An introduction to Quark Flavour Physics

Part 1

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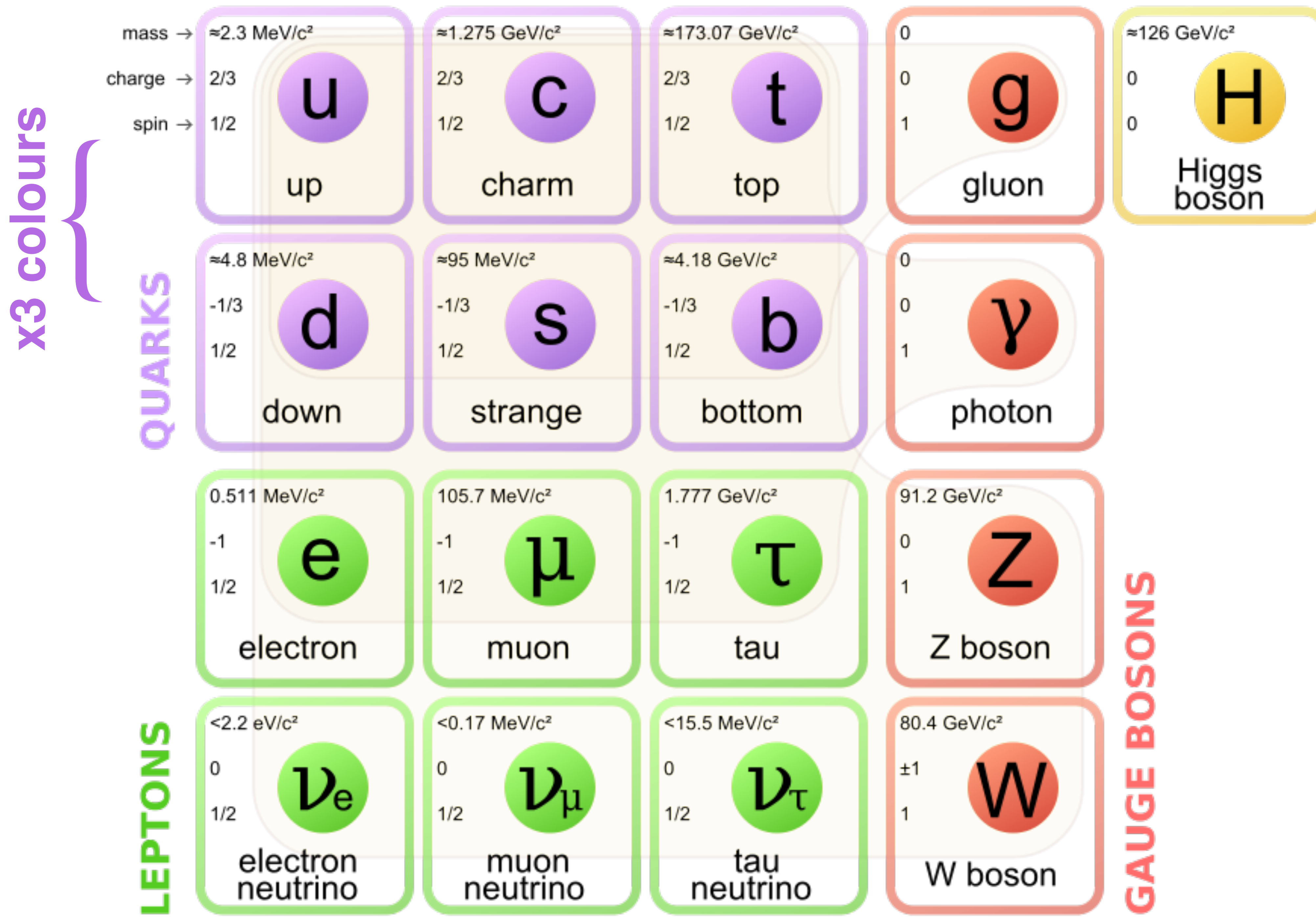
Warwick Week 2019

An introduction to Flavour Physics

- What's covered in these lectures:

1. **An introduction to flavour in the SM.**
   ➡ **A few concepts and a brief history of flavour physics.**
2. CP violation (part 1)
3. CP violation (part 2).
4. Flavour changing neutral current processes.

SM particle content



Flavour in the SM

- Particle physics can be described to excellent precision by a very simple theory:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Gauge}(A_a, \psi_i) + \mathcal{L}_{\rm Higgs}(\phi, A_a, \psi_i)$$

  with:

  - ➡ Gauge terms that deals with the free fields and their interactions by the strong and electroweak interactions.
  - ➡ Higgs terms that gives mass to the SM particles.

Flavour in the SM

- The Gauge part of the Lagrangian is experimentally very well verified,

$$\mathcal{L}_{\rm Gauge} = \sum_{j,\psi} i\bar{\psi}_j \not{D} \psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a} \ .$$

- The fields are arranged a left-handed doublets and right-handed singlets

$$\psi = Q_{\rm L}, u_{\rm R}, d_{\rm R}, L_{\rm L}, e_{\rm R}$$

$$Q_{\rm L} = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \ , \ L_{\rm L} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

- The Lagrangian is invariant under a specific set of symmetry groups: ${\rm SU}(3)_c \times {\rm SU}(2)_{\rm L} \times {\rm U}(1)_Y$

- There are three replicas of the basic fermion families, which without the Higgs would be identical (huge degeneracy).

Flavour in the SM

- The Higgs part of the Lagrangian, on the other hand, is much more ad-hoc. It is necessary to understand the data but is not stable with respect to quantum corrections (often referred to as the Hierarchy problem).

  It is also the origin of the flavour structure of the SM.

- Masses of the fermions are generated by the Yukawa mechanism:

$$\bar{Q}^i_L Y^{ij}_{\mathrm{D}} d^j_{\mathrm{R}} \phi + \ldots \to \bar{d}^i_{\mathrm{L}} M^{ij}_{\mathrm{D}} d^j_{\mathrm{R}} + \ldots$$
$$\bar{Q}^i_L Y^{ij}_{\mathrm{U}} d^j_{\mathrm{R}} \phi_c + \ldots \to \bar{u}^i_{\mathrm{L}} M^{ij}_{\mathrm{U}} u^j_{\mathrm{R}} + \ldots$$

Flavour in the SM

- Can pick a basis in which one of the Yukawa matrices is diagonal, *e.g.*

$$Y_{\rm D} = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \;,\; Y_{\rm U} = V^\dagger \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \;,\; y_i \approx \frac{m_i}{174 \rm GeV}$$

- The matrix V is complex and unitary ($V^\dagger V = 1$).
- To diagonalise both mass matrices it is necessary to rotate u_L and d_L separately. Consequently, V also appears in charged current interactions, $J^\mu_W = \bar{u}_{\rm L}\gamma^\mu d_{\rm L} \rightarrow \bar{u}_{\rm L} V \gamma^\mu d_{\rm L}$
- V is known as the Cabibbo, Kobayashi, Maskawa matrix.

Flavour in the SM

mass eigenstates ≠ weak eigenstates

$$d, s, b \leftrightarrow d', s', b'$$

Lepton and baryon number

- SM Lagrangian is invariant under U(3) symmetries of the left-handed doublets and right-handed singlets if fermions are massless.

$$\mathcal{L}_{\text{Gauge}} = \sum_{j,\psi} i\bar{\psi}_j \not{D} \psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a} \ .$$

- U(3) symmetries are broken by the Yukawa terms, the only remaining symmetries correspond to lepton and baryon number conservation.
- These are “accidental” symmetries, coming from the particle content rather than being imposed.

Free parameters of the SM

- 3 gauge couplings
- Higgs mass and vacuum expectation value

Flavour parameters

- 6 quark masses
- 3 quark mixing angles and one complex phase (in V)
- 3 charged lepton masses
- 3 neutrino masses
- 3 lepton mixing angles and (at least) one complex phase

Why is flavour important?

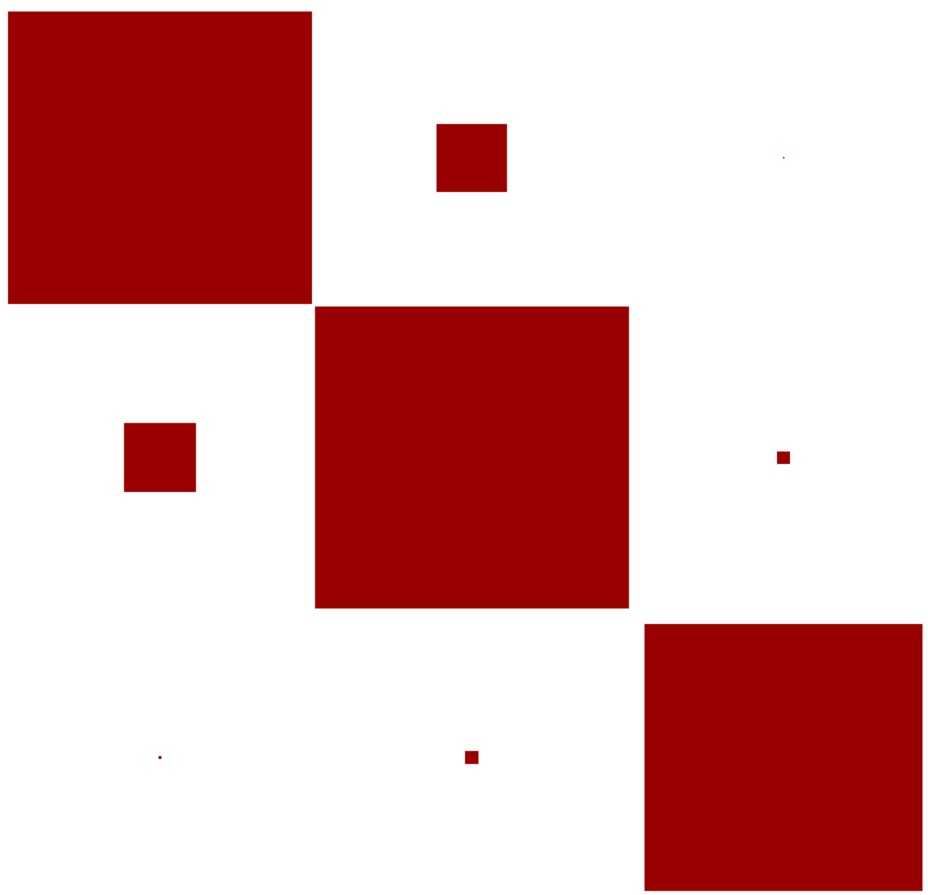
- Most of the free parameters of the SM are related to the flavour sector.
- The flavour sector provides the only source of CP violation in the SM.
- Flavour changing neutral current processes can probe mass scales well beyond those accessible at LHC.
  - ➡ If there are new particles at the TeV-scale, why don't they manifest themselves in FCNC processes (**the so-called flavour problem**)?

Puzzles?

