Flavour Permutation Symmetry and Quark Mixing:

Is the Unitarity Triangle Right?

Paul Harrison*

University of Warwick

SuperB Workshop, Isola d'Elba

1st June 2008

* With Dan Roythorne and Bill Scott, PLB 657 (2007) 210, arXiv:0709.1439 and arXiv:0805.3440

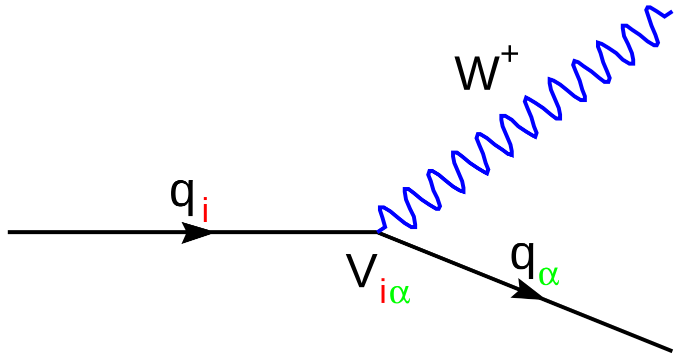
Outline of Talk

- The Flavour Problem
- Plaquette Invariance and Flavour Permutation Symmetry
- Flavour Symmetric Mixing Observables
- Applications
- Conclusions

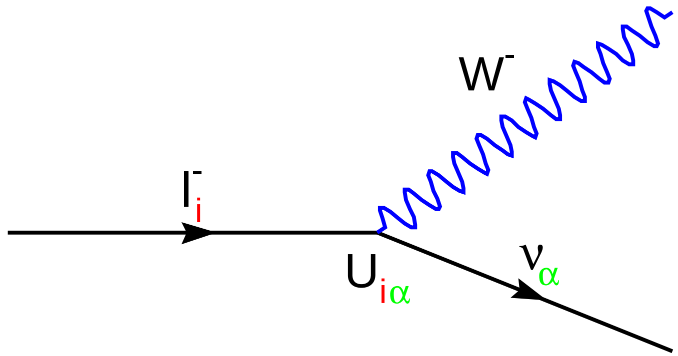
The Flavour Problem

- SM has 20 (22) low energy flavour parameters (out of 25(27))
- They show tantalising structure, eg.

$$|V_{CKM}| \sim \begin{matrix} \\ u \\ c \\ t \end{matrix} \begin{matrix} d & s & b \\ \left(\begin{matrix} \mathcal{O}(1) & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & \mathcal{O}(1) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & \mathcal{O}(1) \end{matrix}\right) \end{matrix}$$



$$|U_{MNS}| \sim \begin{matrix} \\ e \\ \mu \\ \tau \end{matrix} \begin{matrix} \nu_1 & \nu_2 & \nu_3 \\ \left(\begin{matrix} 2/\sqrt{6} & 1/\sqrt{3} & \lesssim 0.2 \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{matrix}\right) \end{matrix}$$



- Are not predictable in the SM - profoundly unsatisfactory.

Masses and Mixings have Common Origin

- in Quark and Lepton mass matrices: $M_{\frac{2}{3}},\ M_{-\frac{1}{3}},\ M_{\ell},\ M_{\nu}$
- Each is product of Higgs vev and Yukawa coupling matrix in Lagrangian

Masses and Mixings have Common Origin

- in Quark and Lepton mass matrices: $M_{\frac{2}{3}},\ M_{-\frac{1}{3}},\ M_\ell,\ M_\nu$
- Each is product of Higgs vev and Yukawa coupling matrix in Lagrangian
- Will work with their Hermitian Squares.
  - eg. for neutrinos: $N = M_\nu M_\nu^\dagger$
  - for charged leptons: $L = M_\ell M_\ell^\dagger$
  - for charged 2/3 quarks: $H_{\frac{2}{3}} = M_{\frac{2}{3}} M_{\frac{2}{3}}^\dagger$ etc.

