A generic C++ implementation of the Pruned DPA for segmentation

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Pruned DPA

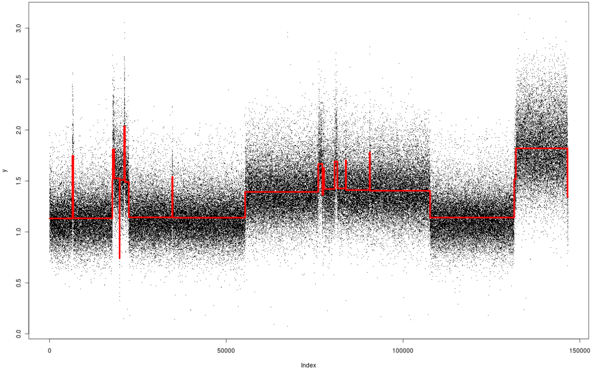
Guillem Rigaill [4]

Pruned dynamic programming for optimal multiple change-point detection

Algorithm for the segmentation of datasets:

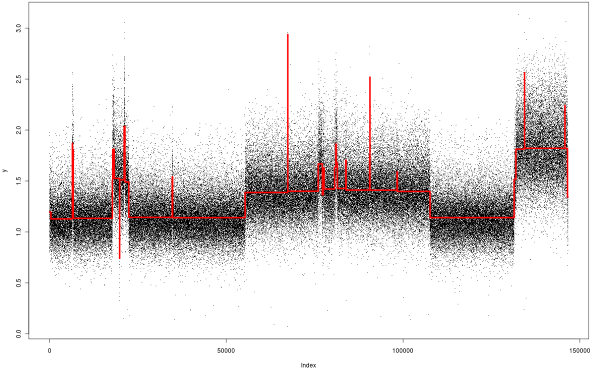
- Exact with respect to given loss
- Fast: empirically in $n \log(n)$
- Returns optimal segmentation in 1 to K_{max} segments
- allows for a vast range of methods for the choice of K

CGH profile, Pruned DPA



- 1 to $K_{max} = 50$
- Runtime = 10.834s
- Lebarbier: [3], Lavielle: [2] : $pen(K) = \beta K \log\left(\frac{n}{K}\right)$
- $K = 28$

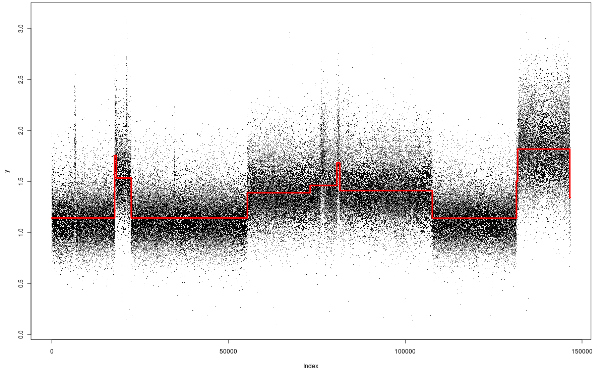
CGH profile, Pruned DPA



- 1 to $K_{max} = 50$
- Runtime = 10.834s

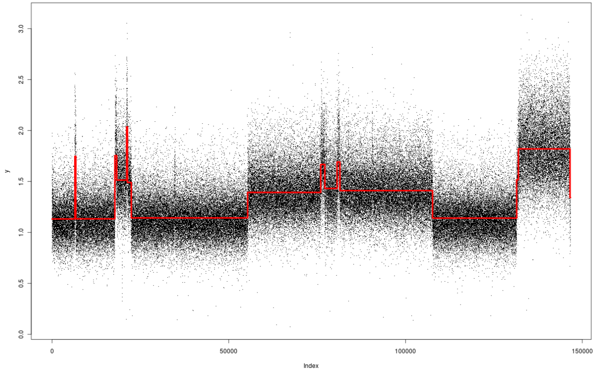
- mBIC (Zhang and Siegmund [5]):
  $pen(K) = \beta K \log\left(\frac{n}{K}\right) + g\left(\sum_{r \in \hat{m}(K)} \log n_r\right)$
- $K = 42$

CGH profile, CART



- 1 to $K_{max} = 50$
- Runtime = 0.171s
- SIC: $pen(K+1) = \frac{1}{2} K \log n$
- $K = 12$

