Bayesian model selection and estimation: Simultaneous mixed effects for models and parameters

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Introduction

Bayesian model selection and estimation (BMSE):
Powerful methods for determining the most likely among sets of competing hypotheses about the mechanisms and parameters that generated observed data, e.g., from experiments on decision-making.

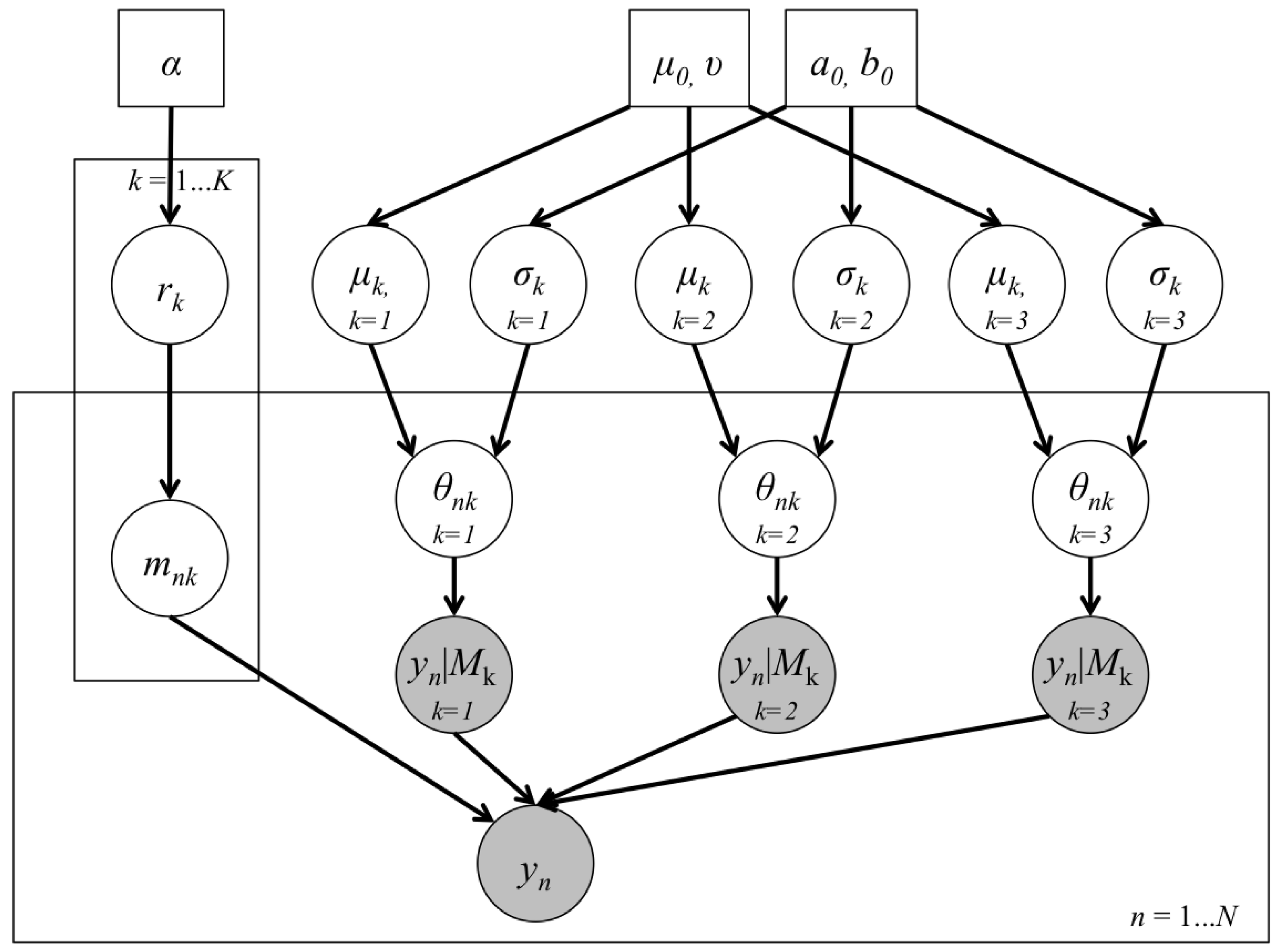
Mixed-effects (or empirical / hierarchical Bayes') models:
Provide full inference in group-studies – with repeated observations for each individual – by adequatly capturing:
- Individual differences (**random effects / posteriors**)
- Mechanisms & parameters common to all individuals (**fixed effects / priors**)

