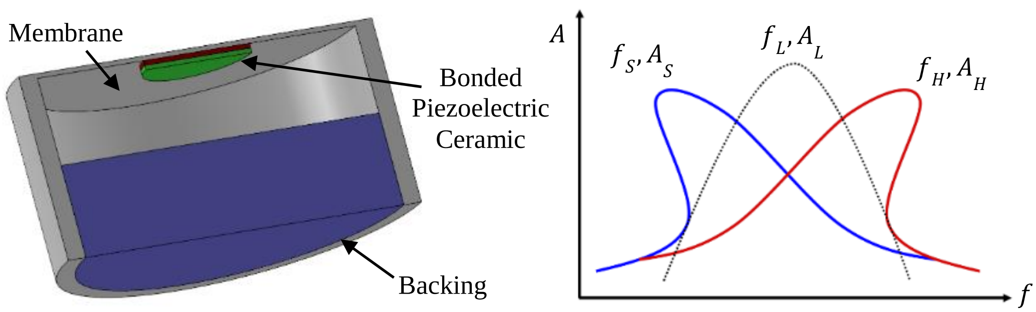
 https://doi.org/10.1121/2.0001095

2019 INTERNATIONAL CONGRESS ON ULTRASONICS
BRUGES, BELGIUM, 3-6 SEPT 2019

1. INTRODUCTION

The flexural ultrasonic transducer (FUT) is a class of ultrasonic measurement device which can efficiently generate and detect ultrasonic waves for applications including proximity measurement and metrology. The traditional form of the transducer comprises a piezoelectric disc bonded with a sufficiently high-strength adhesive to the underside of a metallic membrane. This membrane can be considered as a circular plate with an edge-clamped boundary condition. The high-frequency vibrations of the piezoelectric ceramic disc induce relatively high amplitude vibration motion of the membrane. Although the piezoelectric ceramic disc is a common choice of driver in the FUT, alternative components can be used instead of a piezoelectric ceramic, for example an electromagnetic coil. The FUT also usually includes a backing component fabricated from a material such as silicone, for sealing. A typical schematic of a FUT is illustrated in Fig. 1(a), showing a section-view to illustrate the positioning of the internal components.

Analysis of the dynamic performance of a FUT has largely been undertaken with respect to its linear dynamic response. However, in practical application there are a range of circumstances where a change in excitation or physical properties of the system, in this case the FUT, can produce a subsequent influence on the dynamic response of the FUT which is not in proportion with the change in the excitation conditions or physical properties. This effect can be referred to as dynamic nonlinearity. In general, nonlinearity is a phenomenon which can be found in numerous fields of science and mathematics, relating to mechanical, thermal, and electrical properties of materials and systems[1]. Dynamic nonlinearity can be specifically considered as the disproportionate dynamic response to a given input, in the context of this research. This nonlinearity can be weak, marginally deviating from the linear case, or chaotic. A FUT is typically operated at different excitation voltages depending on the application, and therefore dynamic nonlinearity is essential to consider. Dynamic nonlinearity is schematically illustrated in Fig. 1(b) through two nonlinear paths associated with the amplitude-frequency relationship[2], showing the relationship between amplitude (A) and frequency (f), by increasing the amplitude of vibration. The blue plot which shows a bend towards lower frequencies is indicative of nonlinear softening, where a decrease in resonance frequency can be observed, and the red plot which bends towards higher frequencies indicates an increase in resonance frequency, referred to as nonlinear hardening. The linear response is shown by the black dash plot in the centre. As amplitude is raised, the dynamic response of a transducer may be assumed to be linear until a specific threshold, after which the dynamic response can exhibit either softening or hardening nonlinear behaviour[3]. The peak amplitude locations are highlighted by the coordinates shown in Fig. 1(b), showing how the resonance locations can change. It should be noted that the amplitudes and associated frequency locations which are shown in Fig. 1(b) are for illustrative purposes only and are not expressed to any quantifiable scale.



(a) (b)

Figure 1. (a) Section-view of the FUT, and (b) a comparison between linear and nonlinear dynamic response spectra, showing the form of softening ($_S$) and hardening ($_H$) responses compared to a linear ($_L$) response.

In this study, the nonlinear dynamic response of the FUT is analysed in the context of a typical mode of operation which is exploited in practical application. Dynamic response measurements for different transducers, commercial and custom, are presented to highlight the important factors to consider in the operation of the FUT, and how dynamic nonlinearity can best be considered for optimal FUT performance. It is anticipated that this research will enable a greater understanding of how to design and optimize FUT-based measurement.