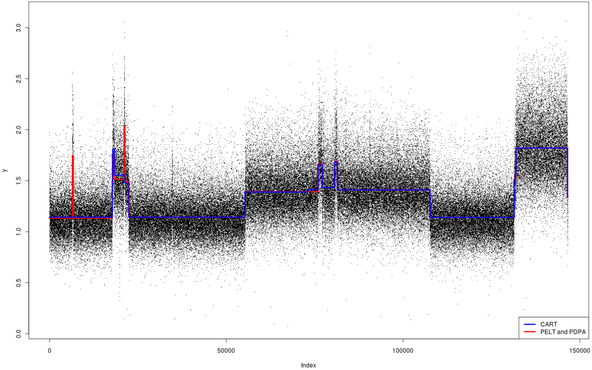
CGH profile, PELT



- Runtime = 14.651s

- SIC: $pen(K+1) = \frac{1}{2} K \log n$
- $K = 17$

Comparison for K=17



Runtimes:

- PDPA = 3.621s
- CART = 0.062s
- PELT = 14.651s

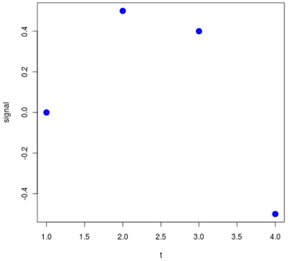
Breakpoints

- PELT and PDPA: same breakpoints
- CART: only 9 out of 16 are identical

An example

Four-point signal

$Y_1 = 0$ $Y_2 = 0.5$ $Y_3 = 0.4$ $Y_4 = -0.5$

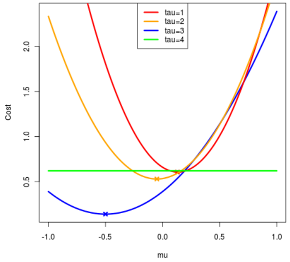


Contrast $\gamma(Y, \mu) = (Y - \mu)^2$

Segmentation in K=2 segments

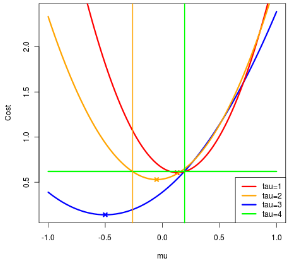
Dynamic Programming approach:

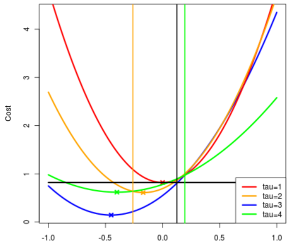
- $\forall t \in \{1, \dots, n\}$ compute cost of segmentation with last breakpoint t as a function of last-segment parameter μ
- $\forall t \in \{1, \dots, n\}$ find minimum of cost function in μ
- identify the minimum of those minimums



Main idea:

- If we add a new point, the values of μ change, but not the best candidates for last breakpoint
- $\Rightarrow$ A beaten candidate can never become optimal again





Pruned DPA: The algorithm

Initialization:
$\forall t \in \{1, n\}$ compute $C_{1,t}$

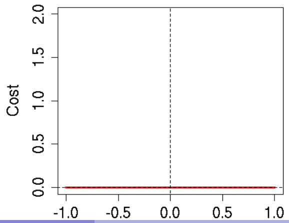
$\tau = K - 1 = 1$
($t = 1$ Signal: $Y_1 = 0$)

Cost function:

- $h^1_{2,1}(\mu) = C_{1,1} = 0$

Set of intervals:

- $S^1_{2,1} = \mathbb{R}(= [-0.5; 0.5])$



New entry:

$$t = 2$$

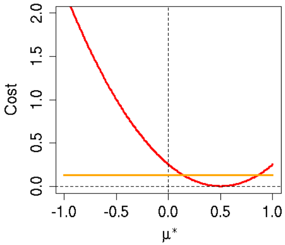
Signal: $Y_1 = 0$ $Y_2 = 0.5$

Cost functions:

- $h^1_{2,2}(\mu) = h^1_{2,1} + (Y_2 - \mu)^2 = 0.25 - \mu + \mu^2$
- $h^2_{2,2}(\mu) = C_{1,2} = 0.125$

Set of intervals:

- $S^1_{2,2} = [0.146; 0.5]$
- $S^2_{2,2} = [-0.5; 0.146]$



New entry:

Signal: $Y_1 = 0$ $Y_2 = 0.5$ $Y_3 = 0.4$

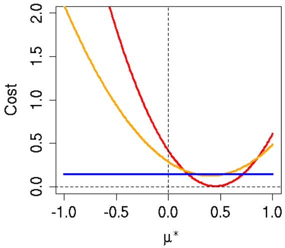
Cost functions:

- $h^1_{2,3}(\mu) = h^1_{2,2} + (Y_3 - \mu)^2 = 0.41 - 1.8\mu + 2\mu^2$
- $h^2_{2,3}(\mu) = h^2_{2,2} + (Y_3 - \mu)^2 = 0.285 - 0.8\mu + \mu^2$
- $h^3_{2,3}(\mu) = C_{1,3} = 0.14$

Set of intervals:

- $S^1_{2,3} = [0.190; 0.5]$
- $S^2_{2,3} = \varnothing$
- $S^3_{2,3} = [-0.5; 0.190]$

$\tau = 2$ is discarded



Last entry:

Signal: $Y_1 = 0$ $Y_2 = 0.5$ $Y_3 = 0.4$ $Y_4 = -0.5$

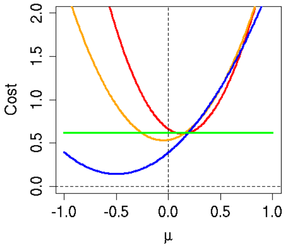
Cost functions:

- $h^1_{2,4}(\mu) = h^1_{2,3} + (Y_4 - \mu)^2 = 0.66 - 0.8\mu + 3\mu^2$
- $h^3_{2,4}(\mu) = h^3_{2,3} + (Y_4 - \mu)^2 = 0.39 + \mu + \mu^2$
- $h^4_{2,4}(\mu) = C_{1,4} = 0.62$

Set of intervals:

- $S^1_{2,4} = \varnothing$
- $S^3_{2,4} = [-0.5; 0.190]$
- $S^4_{2,4} = [0.190; 0.5]$

$\tau = 1$ is discarded



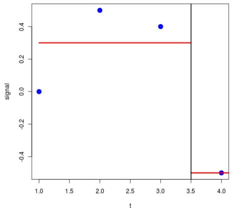
Last step: minimization

- $H_{K,t}(\mu) = \min_{\{K-1<\tau<t\}} \left\{ h^{\tau}_{K,t}(\mu) \right\}$
- $H_{K,t}(\mu) = \begin{cases} 0.39 + \mu + \mu^2 & \text{for} \quad \mu \in [-0.5; 0.190] \\ 0.62 & \text{for} \quad \mu \in [0.190; 0.5] \end{cases}$
- $C_{K,t} = \min_{\mu} \{H_{K,t}(\mu)\}$

$$C_{K,n}(= C_{2,4}) = 0.14$$

$$\tau = 3$$

$$\mu = -0.5$$



From the original DPA to the Pruned DPA

Original DPA: segment additivity $\Theta(Kn^2)$

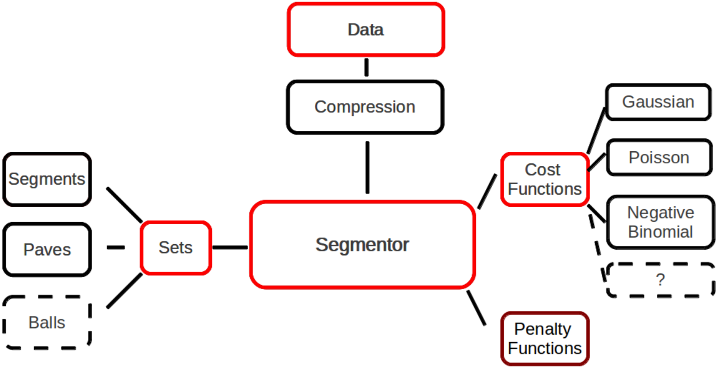
$$C_{k,t} = \min_{\{k-1<\tau<t\}} \left\{ C_{k-1,\tau} + \min_{\mu} \{ \sum_{i=\tau+1}^{t} \gamma(Y_i, \mu) \} \right\}$$

Pruned DPA: point additivity

$$C_{k,t} = \min_{\mu} \left\{ \min_{\{k-1<\tau<t\}} \{ C_{k-1,\tau} + \sum_{i=\tau+1}^{t} \gamma(Y_i, \mu) \} \right\}$$

$$= \min_{\mu} \left\{ \min_{\{k-1<\tau<t\}} \{ C_{k-1,\tau} + \sum_{i=\tau+1}^{t-1} \gamma(Y_i, \mu) + \gamma(Y_t, \mu) \} \right\}$$