Previous models: have assumed mixed-effects
- either *for model parameters*: Huys et al. [1] applied empirical Bayes' via Expectation Maximization (EM) to reinforcement learning models
- or *for the model identity*: Stephan et al. [2] developed a Variational Bayes' (VB) method for treating models as random-effects

Here:
A) We evaluate the empirical Bayes' method assuming mixed-effects for parameters for reinforcement learning models [1]
B) We present a novel Variational Bayes' (VB) model which considers mixed-effects for models and parameters simultaneously

B) Variational Bayes: Methods

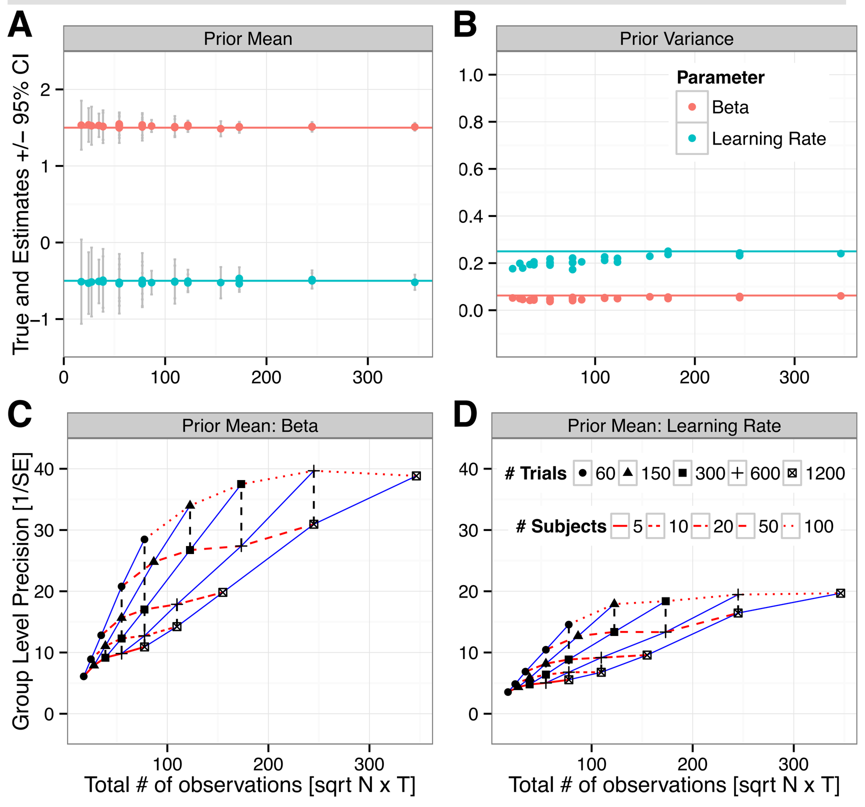
Sufficient statistics approach: combine empirical Bayes [1] with random effects for models [2]
Full random effects inference: Variational Bayes

