Decision Analysis and Political Processes

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Abstract

Decision analysis has been with us for at least half a century. Over that time it has developed from a theoretical paradigm for individual rational choice to a practical tool for individuals, small groups and 'unitary' organisations, which helps them towards a sound decision making mindful of the behavioural characteristics of individuals and group dynamics. Decision analysis has also shown its worth in the context of stakeholder engagement and public participation. The time is right for it to be more widely used across government, but to achieve that it may be necessary for our community of decision analysts to deconstruct our paradigm and recognise more clearly that political processes may need replace the final steps in a decision analysis.

Keywords: Bayesian decision analysis; political process; public participation; stakeholder engagement; System 1 and System 2 Societal Deliberation.

1 Introduction

In 2014 Informs' Decision Analysis Society celebrated 50 years of decision analysis. Some might contest that it is somewhat older, beginning perhaps with Ramsey (1926) or even with Benjamin Franklin's oft-quoted 1772 letter to Joseph Priestley. Whatever the case, decision analysis has been with us some time now. In this essay we want to reflect on its application in societal decision making. Here by 'society' we mean a locality, region or country which has autonomous authority to make decisions. We use the term 'societal decision making' rather than 'public policy analysis' because many public debates concern specific decisions, such as granting planning permission for an industrial complex, rather than more general policy, such as whether to use tax structures to encourage lower fat diets. Nonetheless, we intend that our discussion extends over contexts from the specific to the most general policy.

Our decision analytic paradigm separates beliefs and preferences in its modelling. The former it models with probabilities, the latter with utilities, bringing the two together via expected utility to rank alternatives. We shall refer to this as the Bayesian paradigm. We use the term Bayesian because we want to emphasise the connections between Bayesian statistics and decision analysis.

At its heart the Bayesian paradigm for inference and decision is individualistic: it applies to a single decision maker. Yet with care it can be and has been developed into a tool to articulate small group deliberation and help them decide. A small group might make decisions and commit to action entirely on its own behalf; it might be part of a unitary organisation, which shares common perspectives, beliefs and values and so the group's decision can become that of the organisation; or it may represent a stakeholder group in a much more diverse society in which true consensus on beliefs and values is unlikely. In this last case, the ultimate decision for society is taken by a political process, be it with a small or large 'p'. It is this last case that forms the focus of this paper. How do statistical and decision analyses fit into and support stakeholder engagement and public participation in societal decision making? To answer this question we will need recognise that the practice of decision analysis in such decisions differs in many ways from the idealised and individualistic Bayesian paradigm. Thus the discussion in this paper continues those begun in, *inter alia*, Gregory et al. (2005), French et al. (2007) and Rios Insua and French (2010).

Decision analysts have long recognised that actual individual decision making and the prescription of the normative Bayesian model differ in many ways (see, e.g., Edwards, 1954; Kahneman and Tversky, 1974). Current terminology distinguishes intuitive System 1 Thinking, which drives our unsupported decision behaviour, from the analytic, explicit System 2 Thinking, which decision analysis seeks to promote (Chaiken et al., 1989; Kahneman, 2011). We shall make a further, similar distinction between System 1 Societal Deliberation and System 2 Societal Deliberation: the former referring to the *de facto* informal processes by which organisations and society decide, the latter to the *de jure* formal governance processes by which the law and constitutions expect that they should (Argyris and French, 2017).

In the next section, we begin with the basics and outline the Bayesian paradigm. We note the need to develop decision analyses so that decision makers are helped to move from System1 Thinking towards System 2 Thinking. In Section 3 we discuss the implications of Arrow's Impossibility Theorem for group decision support within organisations, noting that generally within organisations there is broad agreement on values and objectives. In the next two sections, we turn to discuss societal decision making, looking at how the techniques of Bayesian statistics, risk and decision analysis support development of the public understanding of the issues. However, we also note in Section 5 that the full apparatus of subjective expected utility is seldom used. We conclude that our profession need acknowledge the role of political processes not just pragmatically in our work, but explicitly in our methodology.