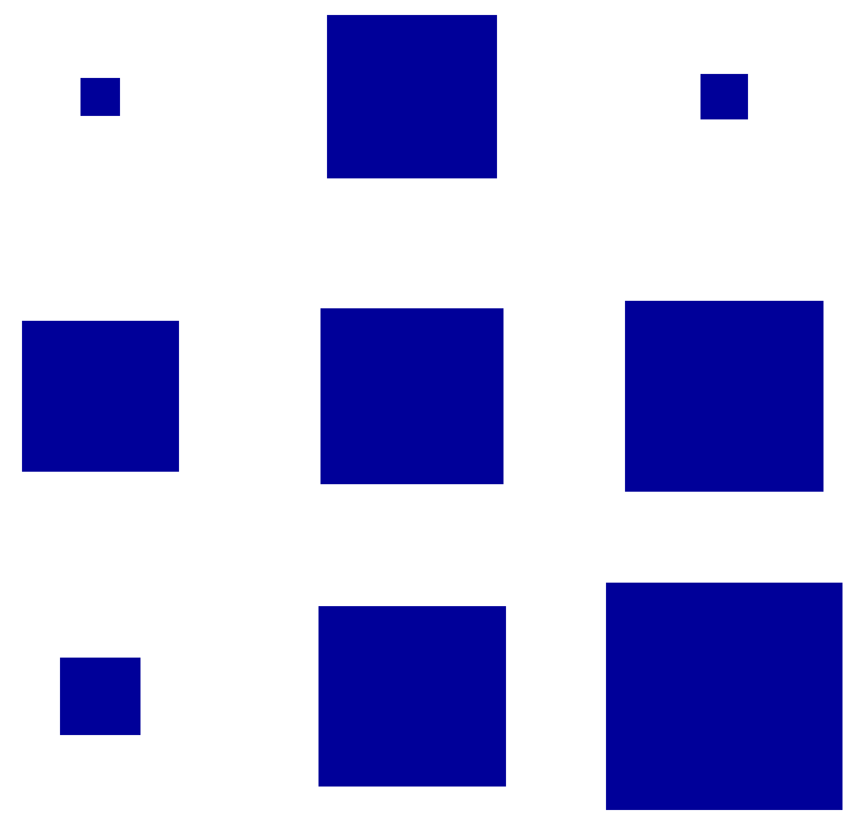
- Why are there as so many parameters and why do they have the values they do?
- Why do we have a flavour structure with three generations?
    - ➡ We know we need 3+ generations to get CP violation. Are there more generations to discover?
- Why do the quarks have a flavour structure that exhibits both smallness and hierarchy?
    - ➡ Why is the neutrino flavour sector so different (neither small nor hierarchical)?

Flavour hierarchy?

CKM matrix for quark sector

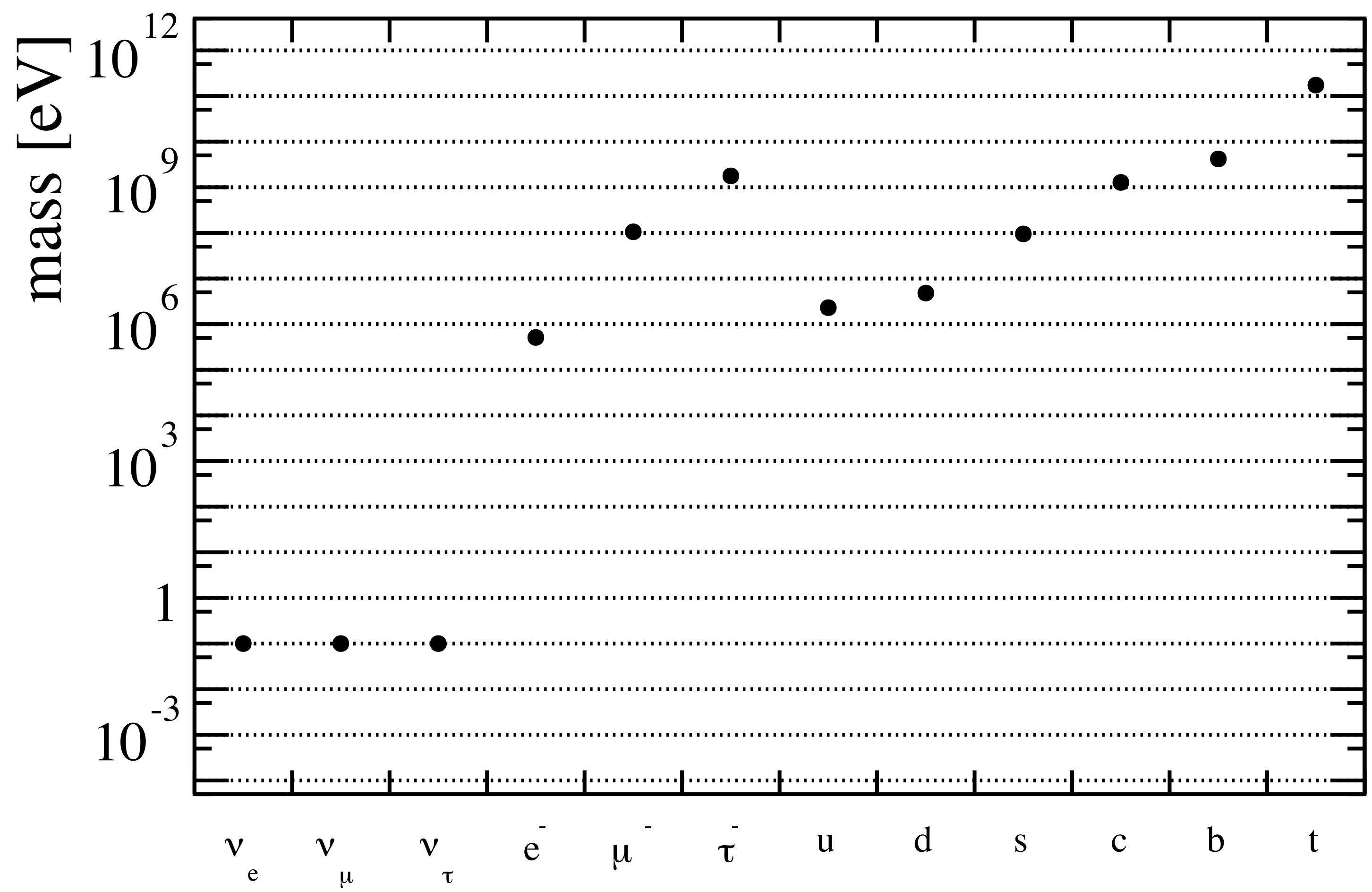


PMNS matrix for neutrino sector



Mass hierarchy?

- Large hierarchy in scale between the masses of the fermions.
- Equivalent to having a large hierarchy in the Yukawa couplings.
- Why is this hierarchy so large? Why is $y_t \sim 1$?



Symmetries and flavour

Historical perspective on flavour in the SM

Isospin

- Proton and neutron have:
  - ➡ Different charges but almost identical masses.
  - ➡ An identical coupling to the strong force.
- In 1932 Heisenberg proposed that they are both part of the an Isospin doublet and can be treated as the same particle with different isospin projections.

$$p(I, I_Z) = (\tfrac{1}{2},\tfrac{1}{2}), \quad n(I,I_Z) = (\tfrac{1}{2},-\tfrac{1}{2})$$

- Pions can be arranged as an Isospin triplet,

$$\pi^+(I, I_Z) = (1,+1),\ \pi^0(I, I_Z) = (1, 0)\ ,\ \pi^-(I, I_Z) = (1,-1)$$

Isospin

- Heisenberg proposed that strong interactions are invariant under rotations in Isospin space (an SU(2) invariance).
  - ➡ Isospin was conserved in strong interactions.
  - ➡ Isospin is violated in weak interactions.

- We now know that this is not the correct model but it is still a very useful concept.

$$\sigma(p + p \to d + \pi^+) : \sigma(p + n \to d + \pi^0) = 2 : 1$$

- It works because $m_u \sim m_d < \Lambda_{QCD}$ and can be used to predict interaction rates.

Clebsch-Gordon coefficients

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

1/2×1/2

m_1 m_2	1, +1
+1/2 +1/2	1

m_1 m_2	1, 0	0, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	1, −1
−1/2 −1/2	1

1×1/2

m_1 m_2	3/2, +3/2
+1 +1/2	1

m_1 m_2	3/2, +1/2	1/2, +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	3/2, −1/2	1/2, −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	3/2, −3/2
−1 −1/2	1

2×1

m_1 m_2	3, +3
+2 +1	1

m_1 m_2	3, +2	2, +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	3, +1	2, +1	1, +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	3, 0	2, 0	1, 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	3, −1	2, −1	1, −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	3, −2	2, −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	3, −3
−2 −1	1

1×1

m_1 m_2	2, +2
+1 +1	1

m_1 m_2	2, +1	1, +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	2, 0	1, 0	0, 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	2, −1	1, −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	2, −2
−1 −1	1

3/2×1

m_1 m_2	5/2, +5/2
+3/2 +1	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	5/2, −3/2	3/2, −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	5/2, −5/2
−3/2 −1	1

2×1/2

m_1 m_2	5/2, +5/2
+2 +1/2	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	5/2, +1/2	3/2, +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	5/2, −1/2	3/2, −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	5/2, −3/2	3/2, −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	5/2, −5/2
−2 −1/2	1

3/2×1/2

m_1 m_2	2, +2
+3/2 +1/2	1

m_1 m_2	2, +1	1, +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	2, 0	1, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	2, −1	1, −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	2, −2
−3/2 −1/2	1

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

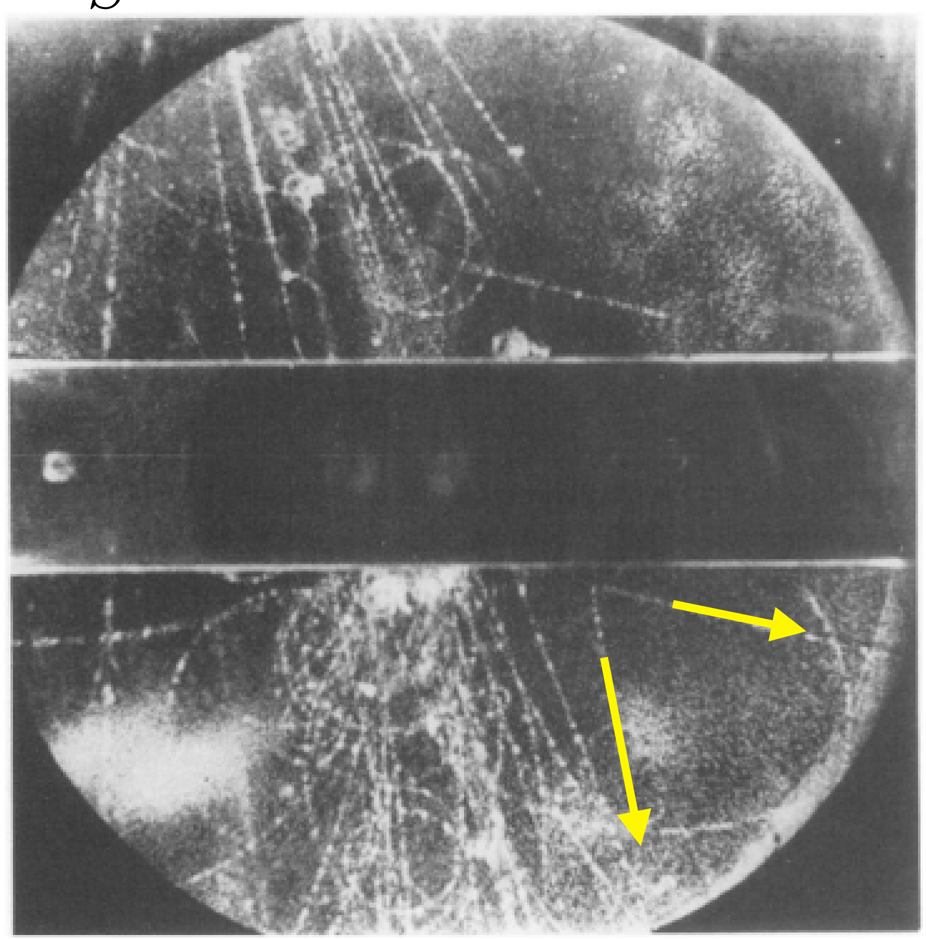
$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