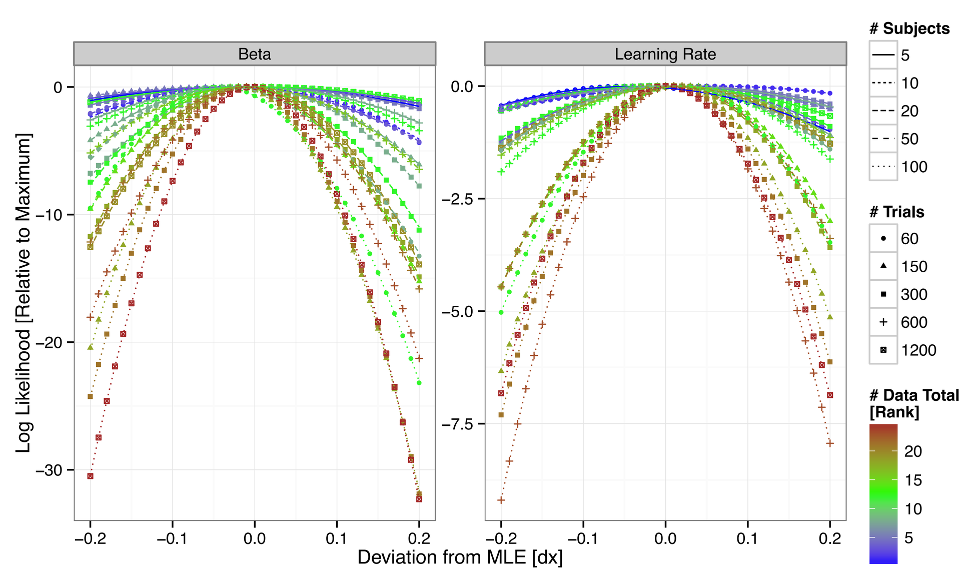


A) empirical Bayes

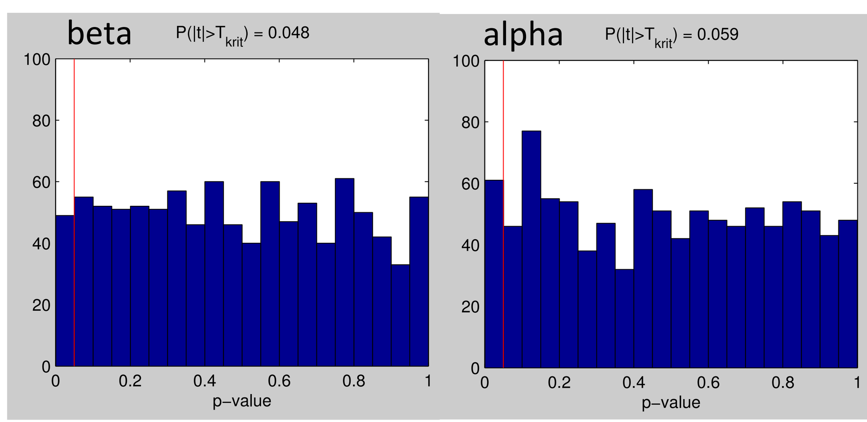
- Generating prior parameters can be recovered from simulated data
- The precision scales with number of data points as theoretically expected

$$P(\mathfrak{X} \mid \mu_\theta, \sigma_\theta) \propto \int_{-\infty}^{\infty} \mathrm{d}\underline{\theta}\, P(\mathfrak{X}, \underline{\theta} \mid \mu_\theta, \sigma_\theta)$$





The likelihood for the prior is approximately Gaussian, providing a basis for a Laplace-Approximation to derive error bars, with normative alpha errors



Generating Model	Fitted model					
	m2b2alr	mr	2b2alr	m2b2al	m	2b2al
m2b2alr	0	337	49	441	1297	531
mr	42	0	428	800	801	1490
2b2alr	12	841	0	280	2678	271
m2b2al	6	452	95	0	514	83
m	40	21	408	45	0	436
2b2al	16	1391	5	18	2271	0

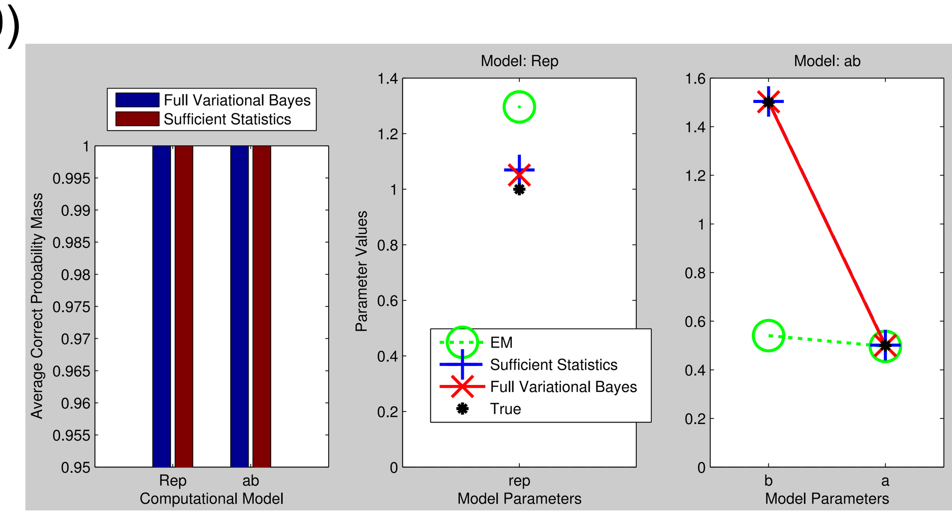
BIC_{int} extracts the true generating model from the data