2 The Bayesian Decision Analysis Paradigm

The Bayesian paradigm may be stated formally as follows. A decision maker has to choose a single action $a \in A$ from a set of possible actions. The consequence $c(a, \theta) \in C$, which will inevitably be multi-dimensional or *multi-attributed*, of her[1] choice depends both on which action she takes and an unknown state of nature θ, which she knows to lie in the set Θ. The attributes in a problem are the dimensions on which describe and model the consequences.

Before choosing the action, the decision maker may observe an outcome $X = x$ of an experiment, which depends on the unknown state θ. Specifically, the observation X is drawn from a distribution $P_X(.|\theta)$. The Bayesian approach has two simple mathematical formulae at its heart: the *subjective expected utility* model (SEU) and *Bayes Theorem*. The former ranks alternative actions $a \in A$ according to:

$$\max_{a\in A} E_\theta\left[u(c(a,\theta))|x\right] = \max_{a\in A} \int_\Theta u(c(a,\theta))p_\theta(\theta|x)d\theta \quad ,$$

where $p_\theta(\cdot \mid x)$ is a probability distribution encoding the decision maker's *current* knowledge, i.e. her *posterior probability* distribution having observed data x about the state of the world θ, and $u(.)$ is a *multi-attribute utility* (MAU) function encoding her preferences for possible consequences, $c(a, \theta)$. Note that the utility function $u(.)$ not only ranks different consequences according to her preferences, higher utilities corresponding to greater preference, but also encodes her attitude to risk or, if you prefer, her feelings about the value of insuring or protecting against specific risks. A

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1 Decision makers will be referred to in the feminine or plural: statisticians, risk and decision analysts in the masculine. Aside from issues of political correctness, the use of distinct pronouns helps present their roles without confusion or needless repetition of 'decision maker' and 'analyst'.

multi-attribute value (MAV) function encodes a decision maker's preferences in balancing varied success on the different attributes describing the consequences without considering her attitude to risk: so $u(.) = r(v(.))$, where the transformation $r(.)$ encodes her valuation of risk.

The term *consequence model* is used here in the sense of the model which predicts the consequences $c(a, \theta)$ of a specific action, a, in the context of a specific state of the world, θ. The development of the consequence model is precisely the scientific modelling process that brings together biological, chemical, economic, environmental, physical and similar understandings of the world and injects these, as appropriate, into the analysis.

Bayes Theorem determines how the decision maker's initial knowledge before observing x, i.e. her *prior probability distribution*, $p_\theta(\cdot)$, is updated in the light of the data to her posterior distribution $p_\theta(\cdot \mid x)$. Stated to within a constant of proportionality:

$$p_\theta\left(\theta \middle| x\right) \underset{\theta}{\propto} p_X\left(x \middle| \theta\right) p_\theta\left(\theta\right),$$

Bayes Theorem and the concept of learning that it encodes underpin Bayesian Statistics and many approaches to machine learning, data-mining and analytics developed in recent decades. Thus the Bayesian approach to decision analysis fully coheres with current approaches and techniques used in statistical analysis and inference.

Figure 1 illustrates the component parts of Bayesian analysis. A full justification of the Bayesian model is provided in, *inter alia*, French and Rios Insua (2000), who describe not just its axiomatic and theoretical basis, but also justifications on the grounds of its feasibility, robustness and transparency. We note though that the Bayesian model is essentially mathematical and our presentation emphasises this. However, its implementation is much more subtle. We argue that it articulates discussion as much as quantitatively ranking alternatives.

The first step in the modelling separates the decision maker's beliefs and understanding of how the world is working and the consequences that her actions may bring, i.e. her Science[2], from her preferences about those consequences, i.e. her Values. We shall return to this separation are several points below. While many non-Bayesians would strongly criticise the mathematical structures indicated in Figure 1, most would accept the separation of the 'Science' and 'Values' and the broad relationships between consequence modelling, statistical inference and decision analysis. Thus many of the qualitative points discussed below apply to their approaches as much as to the Bayesian.