2. METHODOLOGY

A FUT is typically operated at resonance in one of its axisymmetric modes of vibration, commensurate with the literature[4]. These vibration modes can be identified through a combination of mathematical simulation and experimental techniques such as laser Doppler vibrometry (where the Polytec OFV-5000 is used for this investigation). In this research, only the first axisymmetric vibration mode is considered, the (0,0) mode of vibration. In practical application, the excitation signal for a FUT can consist of a burst sinusoid of several cycles, where the drive frequency is close to the resonance frequency. The FUT can be driven effectively at a relatively low voltage, often below 5 $V_{P\text{-}P}$, whilst retaining satisfactory measurement resolution. However, it can be practical to adopt a higher excitation voltage to increase the signal-to-noise ratio. Although it is less common to operate a FUT with a continuous-wave signal, this excitation condition is useful for the study of dynamic nonlinearity based on the relatively high energy transfer compared to a burst input signal. Therefore in this research, both burst and continuous-wave excitation signals are used. A typical response spectrum for burst sinusoid excitation can be found in the literature[4]. Burst sinusoid excitation is exploited in this study specifically to investigate nonlinearity associated with ring-down.

In all of the results shown in this study, the excitation voltage range and measurement increment are specified, all within the nominal range of 4 – 40 $V_{P\text{-}P}$. Also, details of the operating mode of vibration, resonance frequency, and the material type of the FUT cap and membrane are provided, with details of the physical dimensions in each case. The two transducers used in this study are shown in Fig. 2 alongside a schematic of the experimental setup. The FUT designated as Alum (Multicomp) is a commercial-type transducer which is fabricated from aluminium and possesses a membrane diameter of 10 mm. Alum is designed to operate in the (0,0) mode around 40 kHz. The FUT referred to as Titan is custom-made and incorporates a membrane fabricated from titanium whose diameter is around 11.46 mm. Titan operates at approximately 63 kHz in the (0,0) mode of vibration at 20 $V_{P\text{-}P}$.

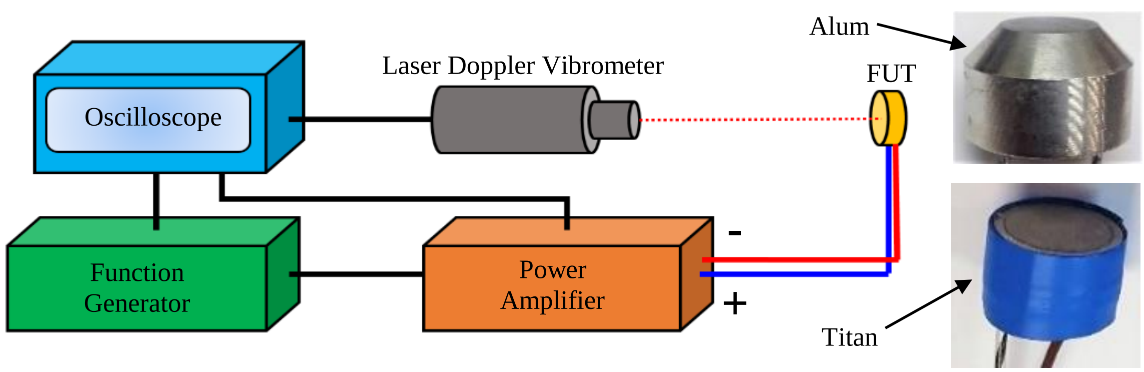
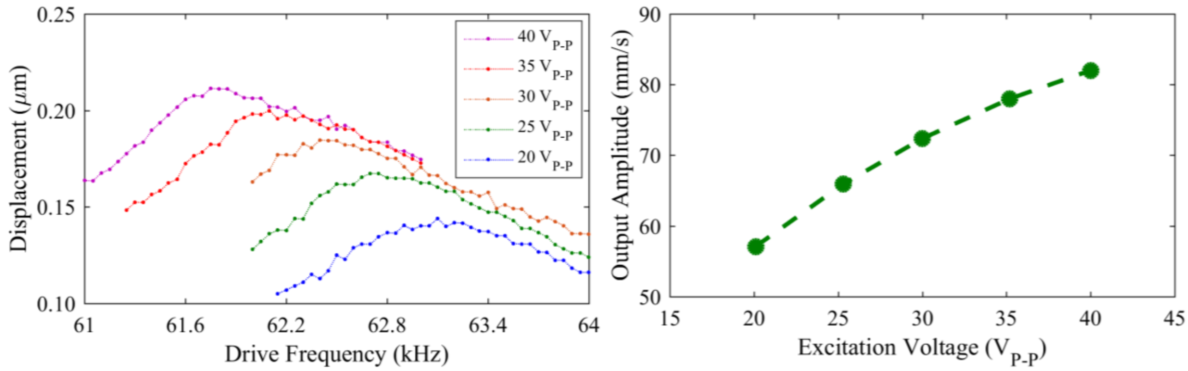


Figure 2. The basic dynamic nonlinearity characterization setup and the FUTs used in this study.