Masses and Mixings have Common Origin

- in Quark and Lepton mass matrices: $M_{\frac{2}{3}},\ M_{-\frac{1}{3}},\ M_{\ell},\ M_{\nu}$
- Each is product of Higgs vev and Yukawa coupling matrix in Lagrangian
- Will work with their Hermitian Squares.
  - eg. for neutrinos: $N = M_{\nu}M_{\nu}^{\dagger}$
  - for charged leptons: $L = M_{\ell}M_{\ell}^{\dagger}$
  - for charged 2/3 quarks: $H_{\frac{2}{3}} = M_{\frac{2}{3}}M_{\frac{2}{3}}^{\dagger}$ etc.
- Diagonalise $U_{\nu}NU_{\nu}^{\dagger} = D_{\nu} \rightarrow$ eigenvalues (ie. masses-squared)
- Diagonalisation different for L and $N \Rightarrow$ mixing, ie. $U_{MNS} \equiv U_{\ell}U_{\nu}^{\dagger}$.

  Analogous for quarks: $U_{CKM} \equiv U_{-\frac{1}{3}}U_{\frac{2}{3}}^{\dagger}$.

The Jarlskogian and Plaquette Invariance

Jarlskog's CP-violating invariant:

$$J = \mathrm{Im}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j})$$

$$\begin{pmatrix} U_{e1} & U^*_{e2} & U_{e3} \\ U^*_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

The Jarlskogian and Plaquette Invariance

Jarlskog's CP-violating invariant:

$$J = \mathrm{Im}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j})$$

$$\begin{pmatrix} U_{e1} & U^*_{e2} & U_{e3} \\ U^*_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Fascinating properties:

- Parameterises CP violation
- Does not depend on which plaquette is used

The Jarlskogian and Plaquette Invariance

Jarlskog's CP-violating invariant:

$$J = \mathrm{Im}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j})$$

$$\begin{pmatrix} U_{e1} & U^*_{e2} & U_{e3} \\ U^*_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Fascinating properties:

- Parameterises CP violation
- Does not depend on which plaquette is used
- Simply related to the mass matrices:

$$J = -i \, \frac{\mathrm{Det}[L, N]}{2 L_\Delta N_\Delta}$$

L_Δ, N_Δ are traces of polynomials in L and N.

Are there Other Plaquette Invariants?

Are there Other Plaquette Invariants?

Yes!

Are there Other Plaquette Invariants?

Yes!

- J samples information uniformly across U

  - it is $flavour - symmetric$ (FS).

Are there Other Plaquette Invariants?

Yes!

- J samples information uniformly across U

  - it is $flavour - symmetric$ (FS).

- Clearly, *any* function of the $U_{\alpha i}$, symmetrised over all *flavour* labels, and reduced to a function of only elements of a single plaquette is plaquette-invariant.

Are there Other Plaquette Invariants?

Yes!

- J samples information uniformly across U

  - it is $flavour - symmetric$ (FS).

- Clearly, *any* function of the $U_{\alpha i}$, symmetrised over all *flavour* labels, and reduced to a function of only elements of a single plaquette is plaquette-invariant.

- Working with observables, find that, like J, FS functions of the mixing matrix can always be expressed as simple functions of the mass matrices.

- Will introduce an elemental set

  – can be used for the flavour-symmetric description of any mixing scheme.

The $S3_\ell \times S3_\nu$ Flavour Permutation Group

- 6 perms. of ℓ flavour indices and 6 of ν "flavour" labels (ie. ν mass eigenstate indices) constitute the $S3_\ell \times S3_\nu$ Flavour Permutation Group (FPG).

The $S3_\ell \times S3_\nu$ Flavour Permutation Group

- 6 perms. of ℓ flavour indices and 6 of ν "flavour" labels (ie. ν mass eigenstate indices) constitute the $S3_\ell \times S3_\nu$ Flavour Permutation Group (FPG).

- Introduce the observable P matrix:

$$P = \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{pmatrix}$$

  – Parameterises mixing (up to the sign of J).

The $S3_\ell \times S3_\nu$ Flavour Permutation Group

- 6 perms. of ℓ flavour indices and 6 of ν "flavour" labels (ie. ν mass eigenstate indices) constitute the $S3_\ell \times S3_\nu$ Flavour Permutation Group (FPG).
- Introduce the observable P matrix:

$$P = \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 2}|^2 & |U_{\tau 3}|^2 \end{pmatrix}$$

  - Parameterises mixing (up to the sign of J).
  - Transforms as a (reducible) $\mathbf{3} \times \mathbf{3}$ (natural representation) of FPG.
  - Rows and columns each sum to unity (a magic square)
    $\Rightarrow$ completely specified by elements of *any* P-plaquette.
  - Each P-plaquette transforms as (irreducible) $\mathbf{2} \times \mathbf{2}$ of FPG.