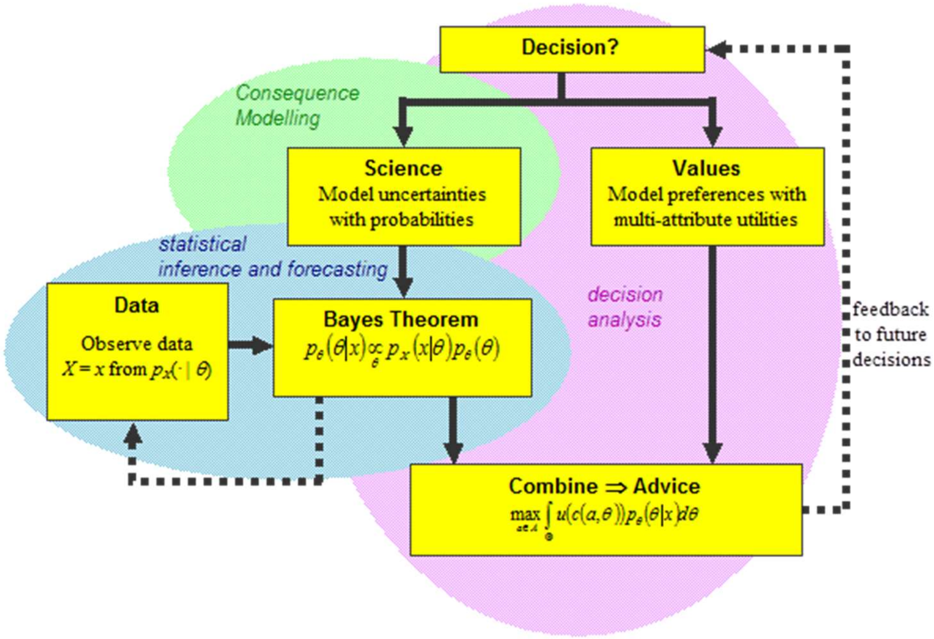

Figure 1: The Bayesian Paradigm (modified from French, 2003)

2 Throughout we shall use the term 'Science' more in its classical meaning of all knowledge. Thus it will refer not just to the knowledge arising in, e.g., the physical and biological sciences but also that arising in, e.g., economics, the humanities and social sciences.

It has been long recognised that there is a tension between descriptive studies of how people *do* reach decisions and normative theories of how they *should*. Edwards (1954) raised the question of whether people actually followed the principles of rationality that were being proposed by Savage, von Neumann and Morgenstern, and others. Over the intervening years, it has become clear empirically that generally they do not (Bazerman, 2006; Hogarth, 1980; Kahneman and Tversky, 1974). Terminology has changed somewhat and currently one talks of *System 1* and *System 2 Thinking* (Chaiken et al., 1989; Kahneman, 2011), the former referring to instinctive thought on the fringes of consciousness, the latter to more conscious, explicit, analytic patterns of thought. System 1 thinking is subject to behavioural biases; indeed, for many years its literature has been referred to under the somewhat pejorative label *heuristics and biases* (Kahneman et al., 1982). In strict terms, System 2 thinking need only be conscious and explicit: it need not be rational in any particular sense; but, for our purposes, we assume that it is and, in particular, that it is Bayesian or a good approximation to that. For important decisions, particularly professional ones, one strives to use rational System 2 thinking. Whether there is a true dichotomy between System 1 and System 2 Thinking or a gradation between subconscious informal and explicit formal thought is moot in behavioural science, but here the simple distinction will serve.

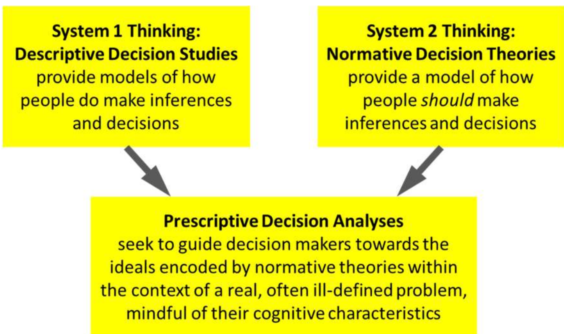

Figure 2: The interplay between System 1 Thinking and System 2 Thinking in prescriptive decision analysis

For individual decision makers, decision analysis has become a process akin to psychotherapy. At its core, the analyst builds and then uses the formalism of subjective probability and utility models to challenge the decision maker to think deeply about her problem. The ultimate expected utility ranking together with sensitivity analyses helps her form her understanding of where the balance of all the factors affecting her choice lies (French, 2003).