The experimental setup shown in Fig. 2 can therefore be used to effectively measure the dynamic nonlinearity in the vibration response of each FUT. The function generator is used to drive the FUT under study around its resonance frequency, where the excitation amplitude can be further increased using the power amplifier if necessary. The laser Doppler vibrometer is used to measure the velocity of the membrane vibration as the excitation frequency is incrementally adjusted around resonance, and this process is repeated for increasing levels of excitation amplitude. The vibration velocity can be converted to displacement for analysis if required.

3. RESULTS

The nonlinearity in the vibration response of Titan was captured in continuous-wave mode around resonance with a resolution of 50 Hz at nominal excitation voltages from 20 $V_{P\text{-}P}$ to 40 $V_{P\text{-}P}$, in measurement increments of 5 $V_{P\text{-}P}$. The amplitude at resonance was monitored, therefore enabling the measurement window to be adjusted. The results are shown in Fig. 3(a). There is a clear softening nonlinearity in the vibration response of Titan, closely aligning with observations for different FUTs reported in the literature[2,3]. There is a resonance frequency reduction of around 1300 Hz based on the difference between the peak amplitudes for a 40 $V_{P\text{-}P}$ excitation voltage compared to 20 $V_{P\text{-}P}$. This can be significant in practical application. The output amplitude, scaled according to the laser Doppler vibrometer sensitivity, is shown in Fig. 3(b) as a function of excitation voltage.



(a) (b)

Figure 3. (a) The softening nonlinearity in continuous-wave mode for Titan, and (b) the relationship between output and excitation amplitudes, scaled to the measurement sensitivity of 10 mm/s/V.

The Titan response aligns closely with literature observations. Therefore, the dynamic nonlinearity associated with the resonant decay, or ring-down, of Alum was undertaken for nominal excitation voltages of 4 V_{P-P} and 20 V_{P-P}, for 150 cycles with a resolution of 100 Hz. Alum was excited around its resonance of 39.50 kHz, after which the zero-crossing points were calculated to determine the change in frequency, before a third-order polynomial fit was produced. The results are an extension to the literature[2], and are shown in Fig. 4.

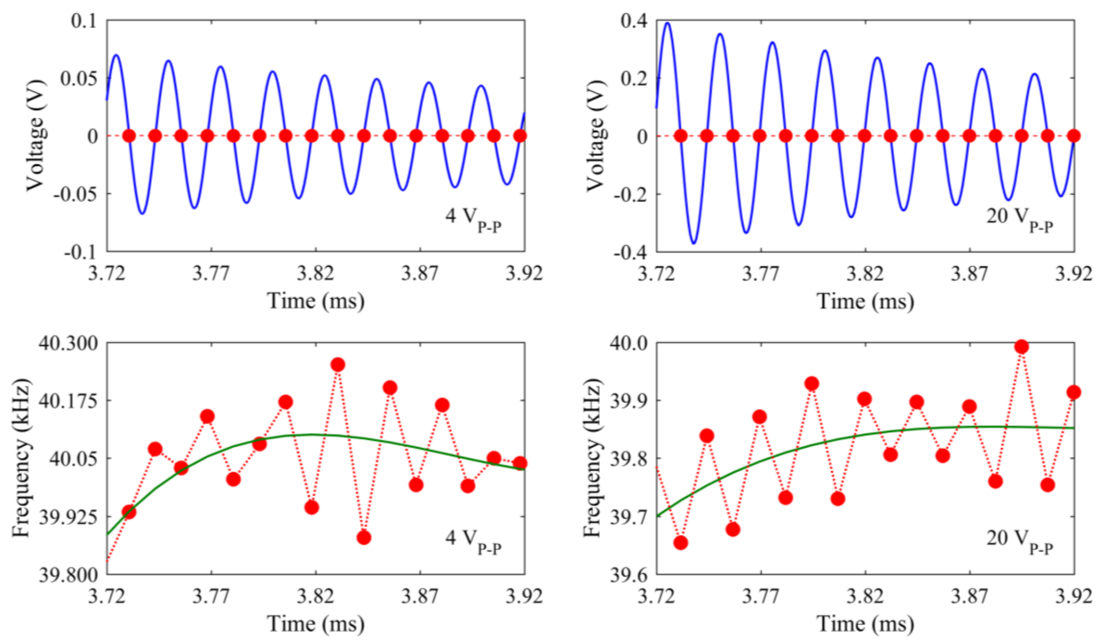


Figure 4. Evidence of nonlinearity in sections of the ring-down response region for Alum.

