Pain-free Bayesian inference of psychometric functions

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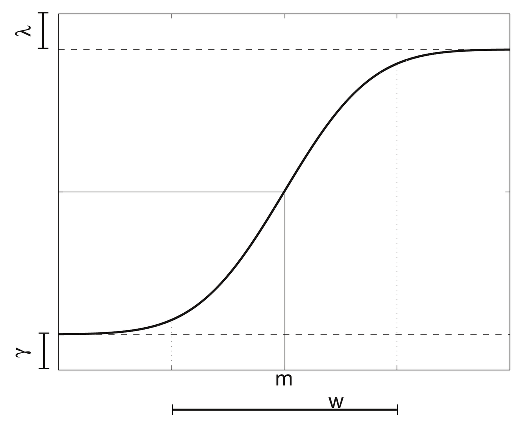
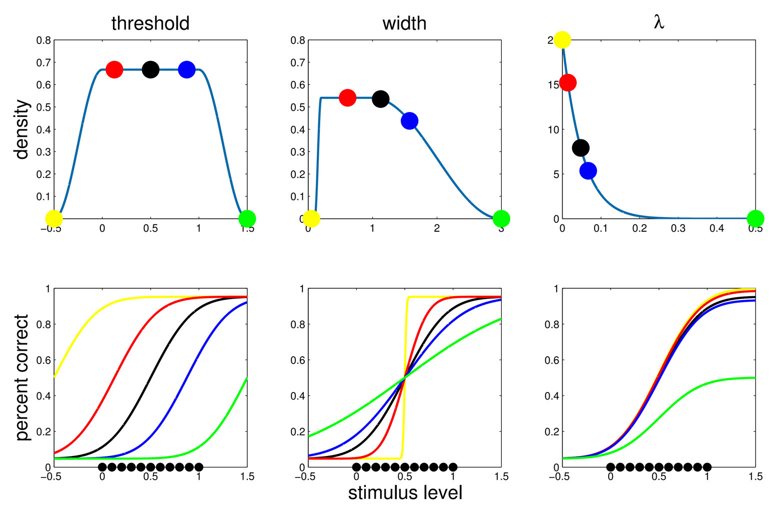
We developed a numerical integration based method for Bayesian Inference on psychometric functions. The method is fast and convenient to use. Furthermore, we provide suitable defaults for all common settings. By fitting a beta-binomial model our method exhibits increased robustness against nonstationary behaviour caused, e.g., by fluctuations in attention or learning. We performed extensive simulations to validate our method and software implementation. Finally we provide generic methods to compare several psychometric functions statistically. All methods are freely available: https://github.com/wichmann-lab/psignifit

We model psychometric functions $\psi(x)$ at a stimulus level x with a sigmoid function $S_{m,w}(x)$ parametrized by its threshold m and width w scaled by upper and lower asymptotes λ and γ:

$$\psi(x) = \gamma + (1 - \gamma - \lambda) S_{m,w}(x)$$

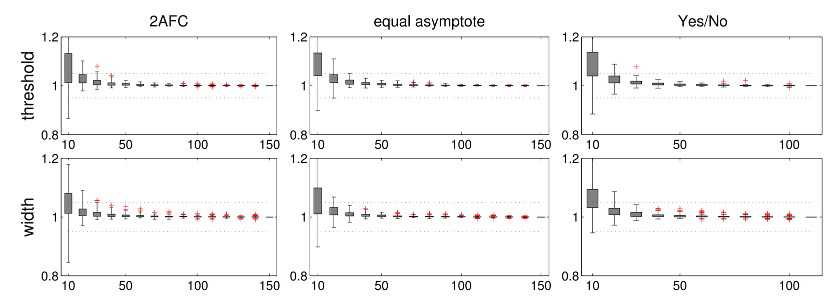




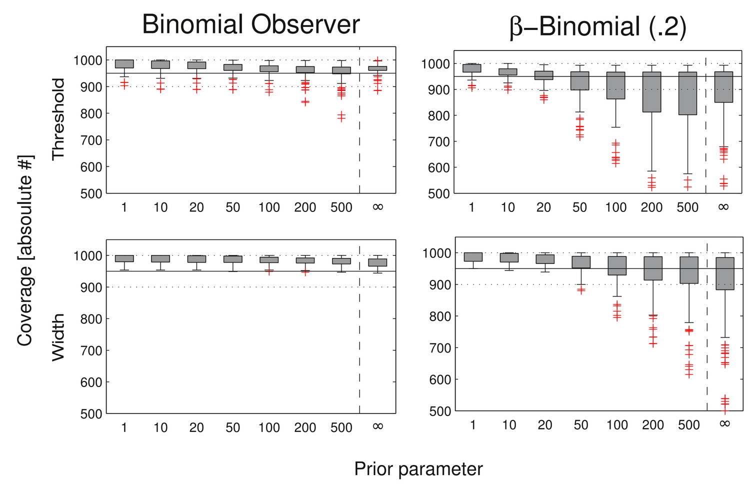
Densities and example functions for our default prior distributions for a psychometric function sampled at the black dots on the x-axis. The filled color circles in the densities in the top row correspond to the psychometric functions of the same color in the lower row. We choose prior distributions based on where the psychometric function is sampled.

Bayesian inference of psychometric function is usually done by MCMC sampling. While being powerful, this typically requires users to have detailed knowledge of MCMC, e.g. to fine-tune the algorithm to datasets, or evaluate chain convergence. Instead we propose to evaluate the full posterior using numerical integration on a grid. Our analysis method is fast—maximally a few seconds for the full 5 parameter model—stable, and independent of user intervention. Defaults for the priors and all other settings making our software easy to use whilst retaining full customisability.

Numerical integration and defaults make our toolbox fast and convenient



Confidence interval size normalized to the size estimated with 150 grid points per dimension against the number of gridpoints in the simulation.



Coverage from simulations for different sampling schemes, dataset sizes and a binomial and a β-binomial observer with $\sigma = .2$. The x axis depicts the strength of the prior on σ. As expected, coverage is good for the binomial observer and depends on the prior for the β-binomial observer.

Non-stationaritites—when the psychometric function is not constant over time—are abundant in experiments and may result from learning or fluctuations in attention. This impairs statistical inference because they increase the variability of the data. To capture this we fit a model which contains a specific non-stationarity, the beta-binomial model. This model assumes that the probability of success for each block is drawn from a beta distribution with variance scaled by σ around the value of the psychometric function:

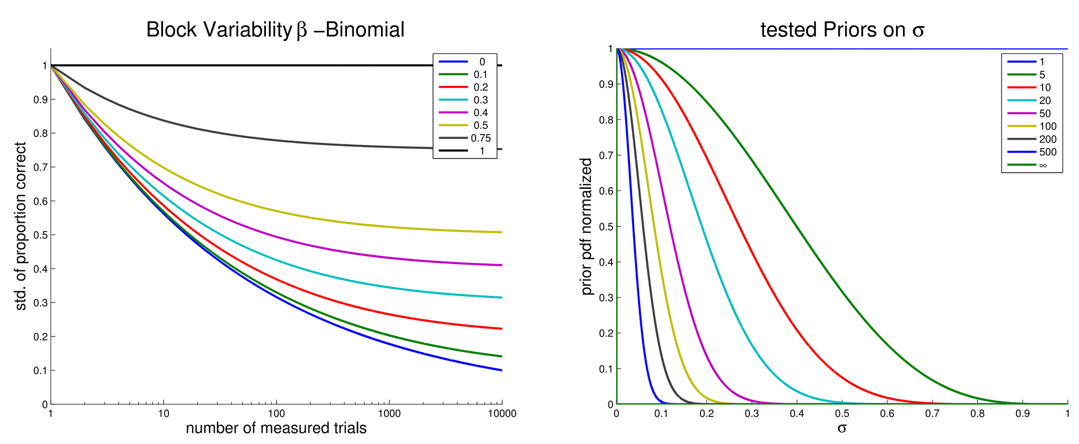
The β-binomial model takes nonstationarities into account, yielding better confidence intervals