The Bayesian model is only part of a much more extensive process which passes, often cycles through many steps from initial formulation through knowledge gathering, modelling and analyses, deliberation and evaluation to final implementation (see, e.g., French et al., 2009). All phases contribute to the process of helping the decision maker move from System 1 Thinking towards System 2 Thinking, bringing structure and challenge to her analysis. Not all decision analyses pass through these phases, however. French et al. (2009) explore the structure of decision analyses in the different contexts of instinctive, operational, tactical and strategic decision making. Broadly their argument is that in well-rehearsed cases such as instinctive or operational decisions the issues are well understood and thus formulation and interpretation need less emphasis. Moreover, there is usually adequate data to fit complex models and perform the entire Bayesian analysis as in Figure 1. For more complex operational and strategic decisions in which there may be novel aspects that require careful exploration, the formulation and appraisal phases assume more importance (see also French, 2013). Here, we focus on strategic decision making in contexts in which there is substantial novelty, meaning that decision makers need to think through their beliefs and preferences as well as many other issues.

3 Supporting Groups

The Bayesian paradigm is individualistic, as so far has been our description of decision analysis. It provides a model of how a rational person should organise her beliefs and preferences and balance these to form a ranking of a number of alternatives. Many decisions, arguably most important decisions, are taken by groups. It would be comforting, therefore, if the Bayesian model extended simply to the context of group decisions, allowing group probabilities and utilities to be constructed and group expected utilities formed. Note that here we are talking about a true group of individuals trying to reach agreement. We are assuming that there is no arbiter or supra decision maker who will interject altruistic values or perceptions that help identify a fair agreement for the group (Keeney and Raiffa, 1976; Raiffa et al., 2002). Sadly, a host of impossibility results and paradoxes show that this is unlikely to be possible if one wishes the method to be democratic and immune from manipulation by one or more members (Arrow, 1963; Hodge and Klima, 2005; Kelly, 1978; Maskin and Sen, 2014; Taylor, 2005). De Finetti (1974; 1975) in his seminal work on probability with the words: “Probability does not exist!” He goes on to argue that probability is a mental construct of an individual used to describe and model her (rational) beliefs. A similar text on group decision making might begin with the words “Group decisions do not exist!”, and then continue to argue that group decisions are social constructs describing the outcome of a social process. That is certainly the course that we shall be taking.

Others, we realise, disagree. Recently Keeney (2013) and Keeney and Nau (2011) argue that one can construct group probabilities and utilities, form group expected utilities and so model rational group decision making in a consistent, logical way. Earlier work in this vein include Bacharach (1975), Harsanyi (1978) and Harsanyi (1979); and there are a host of related discussions in social welfare theory (e.g. Sen, 2017). In doing so they implicitly make assumptions about interpersonal comparisons of utility between the group members. Their quantitative model necessarily resolves diametrically opposed preference between individuals. If the model is cardinal and most are weighted sums, it effectively places the members’ utilities on a common scale. If there were a real supra decision maker, then this could be done on the basis of her judgements; but we are assuming that there is no such person. So the resolution must assume that the group agree on a common numeraire of utility. The literature on social choice and particularly cost benefit analysis abound with discussions and examples showing the impossibly of finding such a common numeraire that allows one to truly compare preferences across individuals (Bedford et al., 2005; French, 1986; Hammond, 1991). In any case, models which construct group expected utilities also assume that all group members honestly reveal their beliefs and preferences and do not game the system, yet results such those of Gibbard (1973), Satterthwaite (1975) and Taylor (2005) suggest that this assumption is deeply flawed.

We, therefore, take the view that groups cannot make decisions, but are social processes that lead to a course of action by translating the wishes of their members in some way. Simplistically, the individual members vote and that leads to a course of action being implemented, though it might be better to say that the members 'behave' rather than use the language of voting. Often a chairperson or facilitator recognises agreement and the groups tacitly accept this by not doing anything to contradict this. Indeed, in some organisational structures the chairperson has all the authority and responsibility to impose his or her choice and the group accepts this, the other members' roles are more advisory or perhaps regulatory to check that some legal requirements are not missed. Moreover, groups usually have an existence that extends beyond the specific decision under consideration. Members may be playing a much longer game involving explicit or implicit horse-trading across several decisions, looking to support friends, or perhaps block the wishes of others. Often there are issues of personal or political career progression at stake. A group is generally a very complex social process and political behaviours may be rife – and political here may have a small or large 'p'!