Generic C++ implementation



Performances on real datasets

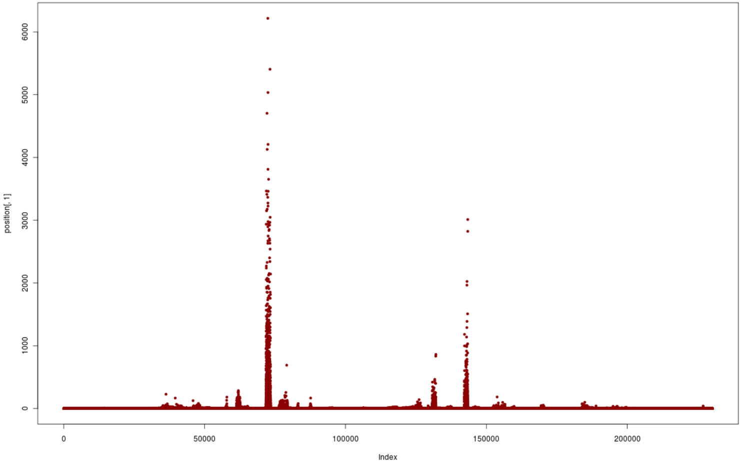
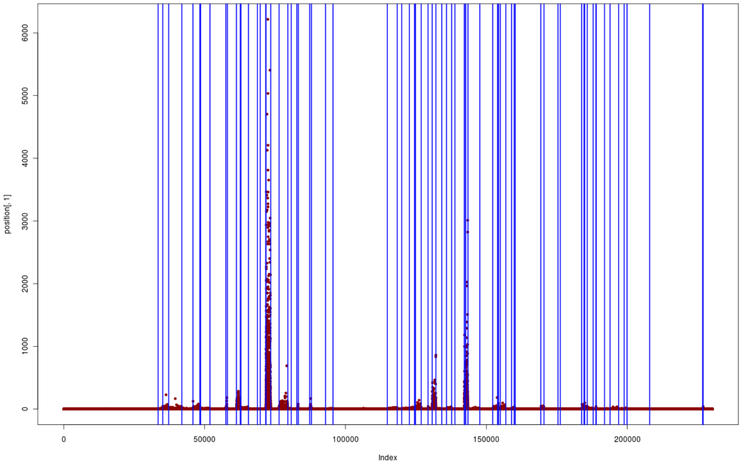
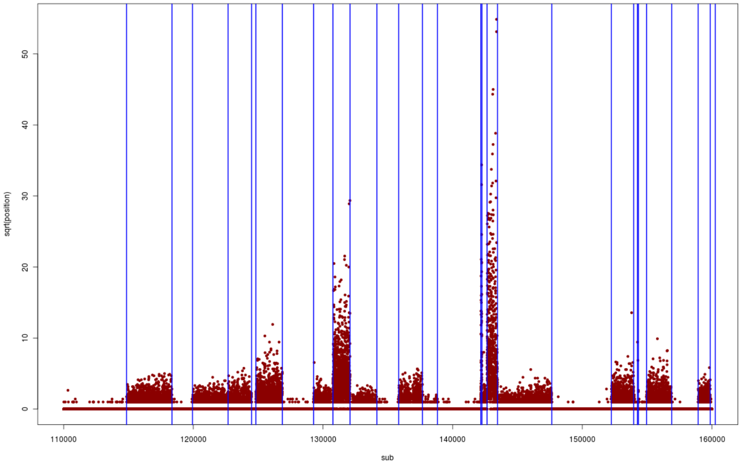


Figure: Chromosome 1, positive strand of S. cerevisiae (yeast)

Performances on real datasets



Performances on real datasets



Performances on real datasets

Neuroblastoma copy number data
Result from Hocking et al. [1]

	errors	FP	FN	Timing(s)
PDPA-Lavielle	2.2	0.6	11.6	2.10
Fused Lasso ($\lambda = f(K)$)	6.7	3.6	18.5	0.08
Circular Binary Segmentation (SD)	11.5	7.6	32.2	51.62
Fused Lasso ($\lambda = cste$)	16.0	12.7	36.6	0.04
Circular Binary Segmentation (default)	40.5	49.3	0.5	1.78
PDPA-mBIC	40.9	49.4	0.0	1.47

Table: Comparison of a few segmentation methods on a real data set. Tuning parameters are learned by Leave-one-out.

More methods are compared in [1]

Conclusions and Perspectives

- Conclusion: PDPA is a fast and exact algorithm that allows the use of:
  - a large range of data type (CGH, Seq-data, etc)
  - a large range of possible contrasts (Quadratic, Poisson, etc)
  - a large range of methods for the choice of K (mBIC, Lavielle, AIC, etc)

- Perspectives:
  - Application to real datasets for the discovery of new transcripts, etc.
  - Theoretical proof of the complexity of the algorithm
  - Implementation of Ridge-type penalties

The End

Thank you!

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