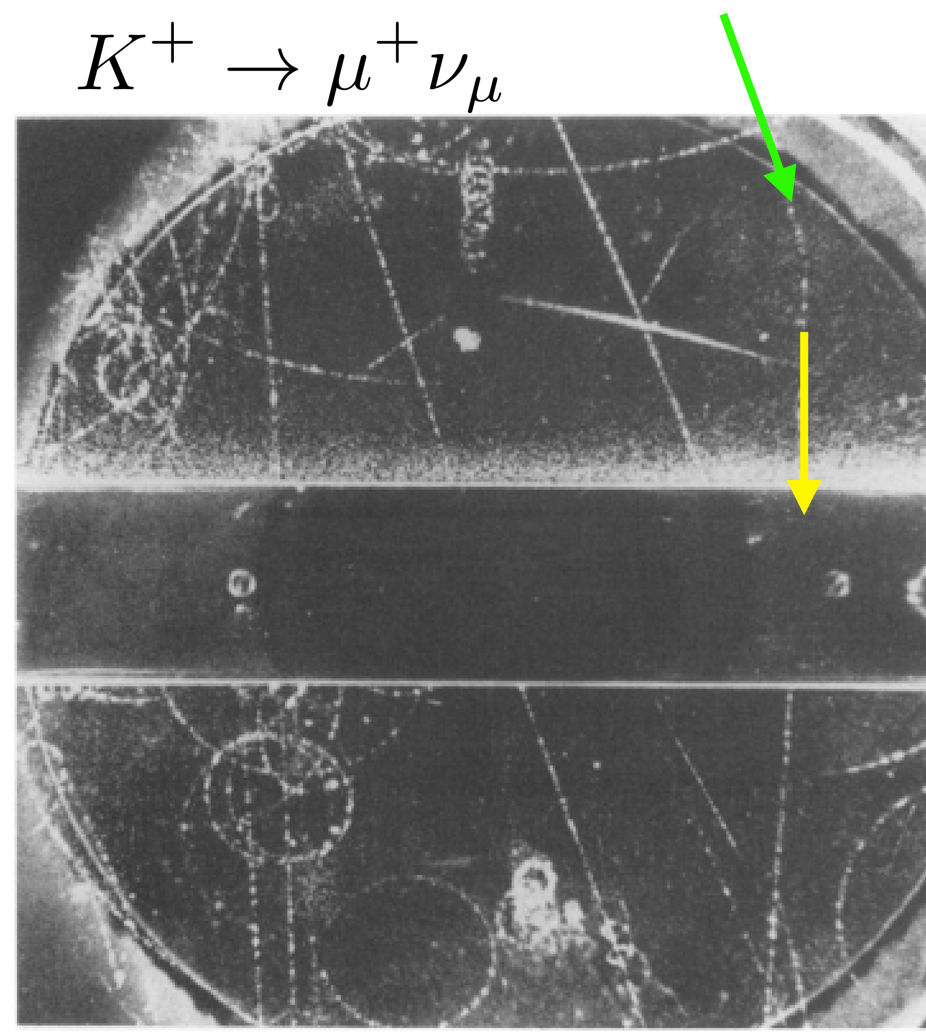
Kaon observation

- In 1947, Rochester and Butler observed two new particles with masses around 500 MeV and relatively long lifetimes.

$K_S^0 \to \pi^+\pi^-$



$K^+ \to \mu^+\nu_\mu$



Quark model

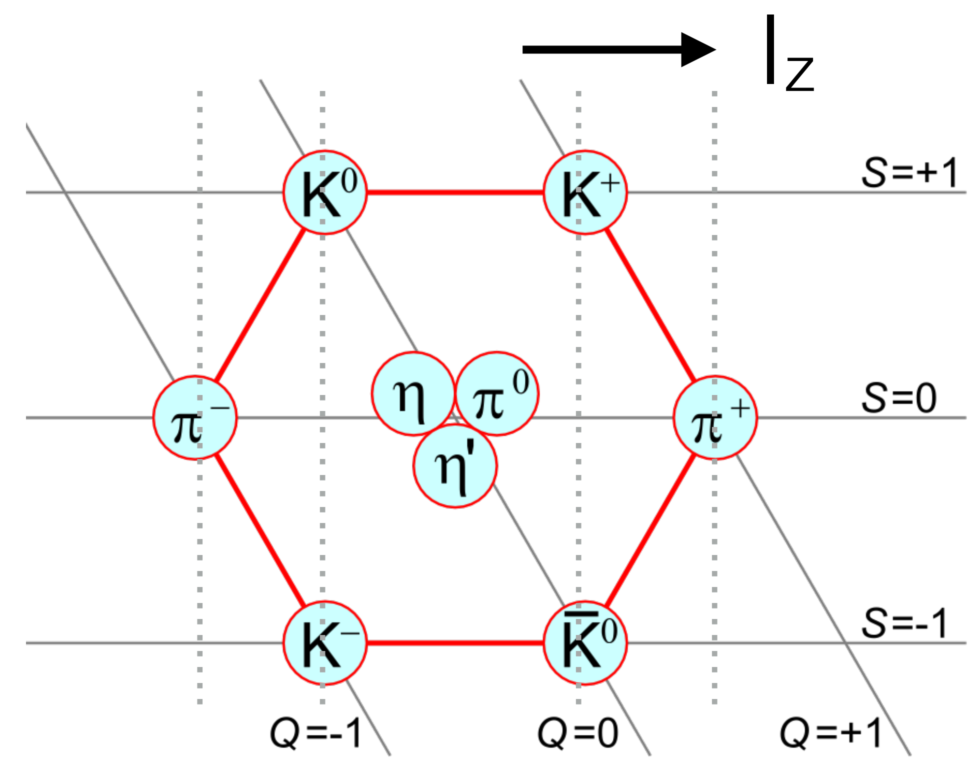
- Concept of quarks introduced by Gell-Mann, Nishijima and Ne'eman to explain the "zoo" of particles.
- Now understood to be real "fundamental" particles.
- Strangeness conserved by the strong interaction but violated in weak decays.
- Organised by

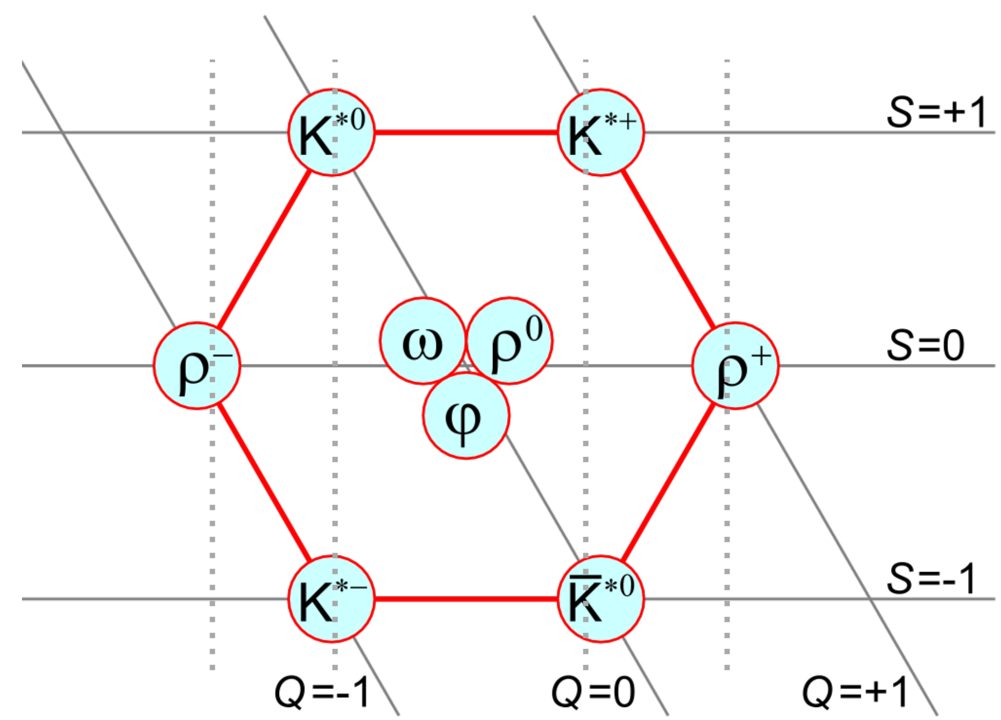
$$Y = B + S \quad Q = e(I_z + Y/2)$$

baryon number

strangeness

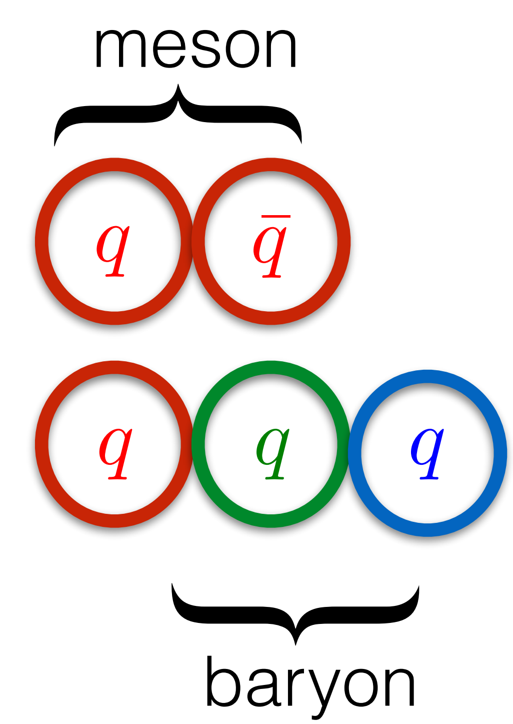
Isospin





Quark model

- Can only make colour neutral objects:
  - ➡ Quark anti-quark or three quark combinations (mesons and baryons).