The fits clearly indicate nonlinear softening as the excitation voltage is increased, showing that even for a burst sinusoidal excitation, nonlinearity can still be detected. It is clear that for forced excitation in steady-state, and the absence of a forced vibration in resonant decay, there is evidence of nonlinearity as the excitation voltage is increased. Even if the resonance shift is not significant, the bandwidth of a FUT can be relatively narrow, therefore a shift of only a few hundred Hz can result in a notable efficiency drop. It is difficult to identify the key contributing factor to dynamic nonlinearity using a limited selection of FUTs, especially if they are all driven via piezoelectric ceramic discs, the characteristics of which have been reported to be nonlinear[3]. However, mathematical relationships can be demonstrated which accurately represent the dynamic nonlinearity of the FUT, showing the potential for future design and optimization of FUT-based measurement systems. The FUT response can be considered through a mass-spring-damper configuration with time-dependent forced vibration[3], as shown by Eq. (1), where B is the forced amplitude, b is the output amplitude, the drive frequency Ω can be equal or different to the resonance frequency ω_0, the perturbation parameter is ϵ, time is denoted by t, the damping term is μ, and the stress coefficients are indicated by α. A solution to Eq. (1) can be given by Eq. (2), where the phase is denoted by γ[3,5].

$$\ddot{x} + 2\epsilon^2\mu\dot{x} + \epsilon\alpha_2 x^2 + \epsilon^2\alpha_3 x^3 + \omega_0^2 x = \epsilon^2 B cos\Omega t \tag{1}$$

$$x = bcos(\Omega t - \gamma) + \frac{1}{2}\epsilon\alpha_2\omega_0^{-2}b^2[-1 + \frac{1}{3}\cos(2\Omega t - 2\gamma)] + O(\epsilon^2) \tag{2}$$

Through these equations, key indicators of dynamic nonlinearity can be demonstrated. The presence of nonlinear softening can be identified through $9\alpha_3\omega_0^2 < 10\alpha_2^2$ and determined through the amplitude and phase parameters. The output amplitude to excitation ratio can be shown to be linear via Eq. (3), and can be extracted from results such as those in Fig. 3(b) for illustration.

$$b(B^{-1}) = (2\omega_0)^{-1} \tag{3}$$

The spectrum shown in Fig. 3(b) can be assumed to be broadly linear, although a wider excitation range can be applied in future since only a limited number of data points have been produced. In general, this research has demonstrated the presence of nonlinear behaviour in the vibration responses of two different FUTs, showing these characteristics to be generally consistent with mathematical theory. Further research should focus on identifying the sources of dynamic nonlinearity by using a wider selection of FUTs, including those not relying on the generation of ultrasound through a piezoelectric ceramic, in order for the significant nonlinear characteristics of these materials to be eliminated.

4. CONCLUSION

This research has demonstrated the existence of dynamic nonlinearity in the vibration responses of two different FUTs. Laser Doppler vibrometry was used to measure the vibration amplitude responses, where characteristic behaviours were found to closely correlate with fundamental mathematical theory. It has been shown that nonlinear softening can be detected for both continuous and burst excitation conditions, and for different dynamic response regions, in this case both steady-state and resonant decay. This is important, since FUTs can be driven in practical application in different forms of excitation. As excitation amplitude level increases, the resonance frequency changes, and if this is not accounted for in the experimental setup, a loss of measurement efficiency is a physical and determinable consequence. It is anticipated that this research will be of importance to improve ultrasonic measurement using the FUT for different applications.

ACKNOWLEDGMENTS

This research is funded by EPSRC grant EP/N025393/1. The authors would like to acknowledge Jonathan Harrington of the University of Warwick for valuable contributions to this research. The project web-site with full details of the research programme can be accessed via the following link:

https://warwick.ac.uk/fac/sci/physics/research/ultra/research/hiffut/

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