Flavour Permutation Symmetry and Fermion Mixing

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* With Dan Roythorne and Bill Scott, PLB 657 (2007) 210, arXiv:0709.1439

Outline of Talk

- Introduction
- What does the SM tell us about flavour?
- What do the Data tell us about flavour?
- Origin of Masses and Mixings in SM
- Plaquette Invariance and Flavour Permutation Symmetry
- Flavour Symmetric Mixing Observables
- Applications
- Conclusions

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But (frustratingly) there still exists no convincing, accepted theory of flavour (and I'm not going to propose one today!).

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This is not our approach.

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- See what the data suggest - an experimentalist's approach.
- Very much work in progress.

Start with the Standard Model

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Distinguished by their charges...

...and by their Masses

$$\mathcal{L}_{Yuk} \sim c_d\,\overline{\psi}_{qL} \cdot \phi \cdot d_R + c_u\,\overline{\psi}_{qL} \cdot \tilde{\phi} \cdot u_R$$
$$+ \; c_e\,\overline{\psi}_{\ell L} \cdot \phi \cdot e_R + c_\nu\,\overline{\psi}_{\ell L} \cdot \tilde{\phi} \cdot \nu_{eR} (\text{+any Majorana mass terms...})$$

with "Yukawa couplings" c_u, c_d, c_e, c_ν free parameters (to be determined by experiment).

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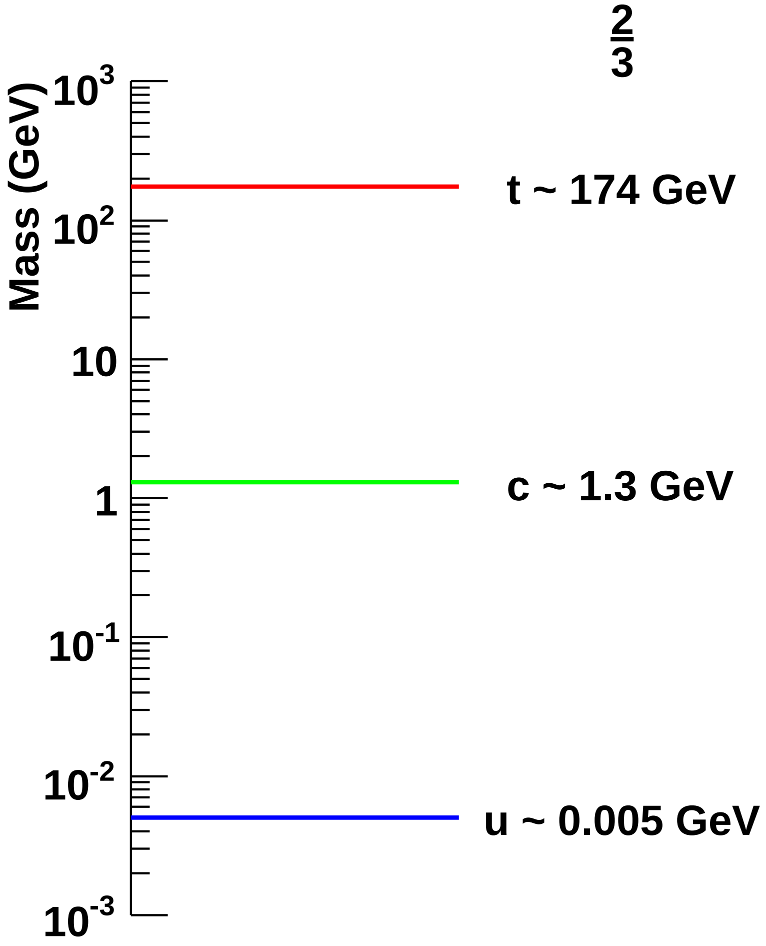
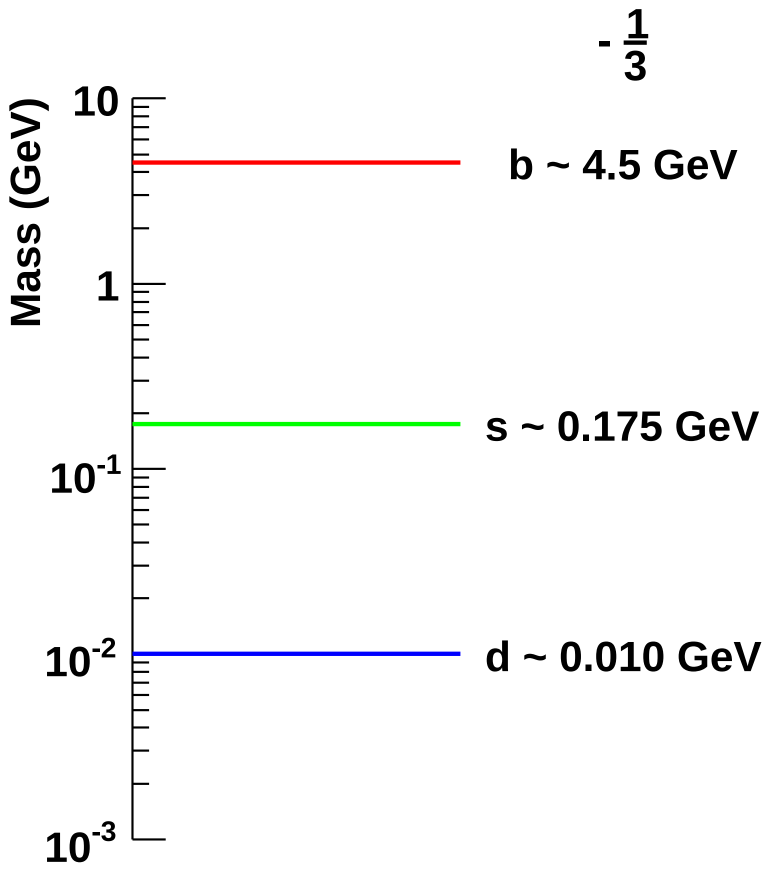
And that's all!