B) Variational Bayes: Results

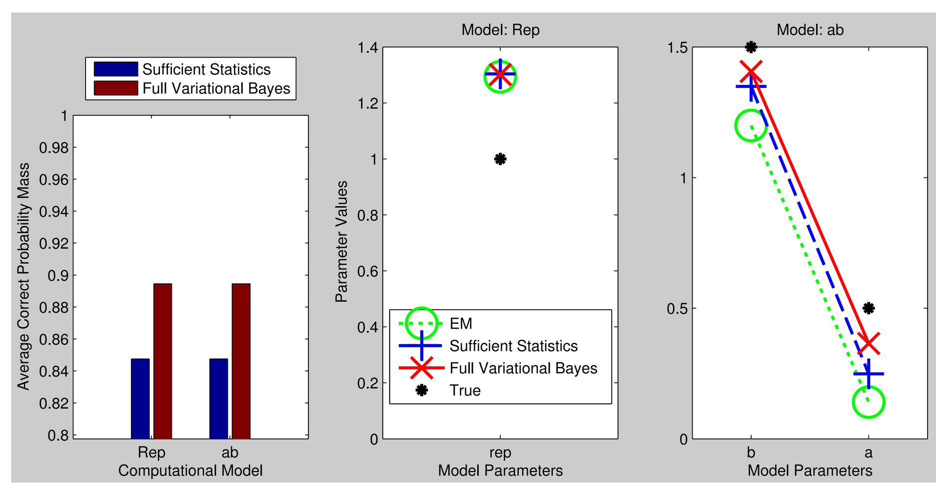
Simulations from known decision processes with N = 90 simulated subjects

Simple RL: ab = simple RL model, assuming 1 state, 2 actions, learning rate (a) inverse noisiness (b) parameters (N=60)
Rep = repetition model (N = 30)

Simple RL + 200 trials
With sufficient data, the correct model can be identified for all subjects and for both methods with high certainty