Singlets Under FPG?

- J is prototype FS observable - invariant under even members of the FPG; flips sign under odd members; ie. $\overline{\mathbf{1}} \times \overline{\mathbf{1}}$.

Singlets Under FPG?

- J is prototype FS observable - invariant under even members of the FPG; flips sign under odd members; ie. $\overline{\mathbf{1}} \times \overline{\mathbf{1}}$.
- Search for other singlets: $\mathbf{1} \times \mathbf{1}$, $\mathbf{1} \times \overline{\mathbf{1}}$ etc.
  - Find simple polynomials of elements of P
  - (anti-)symmetrise over flavour labels

Singlets Under FPG?

- J is prototype FS observable - invariant under even members of the FPG; flips sign under odd members; ie. $\overline{\mathbf{1}} \times \overline{\mathbf{1}}$.
- Search for other singlets: $\mathbf{1} \times \mathbf{1}$, $\mathbf{1} \times \overline{\mathbf{1}}$ etc.
  - Find simple polynomials of elements of P
  - (anti-)symmetrise over flavour labels
- Simple representation theory$\longrightarrow$
  - 1st order in P: $\exists$ no non-trivial singlets
  - 2nd order: one each of $\overline{\mathbf{1}} \times \overline{\mathbf{1}}$ and $\mathbf{1} \times \mathbf{1}$
  - 3rd order: one each of all four singlets
  - $\geq$ 4th order: multiple instances of each
- Will stay at $\leq$ 3rd order. Clearly four are sufficient.

Elemental Set of FS Mixing Observables

Define themselves, up to normalisation (and “offset” in $\mathbf{1} \times \mathbf{1}$ case):

$$\mathcal{G} = \frac{1}{2}\,[\,\sum_{\alpha i}(P_{\alpha i})^2 - 1\,] \qquad \mathcal{F} = \mathrm{Det}P$$

$$\mathcal{C} = \frac{3}{2}\,[\,\sum_{\alpha i}(P_{\alpha i})^3 - \sum_{\alpha i}(P_{\alpha i})^2\,] + 1 \qquad \mathcal{A} = \frac{1}{18}\sum_{\gamma k}(L_{\gamma k})^3$$

where $L_{\gamma k} = (P_{\alpha i} + P_{\beta j} - P_{\beta i} - P_{\alpha j})$.

Elemental Set of FS Mixing Observables

Define themselves, up to normalisation (and "offset" in $\mathbf{1} \times \mathbf{1}$ case):

$$\mathcal{G} = \frac{1}{2}\left[\sum_{\alpha i}(P_{\alpha i})^2 - 1\right] \qquad \mathcal{F} = \mathrm{Det}P$$

$$\mathcal{C} = \frac{3}{2}\left[\sum_{\alpha i}(P_{\alpha i})^3 - \sum_{\alpha i}(P_{\alpha i})^2\right] + 1 \qquad \mathcal{A} = \frac{1}{18}\sum_{\gamma k}(L_{\gamma k})^3$$

where $L_{\gamma k} = (P_{\alpha i} + P_{\beta j} - P_{\beta i} - P_{\alpha j})$.

- $\mathcal{F}$ and $\mathcal{A}$ need no offset (they are anti-symmetric).
  - Reach extremum for no mixing
  - 0 for trimaximal mixing
- All normalised to maximum value $= 1$ (no mixing).
- $\mathcal{G}$ and $\mathcal{C}$ offset to zero for maximal mixing

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

Properties and Values

Observable Name	Order in P	Symmetry: $S3_\ell \times S3_\nu$	Theor. Range	Exptl. Range for Leptons	Exptl. Range for Quarks
$\mathcal{F}$	2	$\overline{\mathbf{1}}\times\overline{\mathbf{1}}$	$(-1,1)$	$(-0.14,0.12)$	$(0.893,0.896)$
$\mathcal{G}$	2	$\mathbf{1}\times\mathbf{1}$	$(0,1)$	$(0.15,0.23)$	$(0.898,0.901)$
$\mathcal{A}$	3	$\overline{\mathbf{1}}\times\overline{\mathbf{1}}$	$(-1,1)$	$(-0.065,0.052)$	$(0.848,0.852)$
$\mathcal{C}$	3	$\mathbf{1}\times\mathbf{1}$	$(-\frac{1}{27},1)$	$(-0.005,0.057)$	$(0.848,0.852)$

Properties and values of FS observables. Experimentally allowed ranges estimated (90% CL) from compilations of current experimental results (neglect any correlations between the input quantities).