And the Data? 3 Families $\Longrightarrow$ 2 Quark Mass Spectra

Charge $= +\frac{2}{3}$



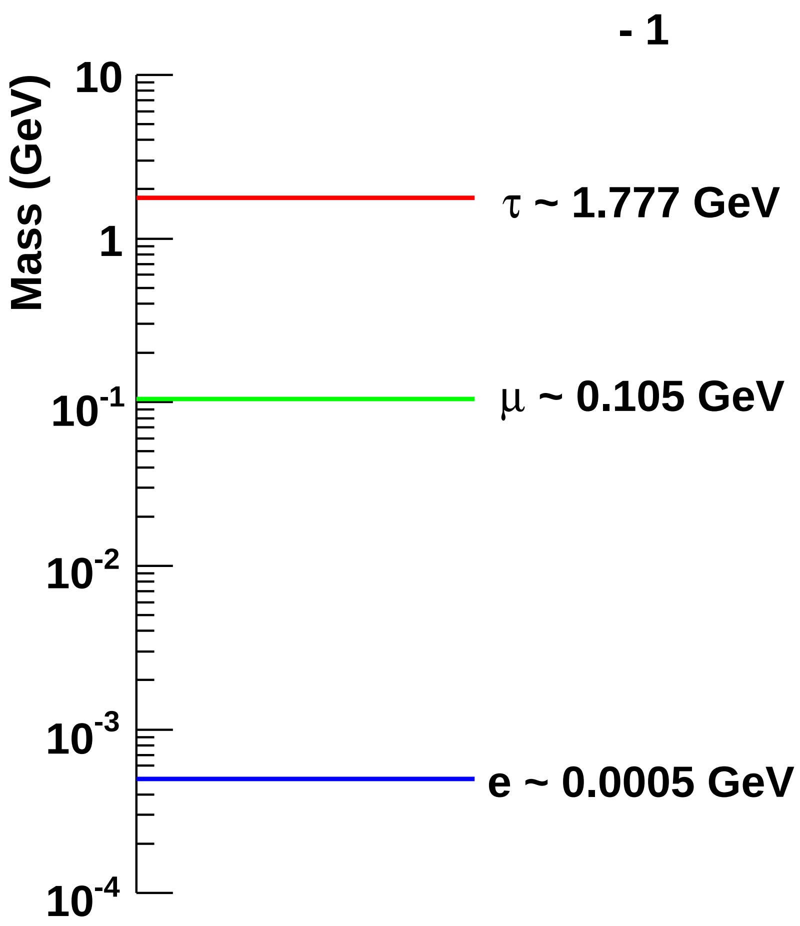
Charge $= -\frac{1}{3}$



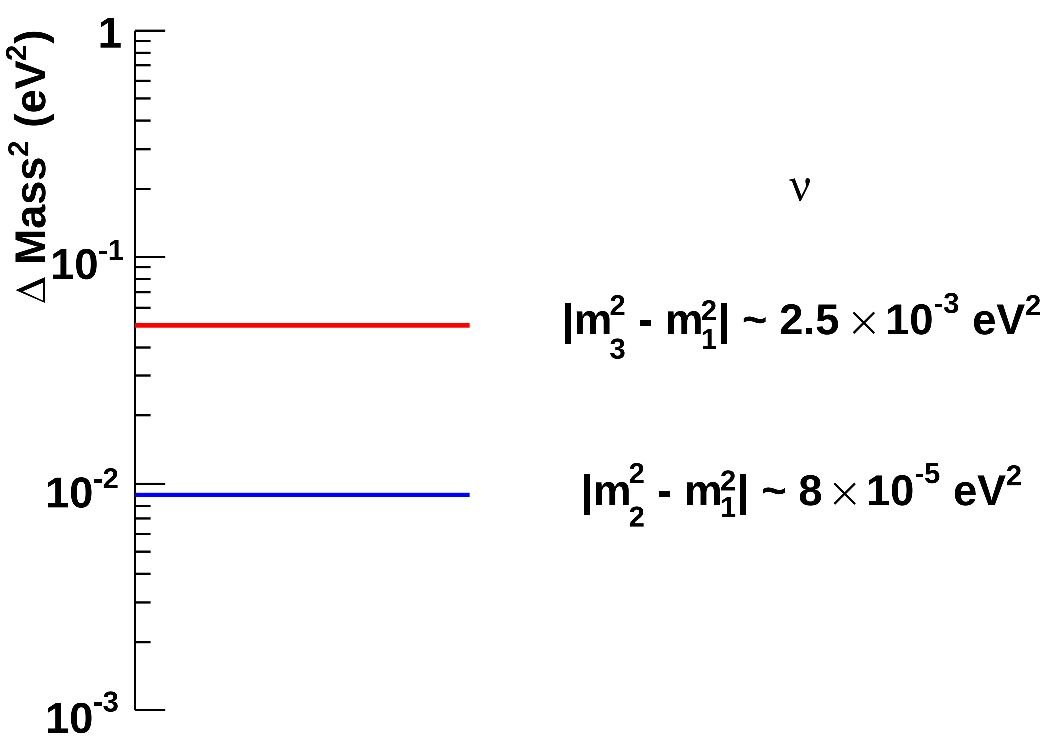
Apparently not a “typical” random, arbitrary set of parameters!