SU(2) flavour mixing

- Four possible combinations from u and d quarks

$$u\overline{u}, d\overline{d}, u\overline{d}, d\overline{u}$$

- Under SU(2) symmetry,
  - ➡ π^0 is a member of an isospin triplet, $\boldsymbol{\eta}$ is a isosinglet

$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d}), \quad \eta = \frac{1}{\sqrt{2}}(u\overline{u} + d\overline{d})$$

SU(3) flavour mixing

- Introducing the strange quark, under SU(3) symmetry we now have an octuplet and a singlet.

$$\pi^0 = \frac{1}{\sqrt{2}}(u\overline{u} - d\overline{d}), \quad \eta_1 = \frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s}), \quad \eta_8 = \frac{1}{\sqrt{6}}(u\overline{u} + d\overline{d} - 2s\overline{s})$$

- Physical states involve a further mixing

$$\eta = \eta_1 \cos\theta + \eta_8 \sin\theta \quad \eta' = -\eta_1 \sin\theta + \eta_8 \cos\theta$$

Quark model

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A SCHEMATIC MODEL OF BARYONS AND MESONS *

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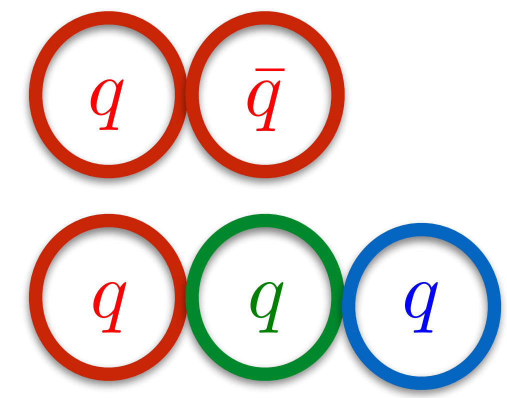
If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

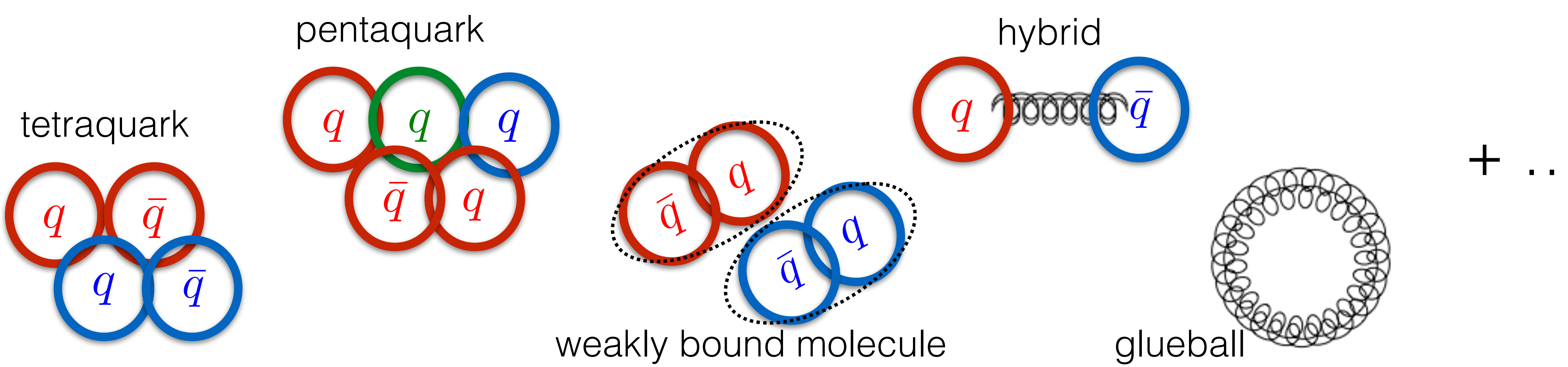
Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

Quark model

- Can only make colour neutral objects:
  - ➡ quark anti-quark or three quark combinations (mesons and baryons). Nearly all known particles fall into one of these two categories.
  - ➡ Can also build colour neutral states containing more quarks (e.g. 4 or 5 quark states).





Exotic charmonium states

- Several exotic states have recently been discovered:
  - ➡ X(3872) by CDF, $Z(4430)^+$ and Y(4140) by Belle etc
- These states decay to charmonia and have masses below the *b*-quark mass.
  - ➡ $Z(4430)^+$ is charged, therefore has minimal quark content $c\bar{c}u\bar{d}$
- States could be weakly bound molecular $D\bar{D}$ states or genuine four quark states or an admixture of the two.

Dalitz plot formalism

- Often analyse three body decays in terms off the Dalitz plot formalism.

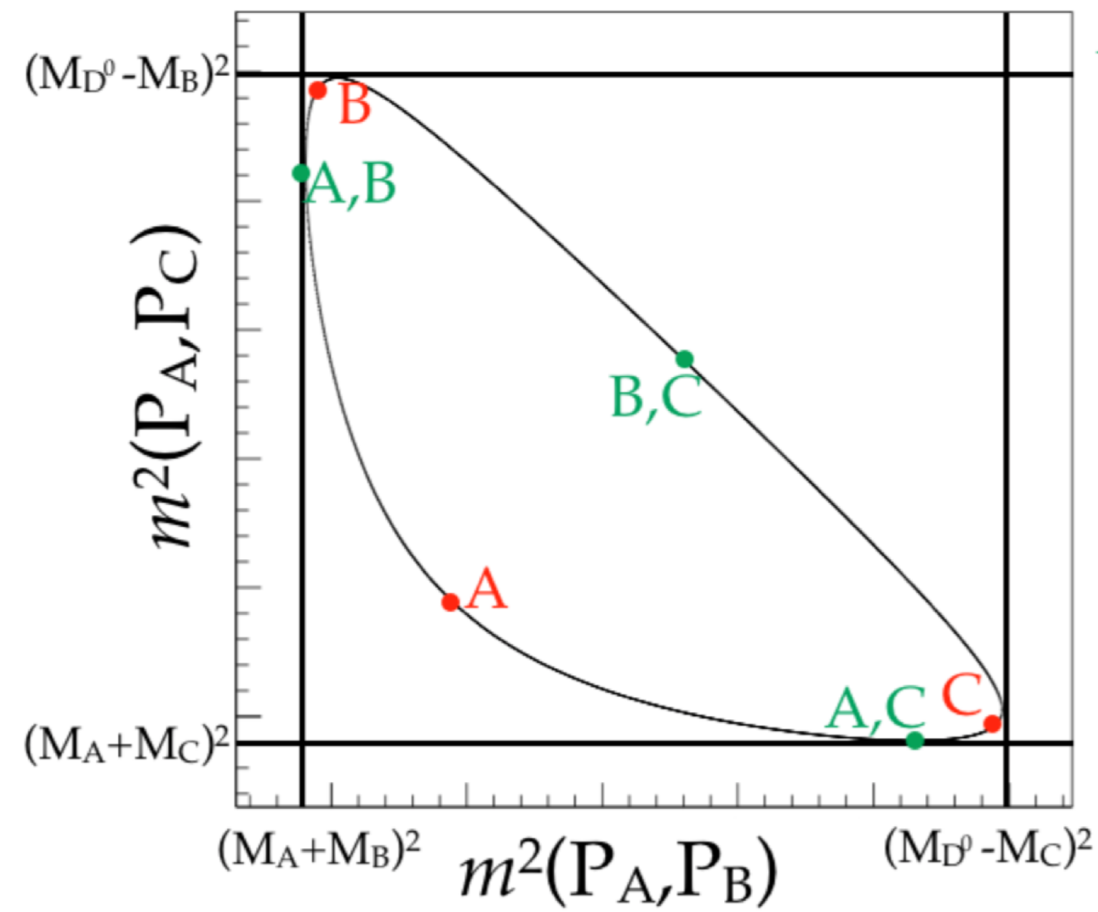
- *n*-body decay rate:

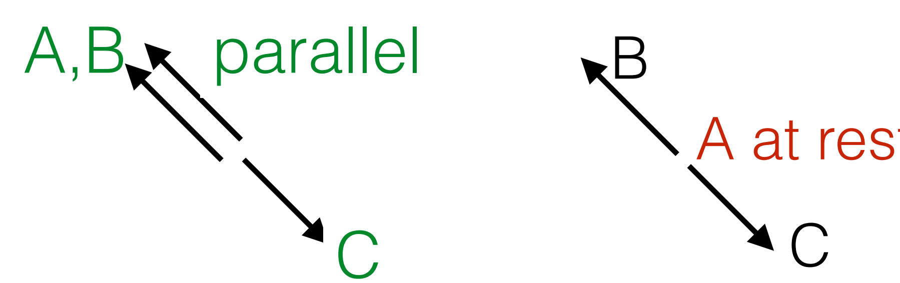
$$\mathrm{d}\Gamma = \frac{(2\pi)^4}{2M}|\mathcal{M}|^2\mathrm{d}\phi(p_1\ldots p_n)$$

- For a 3-body decay:

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\overline{|\mathcal{M}|}^2\mathrm{d}m^2_{12}\mathrm{d}m^2_{23}$$

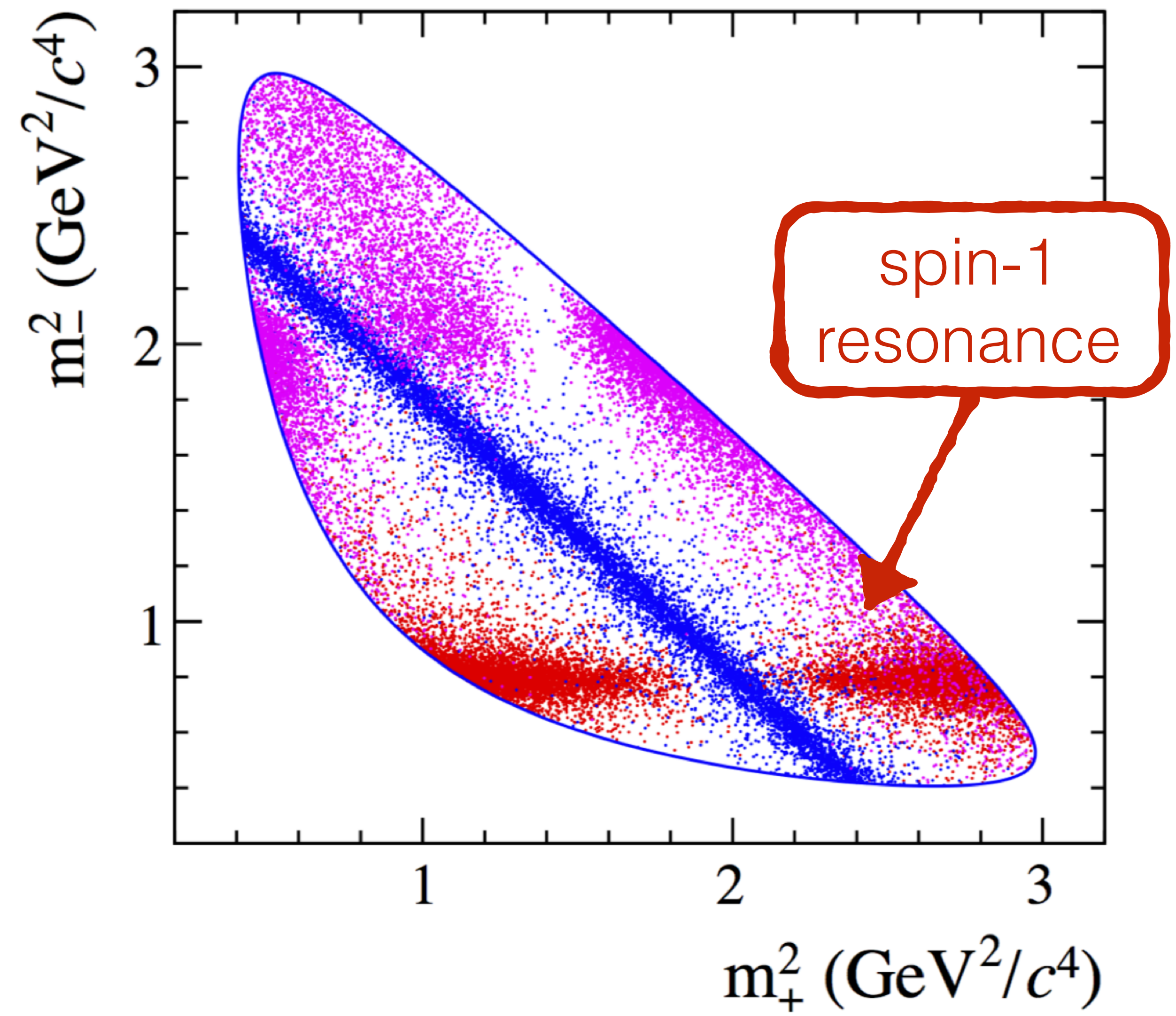
ie 3-body phasespace is flat in the Dalitz plot.