FSMOs in Terms of Mass Matrices

Define reduced (ie. traceless) powers of mass matrices: $\widetilde{L^m} := L^m - \frac{1}{3}\mathrm{Tr}(L^m)$ (similarly for $\widetilde{N^m}$).

Now define Jarlskog-invariant:

$$\widetilde{T}_{mn} := \mathrm{Tr}(\widetilde{L^m}\widetilde{N^n}), \quad m, n = 1, 2.$$

$\widetilde{T}$ Completely equivalent to P (for known lepton masses).

FSMOs in Terms of Mass Matrices

Define reduced (ie. traceless) powers of mass matrices: $\widetilde{L^m} := L^m - \frac{1}{3}\mathrm{Tr}(L^m)$ (similarly for $\widetilde{N^m}$).

Now define Jarlskog-invariant:

$$\widetilde{T}_{mn} := \mathrm{Tr}(\widetilde{L^m}\widetilde{N^n}), \quad m, n = 1, 2.$$

$\widetilde{T}$ Completely equivalent to P (for known lepton masses).

Find:

$$\mathcal{F} \equiv \mathrm{Det}\, P = 3\frac{\mathrm{Det}\,\widetilde{T}}{L_\Delta N_\Delta}; \qquad \left[cf.\ J = -i\,\frac{\mathrm{Det}[L, N]}{2L_\Delta N_\Delta} \right]$$

FSMOs in Terms of Mass Matrices

Define reduced (ie. traceless) powers of mass matrices: $\widetilde{L^m} := L^m - \frac{1}{3}\mathrm{Tr}(L^m)$ (similarly for $\widetilde{N^m}$).

Now define Jarlskog-invariant:

$$\widetilde{T}_{mn} := \mathrm{Tr}(\widetilde{L^m}\widetilde{N^n}), \quad m,n = 1,2.$$

$\widetilde{T}$ Completely equivalent to P (for known lepton masses).

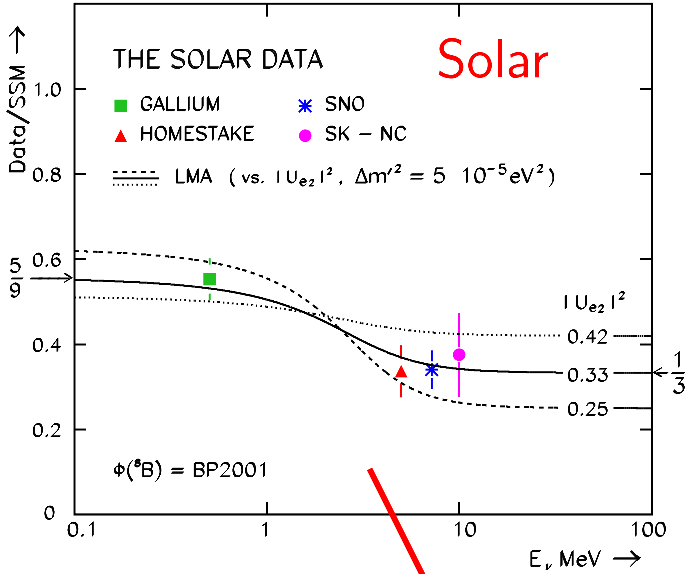
Find:

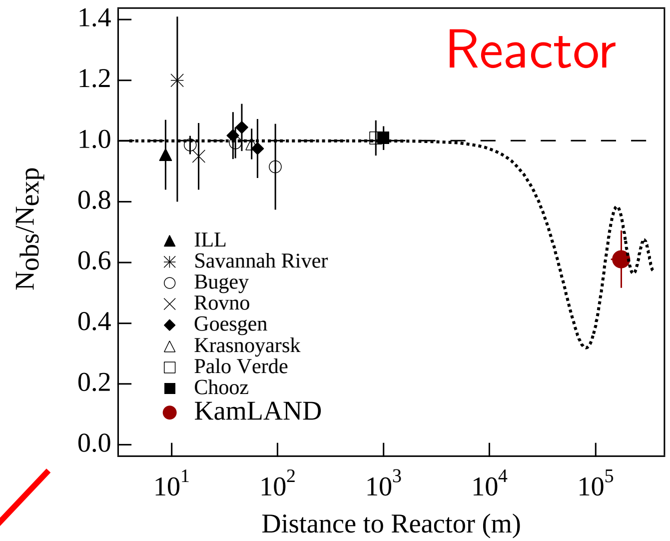
$$\mathcal{F} \equiv \mathrm{Det}\, P = 3\frac{\mathrm{Det}\,\widetilde{T}}{L_\Delta N_\Delta}; \qquad \left[cf.\ J = -i\,\frac{\mathrm{Det}[L,N]}{2L_\Delta N_\Delta}\right]$$

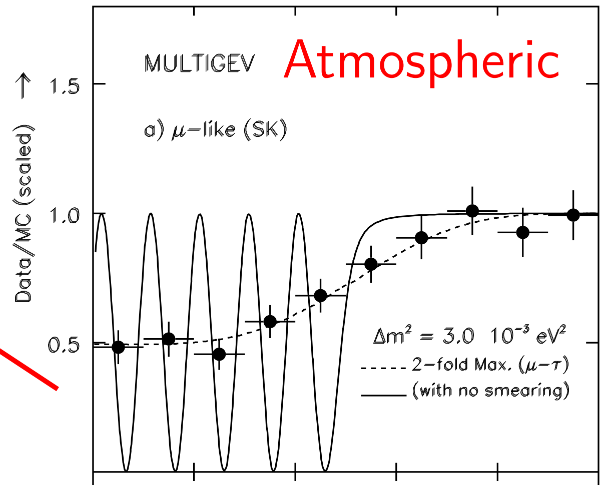
$$\mathcal{G} = \frac{\widetilde{T}_{mn}\,\widetilde{T}_{pq}\,\mathcal{L}^{mp}\,\mathcal{N}^{nq}}{(L_\Delta N_\Delta)^2}; \qquad \mathcal{C},\mathcal{A} = \frac{\widetilde{T}_{mn}\,\widetilde{T}_{pq}\,\widetilde{T}_{rs}\,\mathcal{L}^{(mpr)}_{\mathcal{C},\mathcal{A}}\,\mathcal{N}^{(nqs)}_{\mathcal{C},\mathcal{A}}}{(L_\Delta N_\Delta)^{n_{\mathcal{C},\mathcal{A}}}}$$

The $\mathcal{L}$ ($\mathcal{N}$) are simple functions of traces of $\widetilde{L^m}$ ($\widetilde{N^m}$). $\quad n_{\mathcal{C}}\ (n_{\mathcal{A}}) = 2(3)$.

Application: Flavour-symmetric Descriptions of Mixing







$$|U| \sim \begin{matrix} e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \\ 2/\sqrt{6} & 1/\sqrt{3} & \epsilon \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$\Rightarrow \mathcal{F} = 0, \quad \mathcal{C} = 0, \quad \mathcal{A} = 0, \quad \mathcal{G} = \frac{1}{6}(1 - 3\epsilon^2)^2.$ Where is flavour-symmetry?

Where is Flavour Symmetry?

- It is spontaneously broken

Where is Flavour Symmetry?

- It is spontaneously broken
- There exist 36 equally valid “solutions” of the TBM form, related to each other by allowed permutations of the rows and columns of the mixing matrix.
- eg.

$$|U| \sim \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} 1/\sqrt{3} & 2/\sqrt{6} & \epsilon \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \end{pmatrix} \end{array}$$

Another Application:

Partially Unified FS Description of Quark and Lepton Mixings

Unified Description of quark and lepton mixings is desirable.

Another Application:

Partially Unified FS Description of Quark and Lepton Mixings

Unified Description of quark and lepton mixings is desirable.

What feature do quark and lepton mixing matrices have in common?

Another Application:

Partially Unified FS Description of Quark and Lepton Mixings

Unified Description of quark and lepton mixings is desirable.

What feature do quark and lepton mixing matrices have in common?