The Data - 2 Lepton Mass Spectra

Charge $= -1$



Charge $= 0$

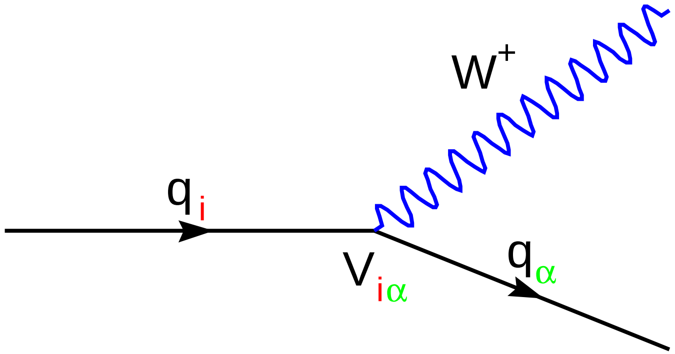


Similarly not a "typical" random, arbitrary set of parameters

The Data - CKM Quark Mixing Matrix

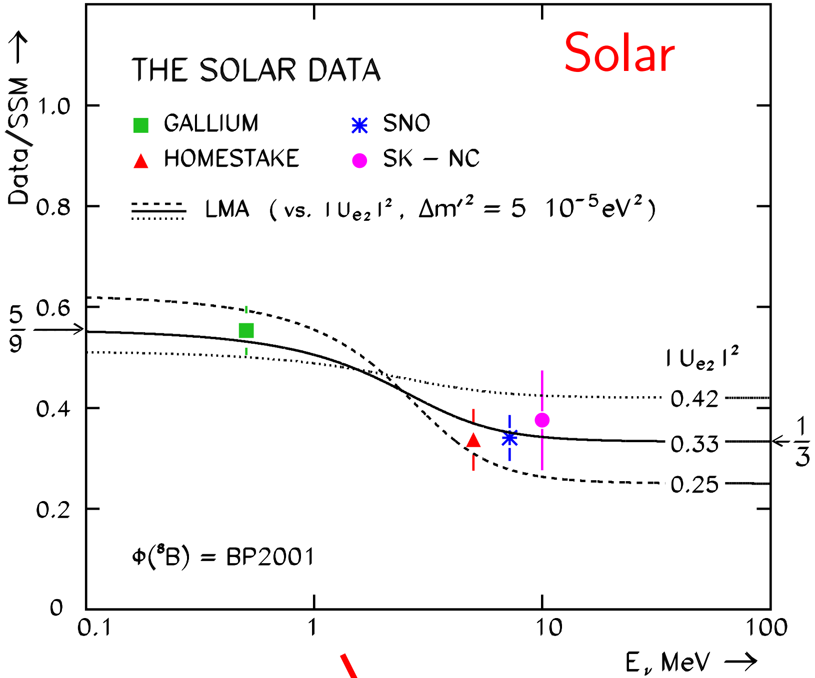
- CKM matrix parameterises couplings of quark mass eigenstates to W.
- Shows tantalising "Wolfenstein" structure:

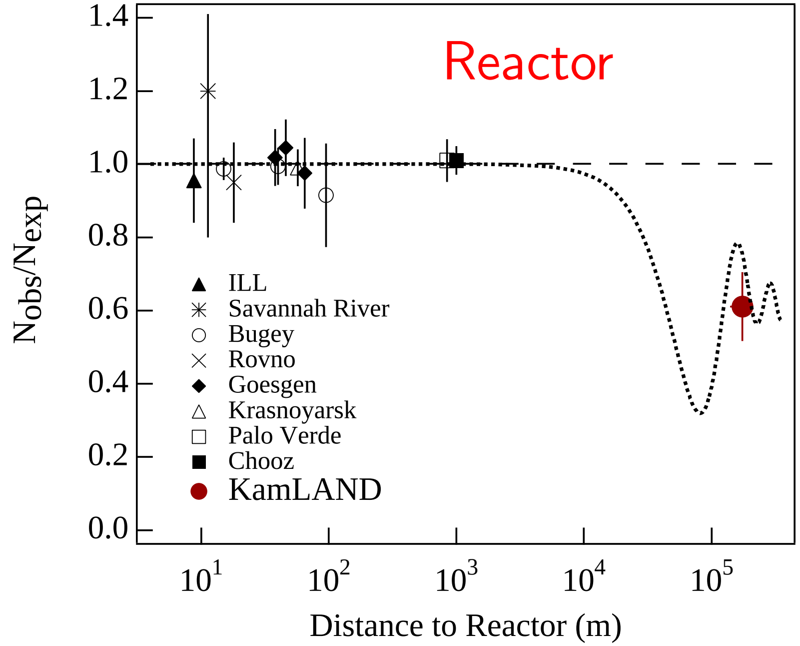
$$|V| \sim \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & \mathcal{O}(1) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & \mathcal{O}(1) \end{pmatrix} \end{array}$$