Taking a group process view, decision analysis has developed a number of approaches for supporting groups of decision makers. Typical of these is the decision conferencing approach in which a facilitator works with the group in a meeting to build a putative decision model and then uses sensitivity techniques to explore differences between individual members (French, 2003; French et al., 2009; Owen, 2015; Phillips, 2007). The putative model is *not* a model reflecting group beliefs and preferences; group judgements are an illusion. It is purely a starting point for discussion and that discussion is articulated through sensitivity explorations. Usually the group reaches a shared understanding and agreement on a way forward, though that agreement may be driven in part by factors outside the immediate context of the decision. Group dynamics are driven not just by the immediate issues being discussed but also by loyalties, historical and future debts and favours and many other factors. Originally decision conferences were conducted face-to-face, but the advent of collaborative computing and Web 2.0 (Coakes et al., 2002; Nunamaker et al., 1991; Rios Insua and French, 2010; Tredinnick, 2006) has seen more and more approaches for conducting the deliberations online with participants taking part from different locations. Many forms of group decision support systems have been developed along such lines.

Decision conferences and similar methods which recognise groups as social processes were first used to support decision making groups in organisations. Organisations may be described in many ways, but one is particularly relevant here. Organisations have common objectives and values, or at least broadly they do. Whether they are profit maximising firms or government departments or NGOs, their members have broadly the same aims, at least in so far as their employment by or membership of the organisation is concerned. Thus, groups within organisations have far more values in common than a randomly selected group from the public at large. This organisational correlation means that decision conferences can generally help groups within organisations reach agreement on what to do. Of course, there will be times when this is not so. One only has to think of boardroom battles over the strategic direction of companies to realise that correlation of values may be far from 100%.

4 Societal Decision Making, Public Participation and Stakeholder Engagement

Global, national, regional and local societies are far less likely to share values than members of an organisation. International crises are often driven by starkly different values and objectives of different countries or groupings. In the worst case, values such as those held by ISIL can lead to civil war and terrorism (Siebert et al., 2015); but thankfully societies almost always solve their differences of opinion though peaceful means: political processes. In the following we have in mind democratic societies, recognising that this covers a multitude of political systems. In autocratic societies single person theories of risk and decision would apply, if such analyses were needed.

There are many definitions of democracy and many forms that it might take. Abraham Lincoln's overused phrase "*government of the people, by the people, for the people*" captures its essence; hence, its overuse. We have already referred to the paradoxes and impossibility results in the literature on voting and social choice which suggest that there can be no rational system of direct democracy in which everybody has a vote on every decision. In any case, with the complexity of modern society there are too many decisions for this to be possible. Thus modern democracies are procedural and representational. The people vote for representatives on local, regional and national governments. Those representatives take the decisions according to constitutionally agreed procedures. Moreover, some decisions are delegated to a range of agencies and regulators under the general oversight of the appropriate governments. Checks and balances along with legal systems ensure that broadly the will of the majority prevails, at least to an extent that avoids civil unrest.

Some 50 years ago, most societal decisions were made without much consultation. The elected representatives in councils and parliament generally got on with the business of governing. Consultation took the form of green and white papers.

Members of the public who were concerned with some aspect of a decision might write to their councillor, parliamentary representative or senator – or, perhaps, pen a letter to the editor of an influential newspaper. True, on some very significant issues there were protests and marches, e.g. nuclear disarmament; but broadly democracies were run through their elected representatives. The instruments of state, its cabinets, parliaments, ministries, regulatory bodies, and regional and local governments made the decisions. The last few decades, however, have seen many challenges to such full representative democracy. In many countries, there has been a growing cynicism and disillusionment, and a recognition of a democratic deficit (Steffek and Kissling, 2007), a clichéd term which seeks to capture a public feeling that its political institutions are no longer representative nor responsive. Disillusion with decision making which shapes the wider environment and social-economic context has not been confined to constitutional political processes. Discomfort with regulation – particularly science-based regulation – has also grown. Events and failures such as Bhopal, Seveso, Chernobyl and BSE (mad-cow disease) led to a growth in public distrust in the authorities and a rejection of many of the strategies proposed to deal with these issues. Business and industry too, as they have become bigger, more multi-national and dominant in their effect on the societies in which they are embedded have lost public confidence.