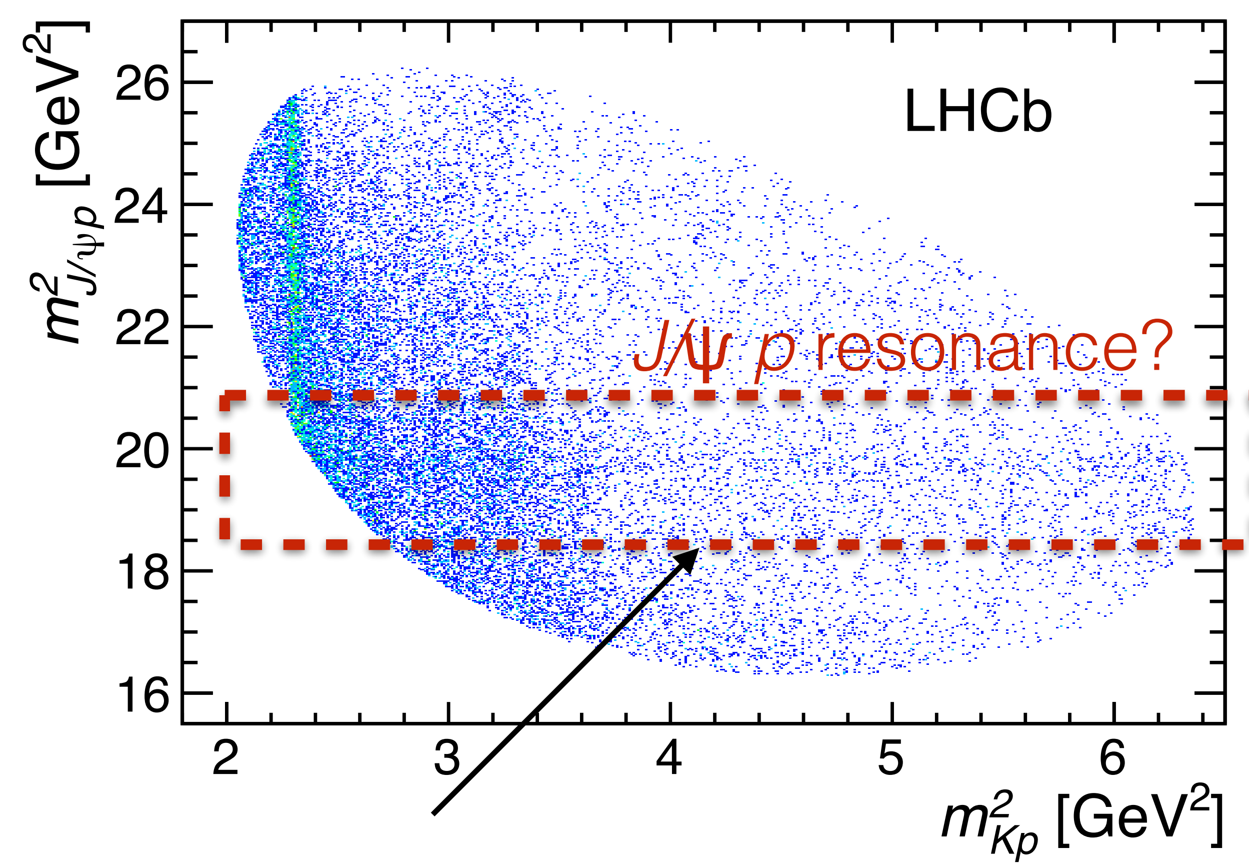
Dalitz plot formalism

- Often analyse three body decays in terms off the Dalitz plot formalism.
- Resonances appear as bands in the Dalitz plot.
  - ➡ Number of lobes is related to spin of the resonance.



Pentaquark discovery

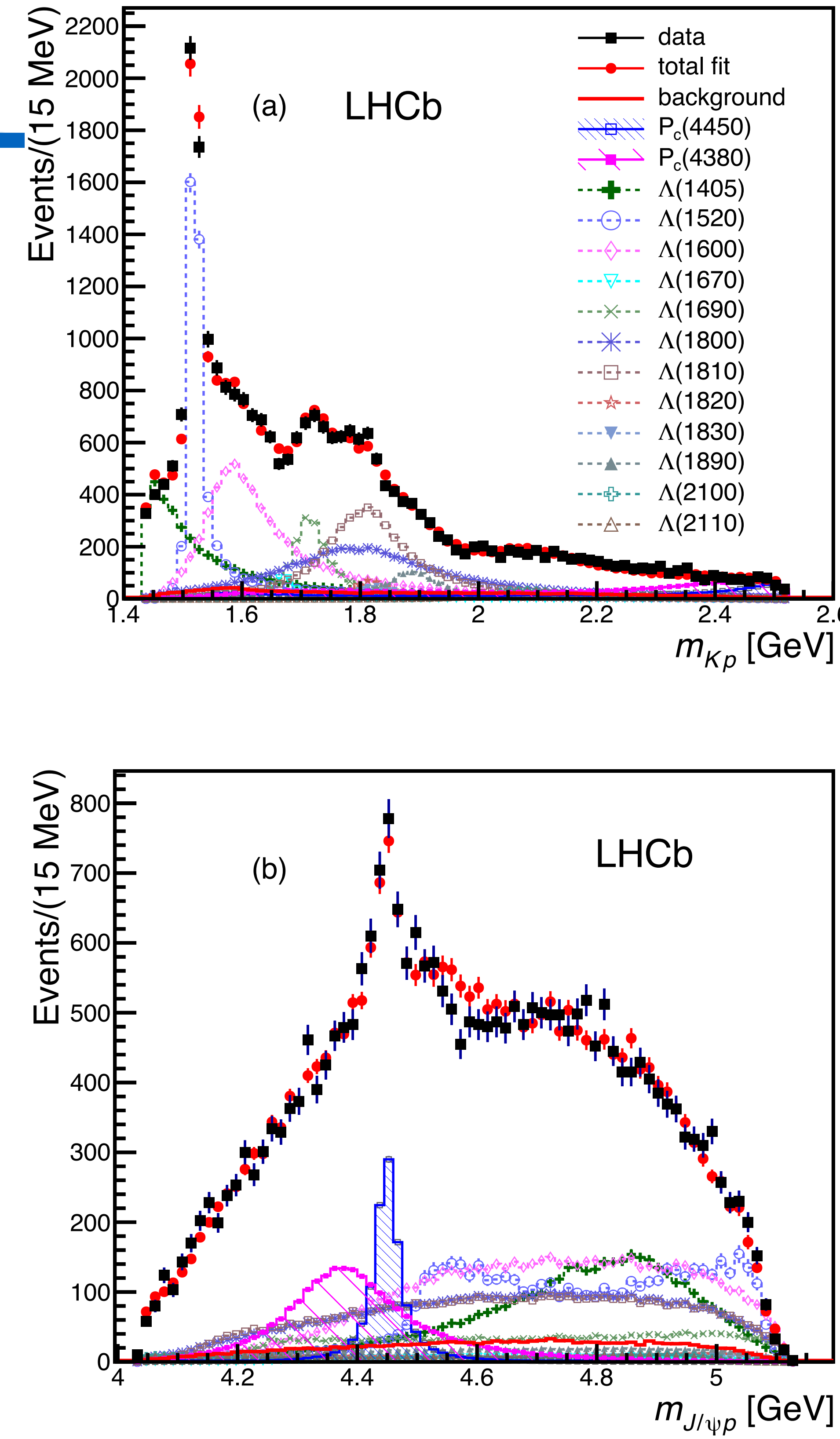
- In $\Lambda_b \to J/\psi\, pK^-$ decays, the LHCb experiment sees a resonant contribution to the $J/\psi\, p$ mass.
- This contribution would have minimal quark content $c\bar{c}uud$
- To understand what the contribution is, need to perform an amplitude analysis of the $J/\psi\, pK^-$ system.



[LHCb, Phys. Rev. Lett. 115 (2015) 072001]

Pentaquark

- Perform an amplitude analysis of the $J/\psi\, pK^-$ system allowing for contributions from all known Λ resonances.
- Data can be described by introducing two new $J/\psi\, p$ states.
  - ➡ States have opposite parity, one is wide with J = 3/2 and the other narrow with J = 5/2.



[LHCb, Phys. Rev. Lett. 115 (2015) 072001]

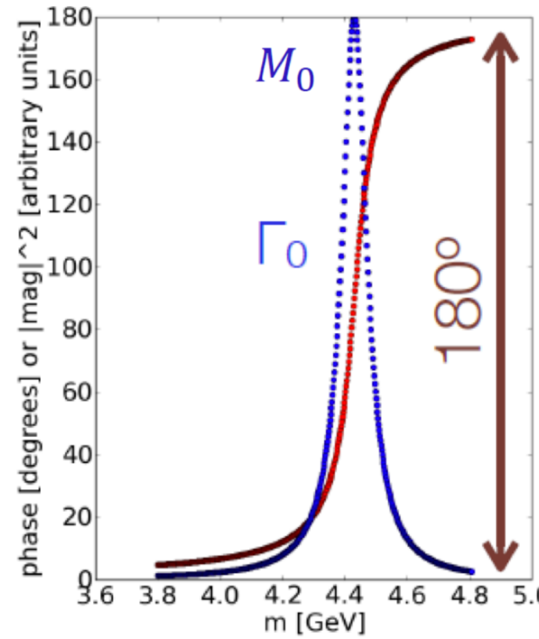
Pentaquark

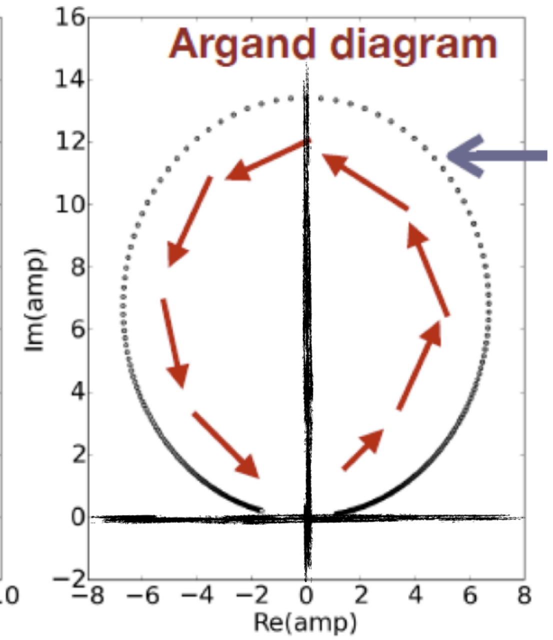
- How can we be sure that these really are genuine new states?
- Resonances should have a pole:
  - ➡ Exploit the phase rotation of a BW.
- From amplitude:

$$\mathcal{A}(t) = \exp(-i(E - E_0)t - \Gamma t/2)$$

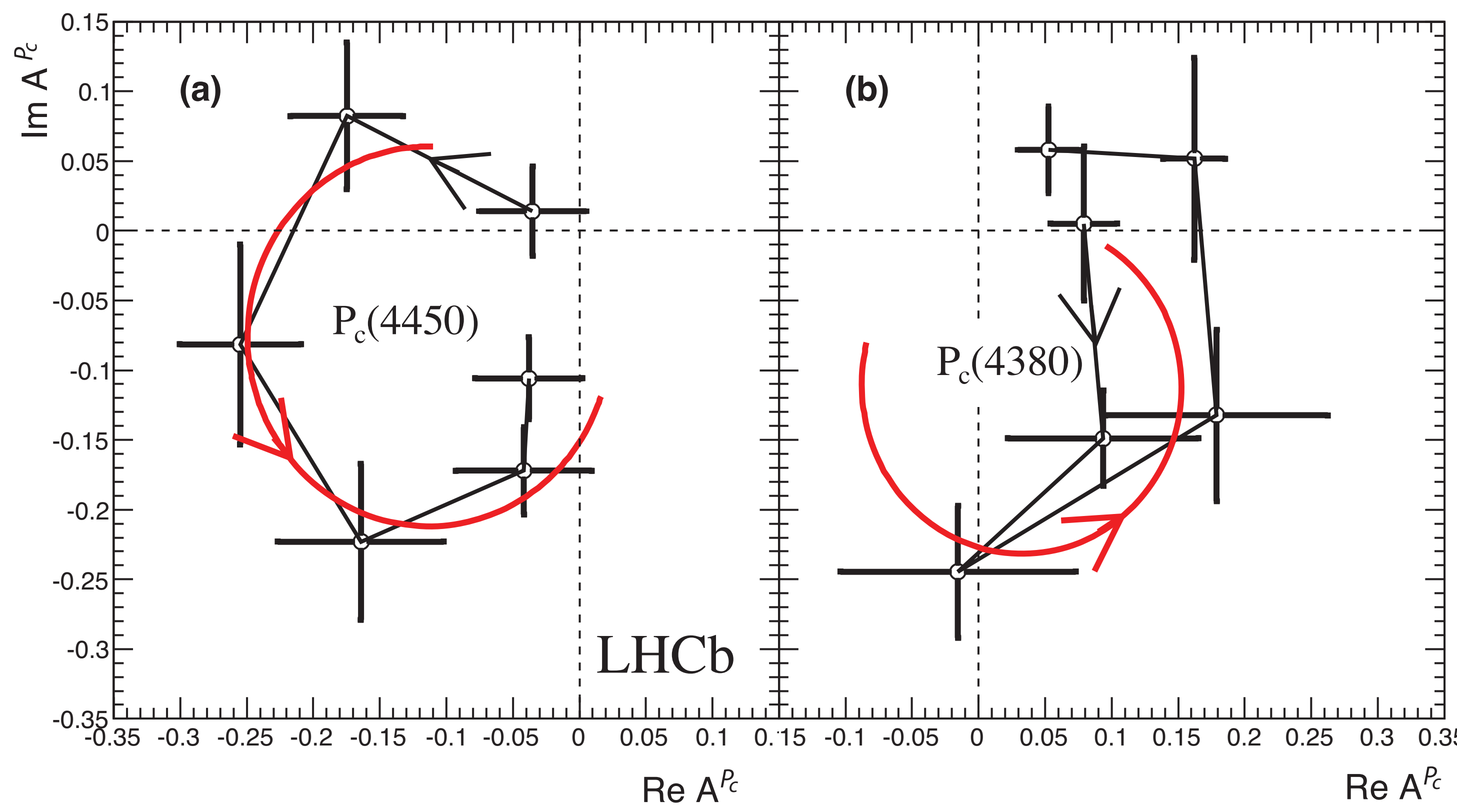
- Fourier transform is:

$$\mathcal{A}(E) \propto \frac{1}{E - E_0 - i\Gamma/2}$$





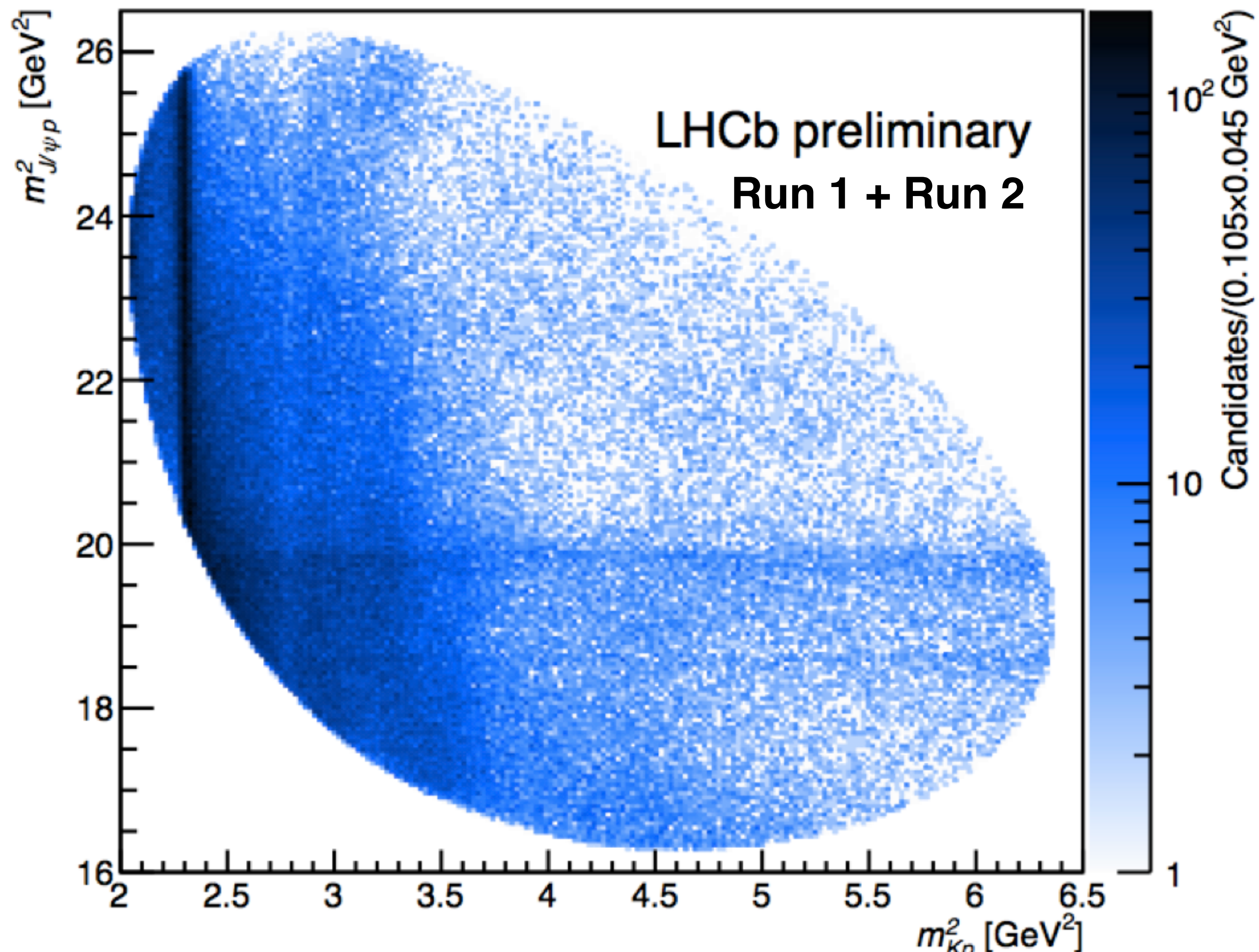
phase rotation



[LHCb, Phys. Rev. Lett. 115 (2015) 072001]

A more complicated picture

- New result combining Run 1 and Run 2 data.
- Shows a new narrow state $P_c(4312)^+$.
- Previous $P_c(4450)^+$ state resolved into two narrower states.
- Two of the states are near the $\Sigma_c^+ \bar{D}^0$ and the $\Sigma_c^+ \bar{D}^{*0}$ thresholds.



[LHCb, arXiv:1904.03947]

Cabibbo angle

- The quark content of the K^+ and K^0 are $(\bar{s}u)$ and $(\bar{s}d)$
- The main decays of the K^+ are

$$K^+ \to \mu^+ \nu_\mu \quad \text{and} \quad K^+ \to \pi^0 e^+ \nu_e$$

i.e. it decays via the charged current interaction.

- The charged current interaction couples to left-handed doublets, therefore need to construct a doublet that allows $s \to u$ and $d \to u$.
- Cabibbo proposed a solution in terms of quark mixing

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}$$

Cabibbo angle

- The quark mixing angle, θ_C, is determined experimentally to be

$$\sin\theta_C \approx 0.22$$

- Cabibbo's proposed solution also explained a discrepancy between the weak coupling constant between muon decays and nuclear decay.

- However, this opened up a new problem, why is

$$\Gamma[K^+ \to \mu\nu] \gg \Gamma[K_L^0 \to \mu^+\mu^-] \quad ?$$

- If the doublet of the weak interaction is the one Cabibbo suggested, can have neural currents

$$J_\mu^0 = \bar{d}'\gamma_\mu(1-\gamma_5)d'$$

which introduces tree level FCNCs.

GIM mechanism

- Expanding, $J^0_\mu = \bar{s}\gamma_\mu(1-\gamma_5)s\sin^2\theta_C + \bar{d}\gamma_\mu(1-\gamma_5)d\cos^2\theta_C$
$$+ \bar{s}\gamma_\mu(1-\gamma_5)d\sin\theta_C\cos\theta_C$$
$$+ \bar{d}\gamma_\mu(1-\gamma_5)s\sin\theta_C\cos\theta_C$$

- So was Cabibbo wrong? Glashow, Iliopoulos and Maiani provided a solution in 1970 by adding a second doublet

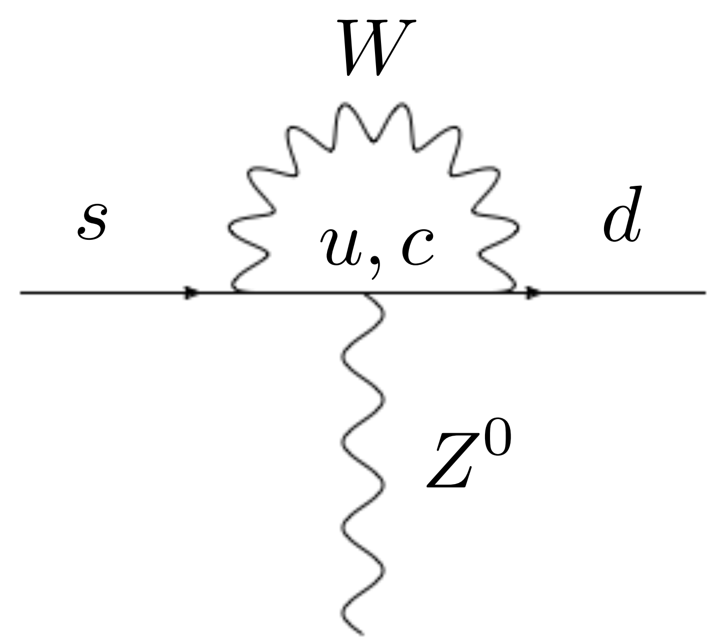
$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix} , \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d\sin\theta_C + s\cos\theta_C \end{pmatrix}$$

- The second doublet exactly cancels the FCNC term.

Quark mixing led to the prediction of the charm quark.

GIM at loop order?

- For “strange” decays still have an effective GIM suppression.