- Each has at least one "small" element.
- What is the flavour-symmetric expression of this?

$$|V_{CKM}| \sim \begin{pmatrix} \sim 1 & \sim \lambda & \sim \lambda^3 \\ \sim \lambda & \sim 1 & \sim \lambda^2 \\ \sim \lambda^3 & \sim \lambda^2 & \sim 1 \end{pmatrix}$$

$$|U_{MNS}| \sim \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & \epsilon \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Another Application:

Partially Unified FS Description of Quark and Lepton Mixings

Unified Description of quark and lepton mixings is desirable.

What feature do quark and lepton mixing matrices have in common?

- Each has at least one "small" element.
- What is the flavour-symmetric expression of this?

$$|V_{CKM}| \sim \begin{pmatrix} \sim 1 & \sim \lambda & \sim \lambda^3 \\ \sim \lambda & \sim 1 & \sim \lambda^2 \\ \sim \lambda^3 & \sim \lambda^2 & \sim 1 \end{pmatrix}$$

$$|U_{MNS}| \sim \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & \epsilon \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

≥ 1 zero $\Rightarrow$ CP-conservation $\Longrightarrow$ $J = 0$. But need two constraints.

FS Condition for One Mixing Zero

After some work, find

$$\implies 2\mathcal{A} + \mathcal{F}(\mathcal{F}^2 - 2\mathcal{C} - 1) = 0 \; \textit{and} \; J = 0$$

- Manifestly FS, but not “neat”

FS Condition for One Mixing Zero

After some work, find

$$\Longrightarrow \; 2\mathcal{A} + \mathcal{F}(\mathcal{F}^2 - 2\mathcal{C} - 1) = 0 \; \textit{and} \; J = 0$$

- Manifestly FS, but not "neat"
- Now, consider K-matrix: $K_{\gamma k} = \mathrm{Re}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j})$ (cf. J).
- Familiar from CP conserving parts of interference terms in penguin dominated rates

FS Condition for One Mixing Zero

After some work, find

$$\Longrightarrow\ 2\mathcal{A}+\mathcal{F}(\mathcal{F}^2-2\mathcal{C}-1)=0 \textit{ and } J=0$$

- Manifestly FS, but not "neat"
- Now, consider K-matrix: $K_{\gamma k}=\mathrm{Re}(U_{\alpha i}U^*_{\alpha j}U^*_{\beta i}U_{\beta j})$ (cf. J).
- Familiar from CP conserving parts of interference terms in penguin dominated rates
- In fact $2\mathcal{A}+\mathcal{F}(\mathcal{F}^2-2\mathcal{C}-1)\equiv 54\,\mathrm{Det}\,K$

FS Condition for One Mixing Zero

After some work, find

$$\Longrightarrow \; 2\mathcal{A} + \mathcal{F}(\mathcal{F}^2 - 2\mathcal{C} - 1) = 0 \; \textit{and} \; J = 0$$

- Manifestly FS, but not "neat"
- Now, consider K-matrix: $K_{\gamma k} = \mathrm{Re}(U_{\alpha i} U^*_{\alpha j} U^*_{\beta i} U_{\beta j})$ (cf. J).
- Familiar from CP conserving parts of interference terms in penguin dominated rates
- In fact $2\mathcal{A} + \mathcal{F}(\mathcal{F}^2 - 2\mathcal{C} - 1) \equiv 54\,\mathrm{Det}\,K$

So

$$\mathrm{Det}\,K = 0 \; \textit{and} \; J = 0$$

is condition we want

FS Condition for One Small Element

- For one small element, relax (slightly) one condition or both.
- For quark mixing, know that $J \neq 0$ (but small)

FS Condition for One Small Element

- For one small element, relax (slightly) one condition or both.
- For quark mixing, know that $J \neq 0$ (but small)
- How about $\mathrm{Det}K$?

$$|\mathrm{Det}\,K_q/(\mathrm{Det}\,K)_{max}| \lesssim 3 \times 10^{-7}$$

- Leptons also consistent with $\mathrm{Det}\,K$ small or zero.

FS Condition for One Small Element

- For one small element, relax (slightly) one condition or both.
- For quark mixing, know that $J \neq 0$ (but small)
- How about $\mathrm{Det} K$?

$$|\mathrm{Det}\, K_q / (\mathrm{Det}\, K)_{max}| \lesssim 3 \times 10^{-7}$$

- Leptons also consistent with $\mathrm{Det}\, K$ small or zero.
- Conjecture: MNS *and* CKM constrained according to the same FS condition:

$$\mathrm{Det} K = 0; \quad J \text{ small.}$$

Unified, partial description of quark and lepton mixings.