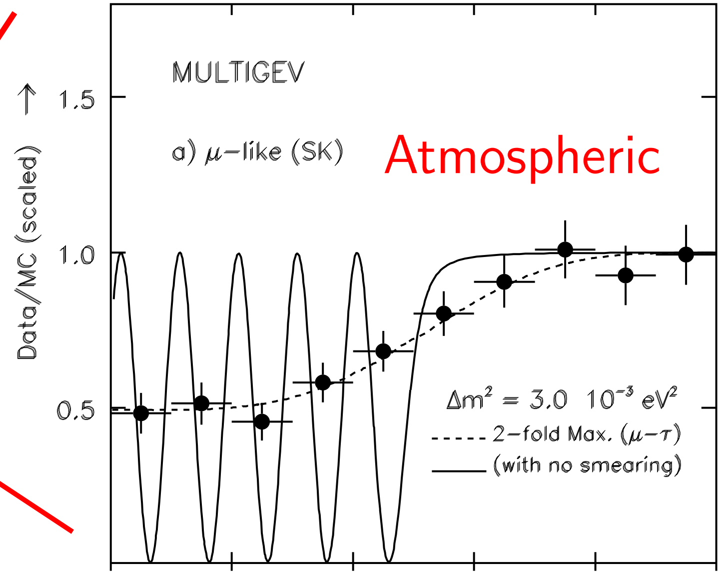


- Elements not predictable in the SM - profoundly unsatisfactory.

Summary of Neutrino Oscillation Data







$$
(|U|^2) = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} \sim \frac{2}{3} & 0.31 \pm 0.04 & < 0.013 \\ \sim \frac{1}{6} & \sim \frac{1}{3} & 0.50 \pm 0.11 \\ \sim \frac{1}{6} & \sim \frac{1}{3} & \sim \frac{1}{2} \end{pmatrix} \end{array}
$$

TriBimaximal Lepton Mixing

So, in leading approximation:

$$
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Or (more realistically):

$$U \simeq \begin{pmatrix} \frac{2}{\sqrt{6}}(1-\epsilon) & \frac{1}{\sqrt{3}}(1+2\epsilon) & U_{e3} \\ -\frac{1}{\sqrt{6}}(1+2\epsilon-\mu+\frac{3}{\sqrt{2}}U_{e3}^*) & \frac{1}{\sqrt{3}}(1-\epsilon-\mu) & \frac{1}{\sqrt{2}}(1+\mu-\frac{1}{\sqrt{2}}U_{e3}) \\ \frac{1}{\sqrt{6}}(1+2\epsilon+\mu-\frac{3}{\sqrt{2}}U_{e3}^*) & -\frac{1}{\sqrt{3}}(1-\epsilon+\mu) & \frac{1}{\sqrt{2}}(1-\mu+\frac{1}{\sqrt{2}}U_{e3}) \end{pmatrix}.$$

with

$$-0.032 < \epsilon < 0.037; \quad -0.18 < \mu < 0.21; \quad |U_{e3}| < 0.11.$$

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  If $\mathrm{Re}(U_{e3}) = 0 = \mu$, then $|U_{\mu i}| = |U_{\tau i}|$ and simulataneous $\mu - \tau$ swap and CP transf. leaves observables invariant (PFH and W.G. Scott PLB 547 (2002) 219).

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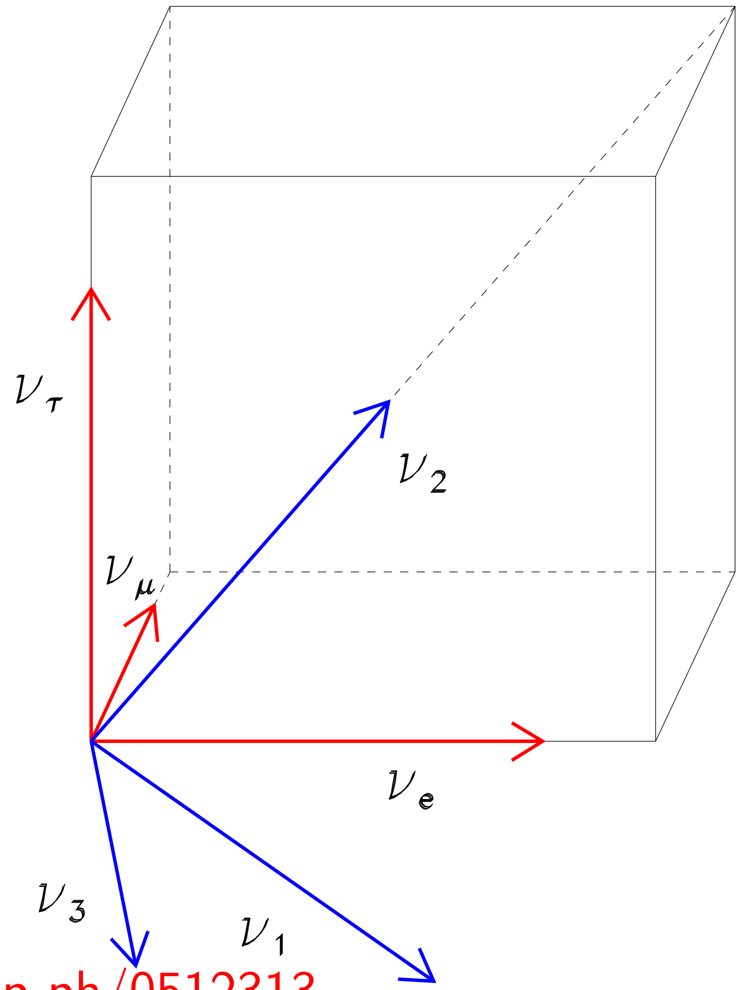
- CP *and* Democracy *and* $\mu - \tau$ Reflection Symmetry