- 2 x 2 unitarity implies

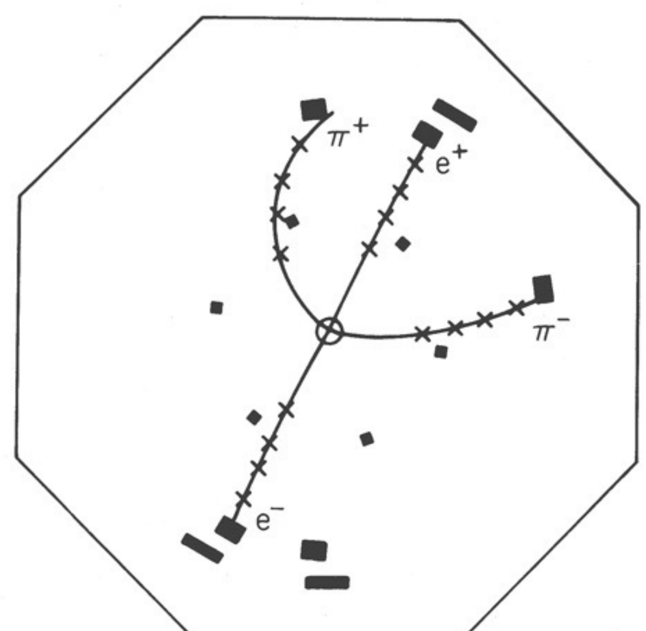
$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} = 0 \qquad \mathcal{A} \approx 0$$

- FCNC decays are very rare,

$$\mathcal{B}(K_L^0 \to \mu^+\mu^-) = (6.8 \pm 0.1) \times 10^{-9}$$

Observation of J/ψ

- Experimental evidence for charm quark came in 1974.
- Discovery of charmonium (J) at Brookhaven in p Be $\rightarrow e^+e^-X$.
- Discovery of charmonium (ψ) at SLAC in $e^+e^- \rightarrow$ hadrons, e^+e^-, $\mu^+\mu^-$



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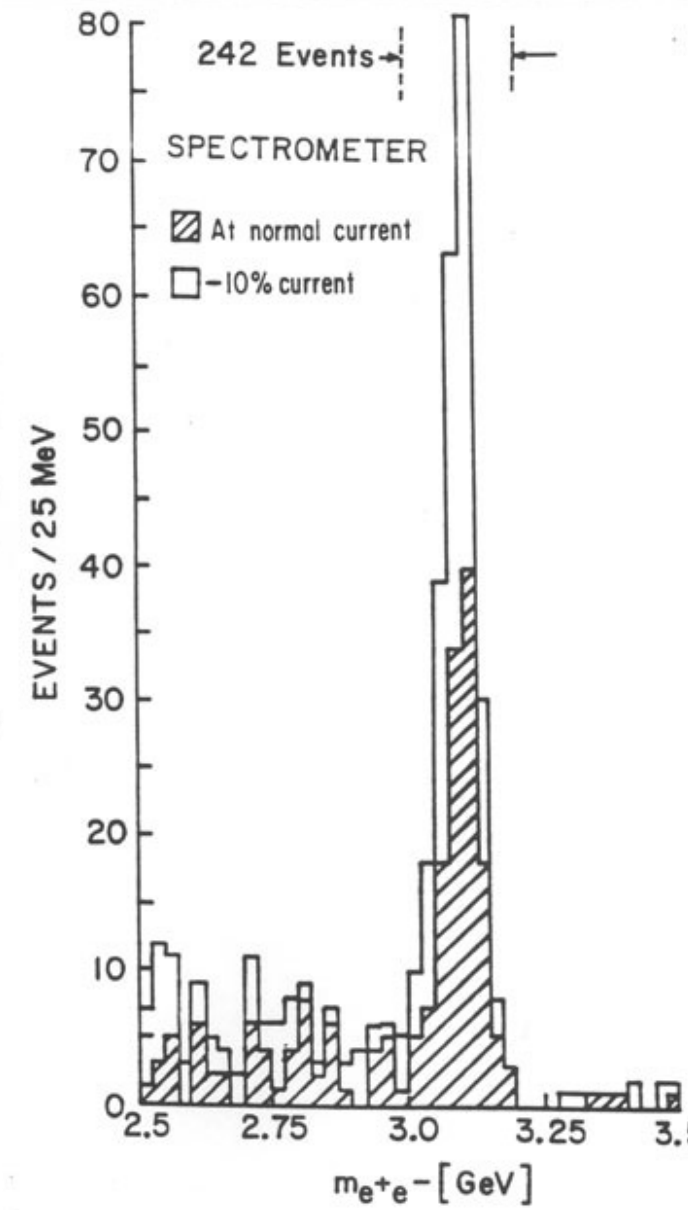


FIG. 2. Mass spectrum showing the existence of J. sults from two spectrometer settings are plotted wing that the peak is independent of spectrometer rents. The run at reduced current was taken two nths later than the normal run.

θ-τ puzzle

- Two decays were found for charged strange mesons

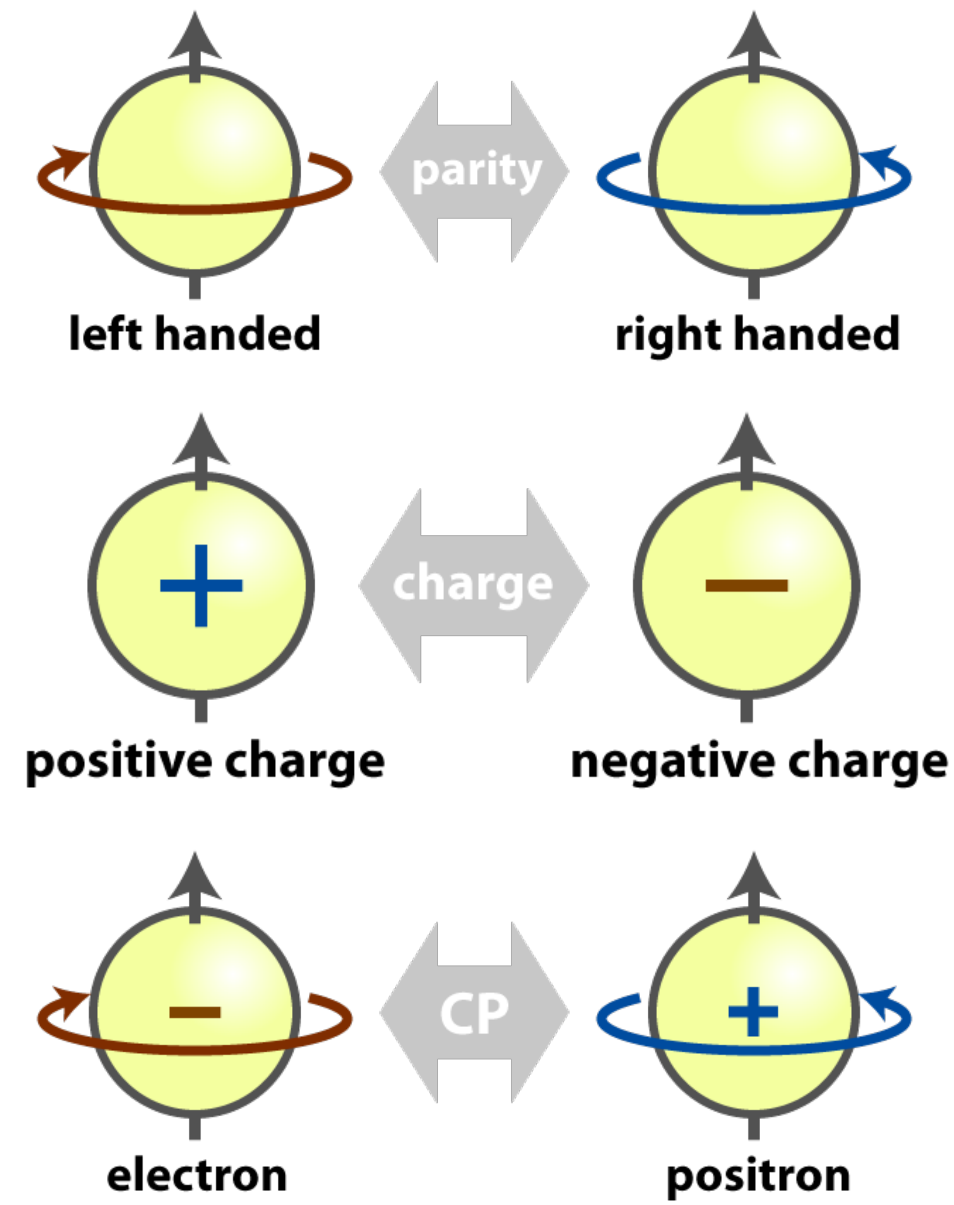
$$\theta^+ \to \pi^+\pi^0$$
$$\tau^+ \to \pi^+\pi^-\pi^+$$

- The θ and τ had the same mass and lifetime, but the parity of 2π and 3π is different.

  ➡ Resolution is that the θ and τ are the same particle and parity is violated in the decay.

C and P

- Prior to 1956, it was thought that the laws of physics were invariant under parity, i.e. mirror image of a process is also a valid physical process.
  - ➡ Shown to be violated in β-decays of Co-60 by C. S. Wu (following an idea by T. D. Lee an C. N. Yang).
- Now know that Parity is maximally violated in weak decays.
  - ➡ No right-handed neutrinos.
- C is also maximally violated in weak decays.
  - ➡ No left-handed anti-neutrino.
- The product CP, distinguishes between matter and anti-matter.



Neutral kaon system

- Ignoring CP violation, the two physical states in the neutral kaon system are

$$|K_1\rangle = \frac{|K^0\rangle - |\overline{K}^0\rangle}{\sqrt{2}} \quad \text{and} \quad |K_2\rangle = \frac{|K^0\rangle + |\overline{K}^0\rangle}{\sqrt{2}}$$

under Parity and Charge Conjugation

$$\mathcal{P}|K^0\rangle = -|K^0\rangle \quad , \quad \mathcal{C}|K^0\rangle = |\overline{K}^0\rangle \quad \text{and} \quad \mathcal{CP}|K^0\rangle = -|\overline{K}^0\rangle$$

- For the physical states

$$\mathcal{P}|K_{1,2}\rangle = -|K_{1,2}\rangle \quad , \quad \mathcal{C}|K_{1,2}\rangle = \mp|K_{1,2}\rangle \quad \text{and} \quad \mathcal{CP}|K_{1,2}\rangle = \pm|K_{1,2}\rangle$$

i.e. they are P, C and CP eigenstates as well.

Neutral kaon system

- What does this tell us about their decays?

$\pi^+\pi^-$

➡ P = +1, C = +1 and CP = +1 } shorter lived K_1

$\pi^+\pi^-\pi^0$

➡ P = -1, C = + 1 and CP = -1 } longer lived K_2

- K_2 decays to 3π but the 2π decay would be forbidden if CP is conserved.

CP violation in the kaon system

- In 1964, Christensen, **Cronin**, **Fitch** and Turlay observed 2π decays of K_2 mesons (K_L).

Observation of CP violation in the kaon system.