$$k_i \sim Binom(n_i, p_i)$$

$$p_i \sim Beta\left(\left(\frac{1}{\sigma^2} - 1\right)\psi(x_i), \left(\frac{1}{\sigma^2} - 1\right)(1 - \psi(x_i))\right)$$

$$\Rightarrow E(p_i) = \psi(x) \qquad \mathrm{Var}(p_i) = \sigma^2 \psi(x)(1 - \psi(x))$$

This model corrects the confidence intervals automatically, when the data is over-dispersed.



Left: Standard deviation of percent correct for differently strong betabinomial observers (normalized to one Bernoulli) trial against the number of trials measured
Right: The different priors we tried for σ, indexed by parameter k: $\beta(\sigma^2, 1, k)$

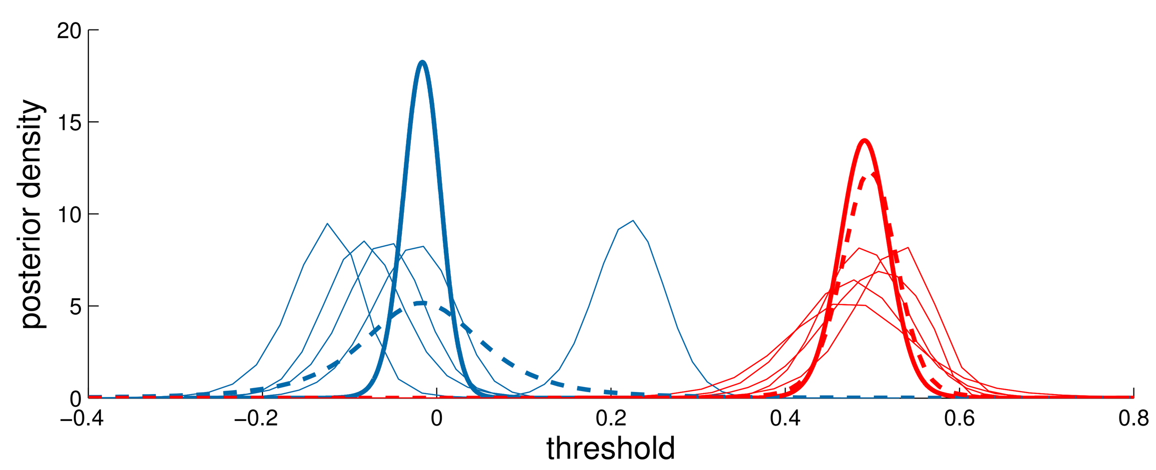
From several posterior distributions of the parameters of individual psychometric functions posteriors on parameters of the measured sample can be calculated, allowing comparisons between single subjects and the measured group. To test whether a group is congruent, e.g. shares some parameter(s) of the psychometric function we provide a Bayesfactor test, comparing the model with shared parameter(s) to the model with individual parameters. Finally we can draw conclusions on any generative model with parameter θ for one or more of the psychometric function parameters α from the marginal on these predicted parameters for each psychometric function.

Bayesian inference of psychometric functions allow more complex conclusions

$$P(\theta|D) \propto P(\theta) \prod_{i=1}^{n} \left[\int P(\alpha_i|\theta) \frac{P(\alpha_i|D_i)}{P(\alpha_i)} d\alpha_i \right]$$

This allows to draw conclusions for population models and to differentiate the variability between subjects from the one from the measurements. As an example we implemented a normal distribution model for the population

How to combine the marginal posteriors from different psychometric functions. The thin lines are the posterior marginals for five single subjects for each of the two groups. The thicker ones are the computed posteriors for the means of the two groups. The dashed lines show the posterior we computed over the mean of the population assuming a normal distribution in each group.



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