Against this background the past quarter century has seen a growth in public participation and stakeholder engagement (Bayley, 2008; Beierle and Cayford, 2002; Renn, 2008). Our societies may not be moving towards direct, deliberative democracy in general, but in specific areas of concern there is undoubtedly wider use of participatory methods in societal decision making, such as citizen juries, deliberative polls and stakeholder workshops. Governments, their agencies and regulators, as well as some businesses and NGOs regularly engage with stakeholders and the public.

How has decision analysis supported this move towards public consultation, engagement and involvement? The most obvious is through stakeholder workshops. During the 1990s when controversial public decisions were faced, there began to be a move towards managing discussions with a broad range of stakeholders through facilitated meetings. Nowadays such meetings are common in the processes leading up a public decision. In many such workshops the discussions are articulated through multi-criteria decision analyses (MCDA). MAV analysis is one of the methods commonly used on MCDA, but other non-Bayesian methods are widely used (Belton and Stewart, 2002). Gregory et al. (2005) and French et al. (2009), inter alia, describe how MAV analyses support stakeholder workshops. Papamichail and French (2013) chart the history of such approaches in nuclear safety and post-Chernobyl remediation. One point that should be noted is that whether or not the MCDA methodology is Bayesian, stakeholder workshops seldom address risk formally through encoding risk attitude and taking expectations against a probability distribution encoding the uncertainties. By and large, stakeholder workshops focus on understanding the stakeholders' perceptions on how conflicting objectives should be balanced; uncertainties are addressed to a much lesser extent.

Initially, stakeholder workshops were real meetings run along the lines of decision conferences. However, other instruments such as citizen juries, focus groups and town hall meetings have been developed for engaging stakeholders and the public. Moreover the advent of the Web and especially the interactivity of Web 2.0 have allowed these instruments to be conducted virtually. All can and, in many cases, do involve some elements of decision analysis to articulate the debate. Decision analysis has been at the heart of much of the movement to stakeholder engagement and public participation (Gregory et al., 2005; Rios Insua and French, 2010).

In parallel with developments in stakeholder engagement which addresses societal values on the right hand side of Figure 1, Bayesian statistical methods have been providing greater support for developing scientific understanding of the uncertainties involved (left hand side of Figure 1). Firstly, with the advent of MCMC and emulation methods and the enormous gains in computational power, Bayesian methods, once intractable now lie at the heart of complex statistical analyses and machine learning algorithms (Conti and O'Hagan, 2010; Gelman et al., 2013; Rogers and Girolami, 2015; Williamson and Goldstein, 2012). Secondly, there have been great strides in the development of structured processes to elicit and model the judgements of panels of experts to offer advice on uncertainty when data is sparse or impossible to collect in the timescales available (Dias et al., 2017; French, 2011; O'Hagan et al., 2006). As we have noted, the Bayesian Paradigm is individualistic, but the processes of Science may be looked as building a consensus on knowledge. A Bayesian analysis may model understanding of an individual scientist, but the building of consensus means that it will be effectively shared by other scientists (Box and Taio, 1973). Sensitivity analysis and robustness studies may be used to investigate residual differences (French, 2003; Rios Insua and Ruggeri, 2000). Thus advisory groups and expert panels which support government, its agencies and the wider society in developing their understanding of the science involved in a set of issues may be supported in a manner entirely compatible with the Bayesian model.

Bayesian methods bring further advantages in that it is natural and seamless to move from inference to forecasting and risk analysis. The posterior distribution provides encodes current uncertainties and allows one to forecast the outcomes of particular decisions along with uncertainty bounds on these. For instance, its form is precisely that needed as input to Monte Carlo risk modelling.

However, we would emphasise again that while information on the scientific uncertainties is provided to stakeholder and public discussion on the right hand side of Figure 1, there is seldom any formal modelling aimed at providing an integrated balance of uncertainties and values; analyses implied by the 'combine ⇒ advice' box are rarely conducted. The use of subjective expected utility in major societal decisions is still to become the rule rather than the exception. Nonetheless, the tools seem to be in place to use the Bayesian Paradigm to develop comprehensive and coherent analyses to completely support societal decisions.