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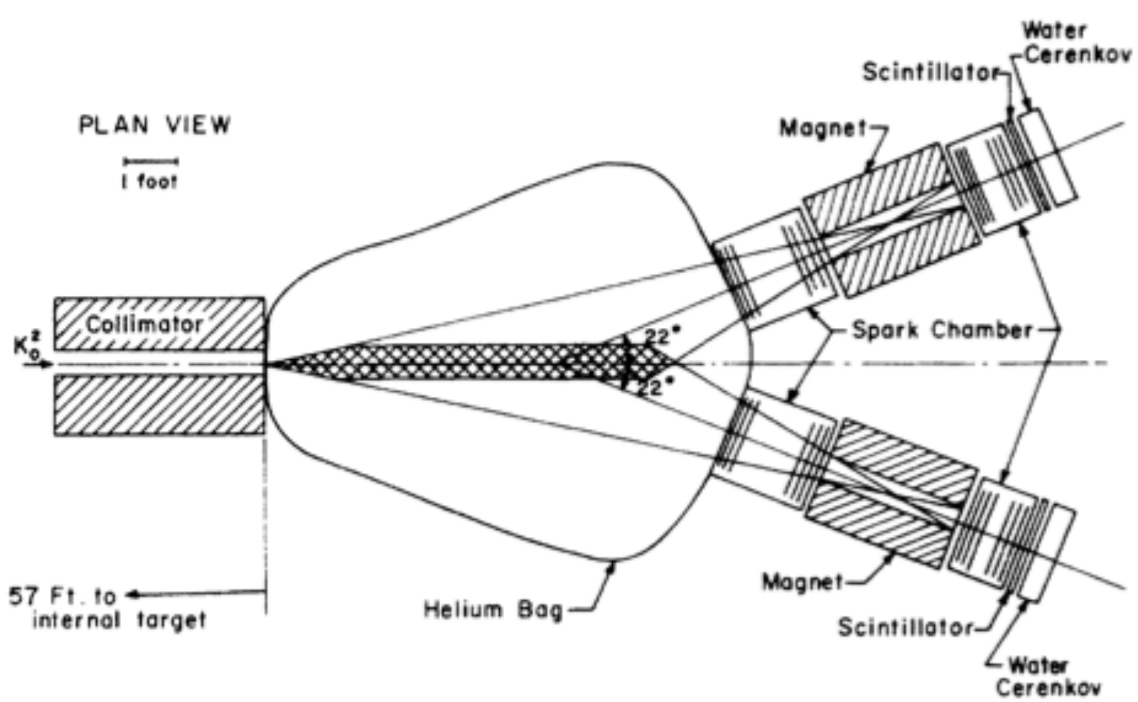
EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)



Fixed target *p*+Be experiment at Brookhaven AGS

The CKM matrix

and CP violation in the SM

$V^{\dagger}V = 1$

- The CKM matrix is a complex 3x3 unitary matrix
  - ➡ 9 magnitudes and 9 phases
- Unitary condition gives 9 constraints, e.g.

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Can absorb phases into external quark fields.
  - ➡ 4 parameters, 3 Euler angles and a **single complex phase**.

NB If there were only two generations, *V* would be a real rotation matrix with no complex phase.

CKM matrix

- Standard form is to express the CKM matrix in terms of three rotation matrices and one CP violating phase,

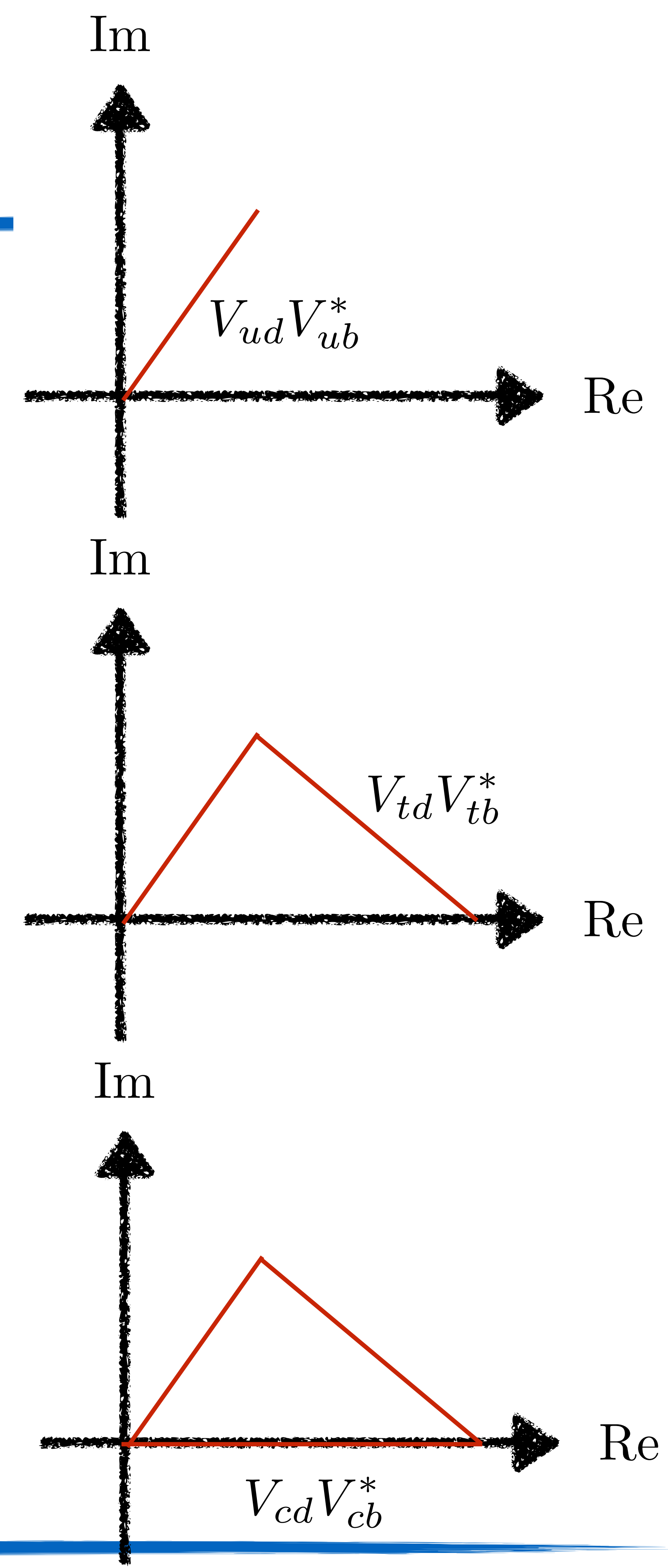
$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{+i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where

$$c_{ij} = \cos\theta_{ij} \text{ and } s_{ij} = \sin\theta_{ij}$$

Unitarity triangles

- Unitarity conditions can be represented by triangles in the complex plane.
  - ➡ Six triangles with the same area.



Jarlskog invariant

- All of the unitarity triangles have the same area, called the Jarlskog invariant.

$$|J| = \mathrm{Im}(V_{ij}V_{kl}V^*_{kj}V^*_{il}) \quad \text{for} \quad i \neq k \text{ and} j \neq l$$

- This is a phase convention independent measure of CP violation in the quark sector.
- In the standard notation $J = c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23}\sin\delta$
- Small size of the Euler angles means J (and CP violation) is small in the SM.

Matter antimatter asymmetry

and the Sakharov conditions

Matter dominated Universe

- From CMB measurements by WMAP + Planck

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \times 10^{-10}$$

- In early (hot) universe expect annihilation to give

$$n_B \approx n_{\bar{B}} \approx n_\gamma$$

The matter anti-matter imbalance is small but far too large to be explained by Electroweak Baryogenesis.

CP violation in the SM (quark sector) is too small due to the small size of the mixing angles and large hierarchy of quark masses.

Sakharov conditions

- Three conditions needed to generate a matter dominated universe from a symmetric initial state were proposed by A. Sakharov in 1967:
    1. C and CP violation.
    2. Baryon number violation.
    3. A system out of thermal equilibrium.

Sakharov conditions

- If we start with an equal amount of matter (M) and anti-matter ($\overline{\mathbf{M}}$) and

$$M \to A$$
$$\overline{M} \to \overline{A}$$

  where A and $\overline{\mathbf{A}}$ have baryon numbers N_A and $-N_A$.

- If C and CP are violated

$$\Gamma[M \to A] \neq \Gamma[\bar{M} \to \bar{A}]$$

  but even with different decay rates, after sufficient time there will be equal amounts of matter/antimatter.

Sakharov conditions

- Can get round this problem by having two (or more) competing process

$$
\begin{aligned}
M &\to A \quad \text{with probability } p \\
M &\to B \quad \text{with probability } 1-p \\
\bar{M} &\to \bar{A} \quad \text{with probability } \bar{p} \\
\bar{M} &\to \bar{B} \quad \text{with probability } 1-\bar{p}
\end{aligned}
$$

- Then

$$
\begin{aligned}
\Delta N &= N_A \cdot p + N_B \cdot (1-p) - N_{\bar{A}} \cdot \bar{p} - N_{\bar{B}} \cdot (1-\bar{p}) \\
&= (p-\bar{p})(N_A - N_B)
\end{aligned}
$$

- Need N_A different from N_B to generate an asymmetry (i.e. baryon number violation as well as CP violation).

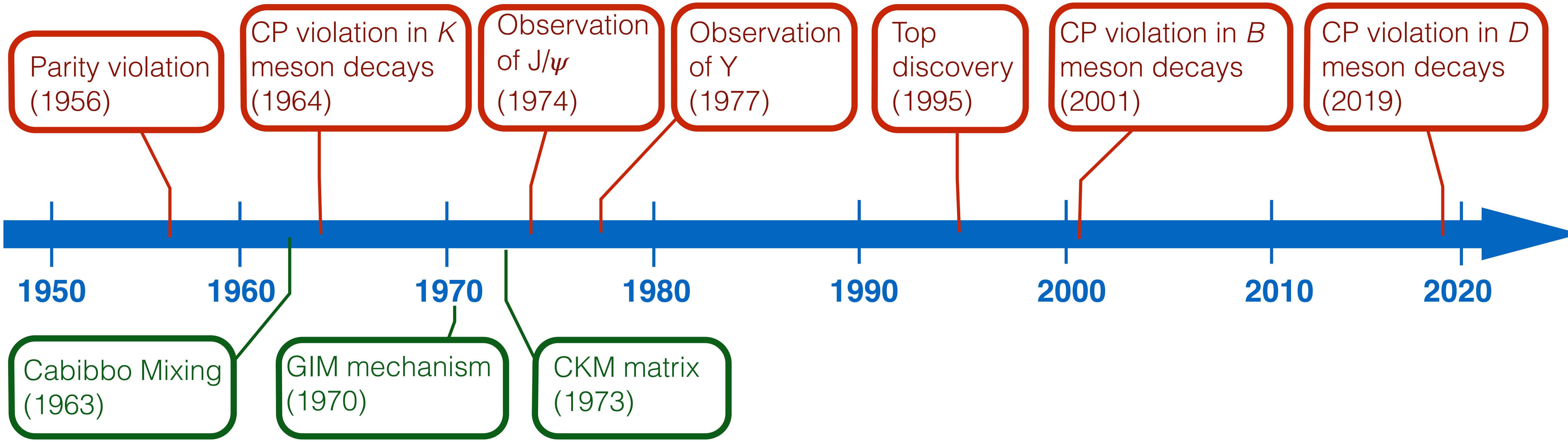
Sakharov conditions

- Even then, the system needs to be out of thermal equilibrium or

$$\Gamma[A \rightarrow B + C] = \Gamma[B + C \rightarrow A]$$

and the asymmetry will be destroyed as fast as it is created.

Recap



Parity violation (1956)
CP violation in K meson decays (1964)
Observation of J/ψ (1974)
Observation of Υ (1977)
Top discovery (1995)
CP violation in B meson decays (2001)
CP violation in D meson decays (2019)
1950
1960
1970
1980
1990
2000
2010
2020
Cabibbo Mixing (1963)
GIM mechanism (1970)
CKM matrix (1973)

Fin