  $\Rightarrow U_{e3} = 0 = \epsilon = \mu \Rightarrow$ TBM mixing.

Discussion of TBM

TBM form is redolent of other symmetries:

- Taking ν_e, ν_μ, ν_τ to define the orientation of a cube, ν_2 lies along its body diagonal
- Same coefficients form the $M = 0$ subset of the $j \times j = 1 \times 1$ set of Clebsch-Gordan Coeffts.



Several authors have built TBM into models,

eg. G. Altarelli and F. Feruglio, hep-ph/0512103
I. de Medeiros Varzielas, S. King and G. Ross, hep-ph/0512313
E. Ma PRD 73:057304, 2006 etc. ($A4$, $D(27)$, $\Sigma(81)$ groups etc.)

The Flavour problem

- The spectra of masses and mixings are today's analogues of the Lyman, Balmer and Paschen series. They show tantalising structure and yet are unexplained, and inexplicable within the prevailing (Standard) Model.
- The Wolfenstein and Tri-bimaximal formulae, provide correct (within exptl. errors) mathematical descriptions of the observed systematics of the mixing spectra, without any explanation at all.

  While based on less precise data, are somewhat akin to the Rydberg formula.

The data demand explanation, but the SM has nothing to say about them.

Origin of Masses and Mixings in the SM

- In SM, they have *common* origin in fermion-Higgs "Yukawa couplings" (multiplied by Higgs Vevs).
- Will focus on leptons – consider 3 families of leptonic $SU(2)$ doublets:

$$\psi_a = \begin{pmatrix} \nu_a \\ \ell_a^- \end{pmatrix}, \quad \psi_b = \begin{pmatrix} \nu_b \\ \ell_b^- \end{pmatrix}, \quad \psi_c = \begin{pmatrix} \nu_c \\ \ell_c^- \end{pmatrix}$$

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- Define also three-component vectors in family-space:

$$\underline{\psi} = \begin{pmatrix} \psi_a \\ \psi_b \\ \psi_c \end{pmatrix} \quad \text{with} \quad \underline{\nu} = \begin{pmatrix} \nu_a \\ \nu_b \\ \nu_c \end{pmatrix} \quad \underline{\ell} = \begin{pmatrix} \ell_a^- \\ \ell_b^- \\ \ell_c^- \end{pmatrix}$$

Origin of Masses and Mixings (contd.)

- Generalise earlier Yukawa couplings to 3 families:

$$\mathcal{L}_{Yuk} = (\overline{\underline{\psi}}_L \cdot \phi)\, Y_\ell\, \underline{\ell}_R + (\overline{\underline{\psi}}_L \cdot \tilde{\phi})\, Y_\nu\, \underline{\nu}_R + H.C.$$

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$$\mathcal{L}_{mass} = \bar{\underline{\ell}}_L\, M_\ell\, \underline{\ell}_R + \bar{\underline{\nu}}_L\, M_D\, \underline{\nu}_R + H.C.$$

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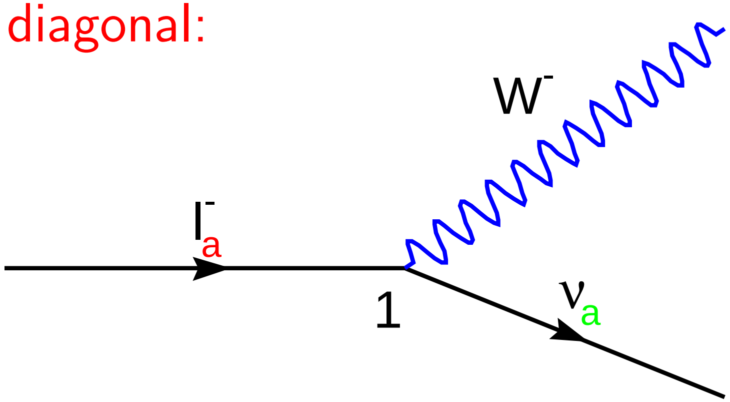
- Charged-current weak interaction (same basis) is diagonal:

$$\mathcal{L}_{cc} = g(\bar{\underline{\ell}}_L \,.\, \underline{\nu}_L) W^+ + H.C.$$



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- Useful to work with Hermitian mass matrices. Define:
  - for charged leptons: $L = M_\ell M_\ell^\dagger$ OR $L = M_\ell U_R^\ell$ etc.
  - for neutrinos: $N = M_\nu M_\nu^\dagger$ OR $N = M_D M_D^\dagger$ OR $N = M_D U_R^\nu$ etc.