5 Decision Analyses as an Input to a Political Process

One of us remembers the early 1970s. Books such as Raiffa (1968), DeGroot (1970) and, above all, Keeney and Raiffa (1976) suggested that Bayesian decision analyses with full use of subjective expected utility models were capable of supporting complex societal decisions. All the parts of the analysis would fit together coherently to provide a rational balanced perspective on all the issues involved in a problem. The siting of Mexico City Airport (De Neufville and Keeney, 1972; Keeney and Raiffa, 1976), a nuclear power station (Keeney and Nair, 1977) and similar studies (Bell et al., 1977) gave hope that these methods would find regular use in a matter of a few years. But that promise has not been fulfilled. Indeed, more careful studies of current approaches to societal decision making suggests that Figure 1 is too simplistic in having a single pathway on each side.

Usually several scientific studies are conducted independently by different government agencies focusing on aspects of the problem to assess the uncertainties. Pressure groups and other bodies may also fund further independent studies. Unaligned academics, researchers and others may offer further perspectives on the uncertainties. Sometimes consensus emerges, but far from always. Science may progress through consensus, but societal risk and decision analyses often concern emerging technologies, environmental risks, business systems or societal behaviours on which scientific consensus has yet to form. In any case, some stakeholders may reject an establishment science perspective and adopt a different worldview. So the single coherent, comprehensive analysis implied by the left hand side of Figure 1 is rarely present. We have noted that stakeholders may hold very different values and that a single comprehensive analysis bringing together all their perspectives is hardly ever conducted, or indeed possible. So there are several competing analyses on each side of the diagram.

Perhaps, we are just being impatient. Democracies evolve slowly. If we accept Informs Decision Analysis Society's chronology, the practical application of decision analysis is barely 50 years old. So it may be too soon to expect its full implementation in societal decision making, drawing together the disparate analyses on each side of the figure and pulling these into a single subjective expected utility analysis. Of course, this single analysis would not be taken as dictating the decision; rather the analysis combined with sensitivity explorations would articulate the deliberations within the decision making body, i.e. cabinet, parliament, or whatever, just as decision analysis guides the decision making of individuals and small groups. But we do not believe that we are being impatient. We simply do not believe that such an approach is practicable; societies are too diverse. Societies live with disagreements rather than resolve them. In our view, the task of decision analysis is not to resolve these disagreements, but to clarify them. It should enable a society's citizens and organs of state understand the different perspectives and values so that political processes can reach and implement a decision that will enable to society to move forward.

6 System 1 Societal Deliberation and System 2 Societal Deliberation

Taking this line of argument, what are the implications for the practice of decision analysis? Firstly, if our role is to clarify the different perspectives on an issue present within a society to inform the political processes which will take the decision, we need to recognise those political processes within our conception of decision analysis. In Figure 2 we recognised that if we are to inform individuals, we should recognise that they seldom act fully in accord with the rationality of the Bayesian paradigm. We had to acknowledge that System 1 Thinking might lead them to quick, but in retrospect ill-judged responses during elicitation and a poor, superficial understanding of the conclusions that might be drawn from the analysis. Interactions within the decision analytic process should be designed to help the decision maker move away from instinctive System 1 Thinking to more explicit, rational System 2 Thinking. Similarly to support decisions in society, decision analysts need to recognise potential tensions between informal societal discussion and more formal debate conducted within political structures laid down by the constitution. To emphasise this, we suggest the following terminology (Argyris and French, 2017):

- *System 1 Societal Deliberation*: In society many of the ways that individuals receive information, discuss issues and express their opinions are informal, such as a chat in a supermarket aisle or Tweets, blogs and similar social media. Free speech in modern democracies means that these means of communication are unstructured and subject to little or no control. They do much to shape opinion and influence voting, but have no formal, defined role in the political system.
- *System 2 Societal Deliberation*: Formal public and stakeholder input to political discussion and debate is much more structured and controlled. Its format is laid down by constitution, precedent and accepted ways of working: green and white papers, town hall meetings, political surgeries, stakeholder workshops, citizens' juries, referendums, etc. In many political systems, an independent judiciary can be asked to review whether consultation processes were fair and inclusive within the bounds set by the constitution and precedent, i.e. within the structures of System 2 Societal Deliberation.