Predictions

- $\mathrm{Det}K = 0; J$ small $\implies$ as $J \to 0$, at least one UT angle $\to 90^\circ$.
- Get precise prediction for one unitarity angle in terms of other mixing elements.
- Doesn't tell us which angle, but the data do.

Predictions

- $\text{Det}K = 0; J \text{ small} \implies$ as $J \to 0$, at least one UT angle $\to 90^\circ$.
- Get precise prediction for one unitarity angle in terms of other mixing elements.
- Doesn't tell us which angle, but the data do.
- For quarks, predict:

$$-\cos\alpha \simeq (90^\circ - \alpha) = \overline{\eta}\lambda^2 = 1^\circ \pm 0.2^\circ$$

cf. $(90^\circ - \alpha) = 0^{\circ}{}^{+3^\circ}_{-7^\circ}$ experimentally.

Predictions

- $\mathrm{Det}K = 0; J$ small $\implies$ as $J \to 0$, at least one UT angle $\to 90^\circ$.
- Get precise prediction for one unitarity angle in terms of other mixing elements.
- Doesn't tell us which angle, but the data do.
- For quarks, predict:

$$-\cos\alpha \simeq (90^\circ - \alpha) = \overline{\eta}\lambda^2 = 1^\circ \pm 0.2^\circ$$

cf. $(90^\circ - \alpha) = 0^{\circ}{}^{+3^\circ}_{-7^\circ}$ experimentally.

- "A sub-1° measurement is possible in $B \to \rho\rho$ and $B \to \pi\pi$ individually"

  (A. Bevan 5th SuperB workshop, Paris, 2007).

Predictions

- $\mathrm{Det}K = 0$; J small $\Longrightarrow$ as $J \to 0$, at least one UT angle $\to 90^\circ$.
- Get precise prediction for one unitarity angle in terms of other mixing elements.
- Doesn't tell us which angle, but the data do.
- For quarks, predict:

$$-\cos\alpha \simeq (90^\circ - \alpha) = \overline{\eta}\lambda^2 = 1^\circ \pm 0.2^\circ$$

cf. $(90^\circ - \alpha) = 0^{\circ}{}^{+3^\circ}_{-7^\circ}$ experimentally.

- "A sub-1° measurement is possible in $B \to \rho\rho$ and $B \to \pi\pi$ individually" (A. Bevan 5th SuperB workshop, Paris, 2007).
- For leptons, predict:

$$|\cos\delta| \simeq |90^\circ - \delta| = 2\sqrt{2}\,\sin\theta_{13}\,\sin\left(\theta_{23} - \frac{\pi}{4}\right) \lesssim 4^\circ$$

Summary

- Have defined flavour-symmetric mass and/or mixing observables.

Summary

- Have defined flavour-symmetric mass and/or mixing observables.
- “Simplest” set defines itself up to normalisation

Summary

- Have defined flavour-symmetric mass and/or mixing observables.
- "Simplest" set defines itself up to normalisation
- Remarkably, leptonic mixing is consistent with 3 of these $= 0$!

Summary

- Have defined flavour-symmetric mass and/or mixing observables.
- "Simplest" set defines itself up to normalisation
- Remarkably, leptonic mixing is consistent with 3 of these $= 0$!
- Used them to implement FS constraints on flavour observables, and make testable conjectures

Summary

- Have defined flavour-symmetric mass and/or mixing observables.
- "Simplest" set defines itself up to normalisation
- Remarkably, leptonic mixing is consistent with 3 of these $= 0$!
- Used them to implement FS constraints on flavour observables, and make testable conjectures
- α_{CKM} should be $(89 \pm 0.2)^\circ$ - UT is *almost* right!

Summary

- Have defined flavour-symmetric mass and/or mixing observables.
- "Simplest" set defines itself up to normalisation
- Remarkably, leptonic mixing is consistent with 3 of these $= 0$!
- Used them to implement FS constraints on flavour observables, and make testable conjectures
- α_{CKM} should be $(89 \pm 0.2)^\circ$ - UT is *almost* right!
- $\delta_{leptons}$ should be within 4° of 90°.