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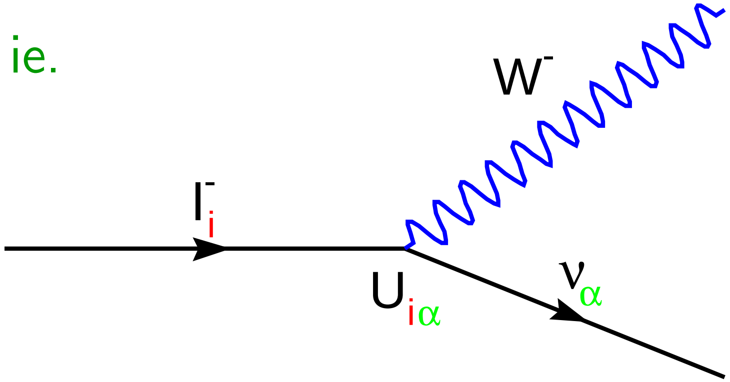
- Diagonalisation different for L and $N \Rightarrow$ mixing, ie.

$$\mathcal{L}_{cc} \;=\; g\,(\underline{\bar{\ell}}_L^{\,m}\, U_{MNS}\, \underline{\nu}_L^m) W^+ + H.C.$$



where $U_{MNS} \equiv U_\ell U_\nu^\dagger$ is the lepton mixing matrix. NB. Analogous for quarks.

Jarlskog (ie. Weak Basis) Covariance

- However L and N are not observable. Under a *simultaneous* change of weak basis:

$$L' = U_J L U_J^\dagger \text{ and } N' = U_J N U_J^\dagger$$

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- Allows, eg. freedom to "push" mixing into neutrinos or charged leptons
- Jarlskog urges (1985): "important results can't be frame-dependent" and reiterates (hep-ph/0606050): "you should be able to formulate it in an invariant form"
- We adopt Jarlskog's prescription as a principle!

Historical Attempts to Constrain Fermion Masses and Mixings

- Inspired by phenomenological relationships in quark mixing, eg.

$$\tan\theta_C \simeq 0.23 \simeq \sqrt{\frac{m_d}{m_s}}; \qquad \tan\theta_{cb} \simeq 0.04 \simeq \sqrt{\frac{m_d}{m_b}}$$

  and common origin of masses and mixings. Many attempts made to relate them.

- Best known is by H. Fritzsch from 1977 and 1978: mass matrices:

$$M^{\frac{2}{3}} = \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix}; \qquad M^{-\frac{1}{3}} = \begin{pmatrix} 0 & \alpha & 0 \\ \alpha^* & 0 & \beta \\ 0 & \beta^* & \gamma \end{pmatrix}$$

- Predicts (after approximations) above θ_C relation, but also

$$\tan\theta_{cb} > \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}} \simeq 0.09, \qquad \text{(now excluded)}.$$

How does Fritzsch *Ansatz* work?

- Mass matrices encode 10 pieces of information (6 masses and 4 mixing parameters) using only 6 magnitudes and one phase, $\Longrightarrow$ 3 constraints.
- Spawned many refinements, some phenomenologically successful.
- Modern models appeal to "flavour symmetries" to enforce zeroes or other "textures".
- Many build-in supersymmetry and/or complicated Higgs structures etc.

How does Fritzsch *Ansatz* work?

- Mass matrices encode 10 pieces of information (6 masses and 4 mixing parameters) using only 6 magnitudes and one phase, $\Longrightarrow$ 3 constraints.
- Spawned many refinements, some phenomenologically successful.
- Modern models appeal to "flavour symmetries" to enforce zeroes or other "textures".
- Many build-in supersymmetry and/or complicated Higgs structures etc.
- However, generally assume that $\exists$ a "special" basis, in which the mass matrices take some "particular" form (which limits number of parameters to < 10).
- "Specialness" of this basis is never explained.
- Since 1905, we have been alert to excessive reliance on a given frame or basis.
- We seek, instead, a Jarskog-invariant description of mixing.

Flavour-Symmetric Jarlskog Invariants

- "Usual" observables are not flavour-symmetric. eg. the electron mass, m_e, (electron label) and $|U_{e3}|$ (flavour and mass labels).
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- Examples are (in an arbitrary weak-basis, primes dropped):

$$\mathrm{Tr}(L) = m_e + m_\mu + m_\tau; \quad \mathrm{Tr}(L^2) = m_e^2 + m_\mu^2 + m_\tau^2; \quad \mathrm{Tr}(L^3) = m_e^3 + m_\mu^3 + m_\tau^3$$

which form a complete set.

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which form a complete set.

- All traces of polynomials of L and N are invariant (and FS), eg.:

$$\mathrm{Tr}(L'N') = \mathrm{Tr}(ULU^\dagger UNU^\dagger) = \mathrm{Tr}(ULNU^\dagger) = \mathrm{Tr}(LNU^\dagger U) = \mathrm{Tr}(LN)$$

etc.

- But typically depend in a complicated way on masses and mixing matrix elements.