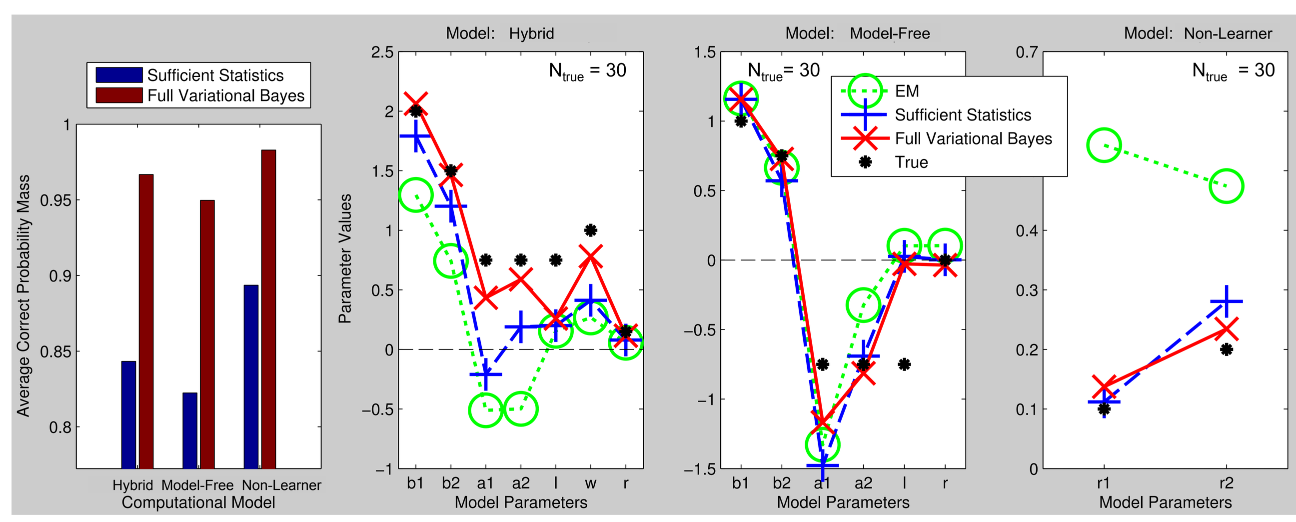


Simple RL + 20 trials
With scarce data per subject, the full VB method improves model comparison compared to the sufficient statistics approach

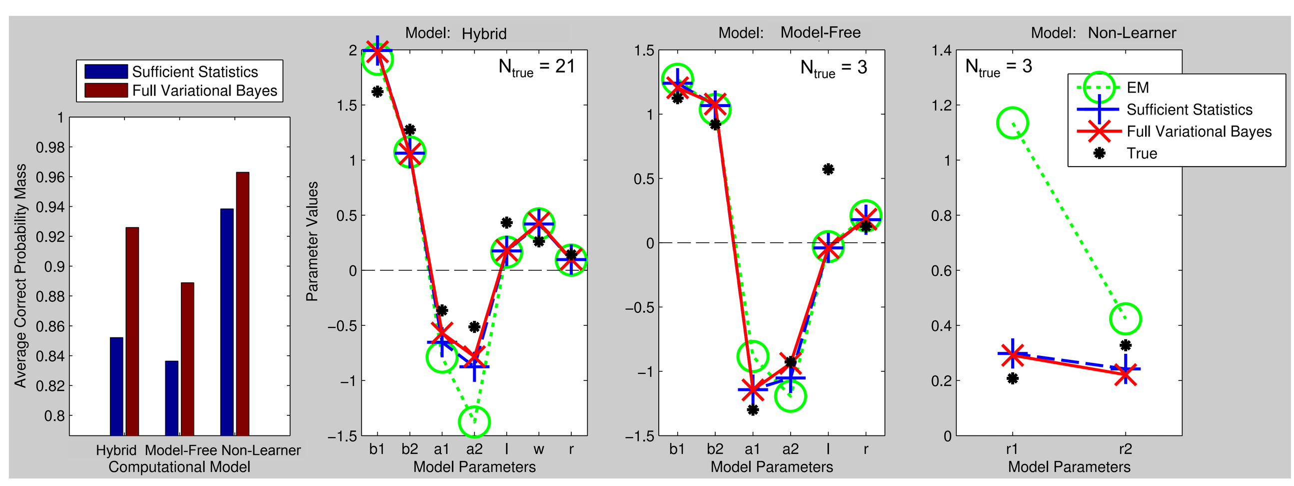


2step: Hybrid = model-based + model-free; model-free; non-learner [3]

2step + 201 trials
Using three similar models with differing model parameters in the 2step task also yields posterior uncertainty, and the full VB performs best



2step + 201 trials + parameters based on real data [4]
The advantage for the full VB is visible also for parameters obtained from real (observed) data



Conclusions

- Fitting empirical Bayes' models of reinforcement learning using Expectation Maximization (EM) [1] exhibits desirable normative properties
- Our new Variational Bayes method suggests that we can and should understand the heterogeneity and homogeneity observed in group studies of decision-making by investigating contributions of both, the underlying mechanisms and their parameters
- We find increased accuracy in Bayesian model comparison for our new VB method compared to previous approaches [1, 2]
- We expect that this new mixed-effects method will prove useful for a wide range of computational modeling approches in group studies of cognition and biology

References

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[4] Schad, D. J., Jünger, E., Garbusow, M., Sebold, M., Bernhardt, N., Javadi, A. H., et al. (2014). Individual differences in processing speed and working memory capacity moderate the balance between habitual and goal-directed choice behaviour. *Submitted*.

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