The Jarlskogian and Plaquette Invariance

Jarlskog's CP-violating invariant:

$$J = \mathrm{Im}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j})$$

$$\begin{pmatrix} U_{e1} & U^*_{e2} & U_{e3} \\ U^*_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

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Fascinating properties:

- Parameterises CP violation
- Does not depend on which plaquette is used
- Simply related to the mass matrices:

$$J = -i \, \frac{\mathrm{Det}[L, N]}{2 L_\Delta N_\Delta}$$

L_Δ, N_Δ are traces of polynomials in L and N.

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- Working with observables, find that, like J, FS functions of the mixing matrix can always be expressed as simple functions of the mass matrices.

- Will introduce an elemental set

  – can be used for the flavour-symmetric description of any mixing scheme.

The $S3_\ell \times S3_\nu$ Flavour Permutation Group

- 6 perms. of ℓ flavour indices and 6 of ν “flavour” labels (ie. ν mass eigenstate indices) constitute the $S3_\ell \times S3_\nu$ Flavour Permutation Group (FPG).

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$$P = \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\ |U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2 \end{pmatrix}$$

  – Parameterises mixing (up to the sign of J).

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  – Parameterises mixing (up to the sign of J).

  – Transforms as a (reducible) $\mathbf{3} \times \mathbf{3}$ (natural representation) of FPG.

  – Rows and columns each sum to unity (a magic square)
  $\Rightarrow$ completely specified by elements of *any* P-plaquette.

  – Each P-plaquette transforms as (irreducible) $\mathbf{2} \times \mathbf{2}$ of FPG.

Singlets Under FPG?

- J is prototype FS observable - invariant under even members of the FPG; flips sign under odd members; ie. $\overline{\mathbf{1}} \times \overline{\mathbf{1}}$.

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  - Find simple polynomials of elements of P
  - (anti-)symmetrise over flavour labels
- Simple representation theory$\longrightarrow$
  - 1st order in P: $\exists$ no non-trivial singlets
  - 2nd order: one each of $\overline{\mathbf{1}} \times \overline{\mathbf{1}}$ and $\mathbf{1} \times \mathbf{1}$
  - 3rd order: one each of all four singlets
  - $\geq$ 4th order: multiple instances of each
- Will stay at $\leq$ 3rd order. Clearly four are sufficient.

Elemental Set of FS Mixing Observables

Define themselves, up to normalisation (and "offset" in $\mathbf{1} \times \mathbf{1}$ case):

$$\mathcal{G} = \frac{1}{2}\,[\,\sum_{\alpha i}(P_{\alpha i})^2 - 1\,] \qquad \mathcal{F} = \mathrm{Det}P$$

$$\mathcal{C} = \frac{3}{2}\,[\,\sum_{\alpha i}(P_{\alpha i})^3 - \sum_{\alpha i}(P_{\alpha i})^2\,] + 1 \qquad \mathcal{A} = \frac{1}{18}\sum_{\gamma k}(L_{\gamma k})^3$$

where $L_{\gamma k} = (P_{\alpha i} + P_{\beta j} - P_{\beta i} - P_{\alpha j})$.

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- $\mathcal{F}$ and $\mathcal{A}$ need no offset (they are anti-symmetric).
  - Reach extremum for no mixing
  - 0 for trimaximal mixing
- All normalised to maximum value $= 1$ (no mixing).
- $\mathcal{G}$ and $\mathcal{C}$ offset to zero for maximal mixing

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Properties and Values

Observable Name	Order in P	Symmetry: $S3_\ell \times S3_\nu$	Theor. Range	Exptl. Range for Leptons	Exptl. Range for Quarks
$\mathcal{F}$	2	$\overline{\mathbf{1}}\times\overline{\mathbf{1}}$	$(-1, 1)$	$(-0.14, 0.12)$	$(0.893, 0.896)$
$\mathcal{G}$	2	$\mathbf{1}\times\mathbf{1}$	$(0, 1)$	$(0.15, 0.23)$	$(0.898, 0.901)$
$\mathcal{A}$	3	$\overline{\mathbf{1}}\times\overline{\mathbf{1}}$	$(-1, 1)$	$(-0.065, 0.052)$	$(0.848, 0.852)$
$\mathcal{C}$	3	$\mathbf{1}\times\mathbf{1}$	$(-\frac{1}{27}, 1)$	$(-0.005, 0.057)$	$(0.848, 0.852)$

Properties and values of FS observables. Experimentally allowed ranges estimated (90% CL) from compilations of current experimental results (neglect any correlations between the input quantities).

FSMOs in Terms of Mass Matrices

Define reduced (ie. traceless) powers of mass matrices: $\widetilde{L^m} := L^m - \frac{1}{3}\mathrm{Tr}(L^m)$ (similarly for $\widetilde{N^m}$).

Now define Jarlskog-invariant:

$$\widetilde{T}_{mn} := \mathrm{Tr}(\widetilde{L^m}\widetilde{N^n}), \quad m, n = 1, 2.$$

$\widetilde{T}$ Completely equivalent to P (for known lepton masses).

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Find:

$$\mathcal{F} \equiv \mathrm{Det}\, P = 3\frac{\mathrm{Det}\,\widetilde{T}}{L_\Delta N_\Delta}; \qquad \left[cf.\ J = -i\,\frac{\mathrm{Det}[L, N]}{2L_\Delta N_\Delta} \right]$$

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$$\mathcal{G} = \frac{\widetilde{T}_{mn}\,\widetilde{T}_{pq}\,\mathcal{L}^{mp}\,\mathcal{N}^{nq}}{(L_\Delta N_\Delta)^2}; \qquad \mathcal{C},\mathcal{A} = \frac{\widetilde{T}_{mn}\,\widetilde{T}_{pq}\,\widetilde{T}_{rs}\,\mathcal{L}^{(mpr)}_{\mathcal{C},\mathcal{A}}\,\mathcal{N}^{(nqs)}_{\mathcal{C},\mathcal{A}}}{(L_\Delta N_\Delta)^{K_{\mathcal{C},\mathcal{A}}}}$$

The $\mathcal{L}$ ($\mathcal{N}$) are simple functions of traces of $\widetilde{L^m}$ ($\widetilde{N^m}$). $\quad K_{\mathcal{C}}\ (K_{\mathcal{A}}) = 2(3)$.

Application: Flavour-symmetric Descriptions of Mixing

MNS Matrix:

$$
|U| \sim \begin{array}{c} \\ e \\ \mu \\ \tau \end{array}
\begin{array}{c}
\begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\
\begin{pmatrix}
2/\sqrt{6} & 1/\sqrt{3} & \epsilon \\
1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\
1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}
\end{array}
$$

$$\Rightarrow \mathcal{F} = 0, \quad \mathcal{C} = 0, \quad \mathcal{A} = 0, \quad \mathcal{G} = \tfrac{1}{6}(1 - 3\epsilon^2)^2.$$

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Where is flavour-symmetry?

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- There exist 36 equally valid "solutions" of the TBM form, related to each other by allowed permutations of the rows and columns of the mixing matrix.
- eg.

$$|U| \sim \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} 1/\sqrt{3} & 2/\sqrt{6} & \epsilon \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{pmatrix} \end{array}$$

FS Descriptions of Several Mixing Schemes

Particular mixing schemes and their corresponding descriptions in terms of constraints on FS mixing observables, and symmetries.

Mixing Ansatz	$\mathcal{F}$	$\mathcal{G}$	$\mathcal{C}$	$\mathcal{A}$	Corresponding Symmetries	$18J^2$	$\mathcal{B}$	$\mathcal{D}$
No Mixing	1	1	1	1	CP	0	0	0
Tribimaximal Mixing*	0	$\frac{1}{6}$	0	0	Dem., μ-τ, CP	0	0	$\frac{1}{12\sqrt{3}}$
Trimaximal Mixing	0	0	0	0	Dem., μ-τ	$\frac{1}{6}$	0	0
S3 Group Mixing*	0	–	0	–	Democracy	–	0	–
Two Equal P-Rows*	0	–	–	0	e.g. μ-τ	–	0	–
Two Equal P-Columns	0	–	–	0	e.g. 1-2	–	–	0
Altarelli-Feruglio*	0	–	$\frac{6\mathcal{G}-1}{8}$	0	μ-τ, CP	0	0	–
Tri-χmaximal Mixing*	0	–	0	0	Dem., μ-τ	–	0	–
Tri-ϕmaximal Mixing*	0	$\frac{1}{6}$	0	–	Dem., CP	0	0	–
Bi-maximal Mixing	0	$\frac{1}{8}$	$-\frac{1}{32}$	0	CP, μ-τ, 1-2	0	0	0

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- Setting for example, $\mathcal{F}^2 + \mathcal{C}^2 + \mathcal{A}^2 + J^2 = 0$ provides a FS, JI constraint on the mass matrices (and, by extrapolation, on a combination of vevs times Yukawas).
- Such constraints may be "easily" implemented in BSM models, and allow the spontaneous breaking of flavour symmetry.

A Further Application

- What feature do quark and lepton mixing matrices have in common?
- Each has at least one "small" element.
- What is the flavour-symmetric expression of this?

$$|V_{CKM}| \sim \begin{pmatrix} \sim 1 & \sim \lambda & \sim \lambda^3 \\ \sim \lambda & \sim 1 & \sim \lambda^2 \\ \sim \lambda^3 & \sim \lambda^2 & \sim 1 \end{pmatrix}$$

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≥ 1 zero $\Rightarrow$ CP-conservation $\implies J = 0$.

But need two constraints - find:

$$2\mathcal{A} + \mathcal{F}(\mathcal{F}^2 - 2\mathcal{C} - 1) = 0 \; and \; J = 0 \implies \text{at least one zero}$$

A Further Application (Cont.)

- NB. Consider K-matrix: $K_{\gamma k} = \mathrm{Re}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j})$ (cf. J).

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- $\Longrightarrow$ FS prediction: if $\mathrm{Det}K = 0$, then as $J \to 0$, at least one UT angle $\to 90^\circ$.

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For quarks: $\mathrm{Det}K = 0 \implies (90° - \alpha) = \overline{\eta}\lambda^2 = 1° \pm 0.2°$

cf. $(90° - \alpha) = 0°{}^{+3°}_{-7°}$ experimentally.

For leptons: $\mathrm{Det}K = 0 \implies (90° - \delta) = \frac{|U_{e3}|}{\sqrt{2}} \lesssim 8°$ (good for CPV).

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- More speculatively, such quantities could be used to implement models of spontaneous flavour violation.