As with System 1 and System 2 Thinking, we do not suggest that this is a strict dichotomy, but rather a gradation from completely informal, uncontrolled and unplanned conversations through to structured debate and consultation within constitutionally agreed frameworks. Where, for instance, but in the middle does public broadcasting fit? While it is usually free to choose issues and how they are explored, it may be required by the state to ensure that a full breadth of views are expressed. In using the dichotomous terminology to simplify our overall argument, we recognise that we skim over subtleties that will arise from the exact nature of a particular deliberation mechanism.

With this distinction in mind, we suggest that societal decision analysis should be seen as seeking to guide decision makers, stakeholders and the public towards the ideals encoded by normative theories, both of decision making and political constitution, within the context of a real, often ill-defined problem, mindful of their cognitive characteristics and informal deliberative mechanisms. See Figure 3. Note that the analysis, its presentation and the decision process should seek to provide support both at the individual level (top part of figure) and at the level of stakeholder groups and society itself (lower part of figure). The former support is aimed at ensuring each individual is informed and expresses his or her opinion in a manner which leans towards System 2 Thinking and guards against the superficial, instinctive irrationality 'traps' of System 1 Thinking. The latter support is aimed at ensuring that appropriate formal discussion and debate take place and that the ultimate constitutionally legitimate decision process – the System 2 Societal Deliberation – is well-informed, while providing information and guidance to and learning from informal social debate – the System 1 Societal Deliberation.

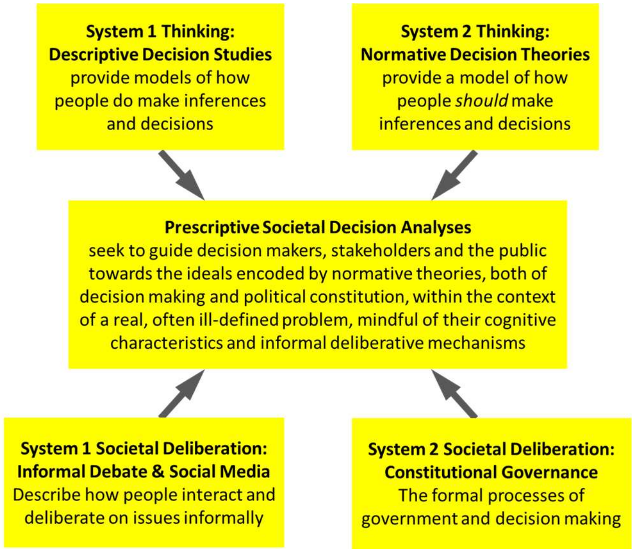

Figure 3: The interplay between System 1 Thinking, System 2 Thinking, System 1 Societal Deliberation and System 2 Societal Deliberation in prescriptive societal decision analysis

We have developed the distinction between the informal social processes of System 1 Societal Deliberation and the formal governance structures of System 2 Societal Deliberation in the context of societal decisions. But the same distinction applies in group and organisational decision making. It is just far less apparent. Common objectives, the involvement of fewer individuals, perhaps shorter timescales and tighter focus mean that less effort is needed in managing informal and formal deliberations. The distinction is neither as obvious nor as important in coming to a decision. That said, analysts facilitating decision conferences generally seek to understand both formal and informal relationships, power structures and accountabilities within the group (Eden and Radford, 1990).

7 Implications and Challenges for Societal Decision Analysis

7.1 Designing societal deliberation processes

Our view of societal decision making raises many challenges for decision analysis, ones that we might address poorly if we see our ultimate goal as a full implementation of the Bayesian Paradigm of Figure 1. Firstly, we may seek decision processes which draw in and reasonably reflect many of the concerns, issues and imperatives present in informal System 1 Societal Deliberations while ensuring the actual decision is taken legitimately within the governance structures required by System 2 Societal Deliberation, but it is far from clear how we should do so. Bayley and French (2011) note that while there is a lot of evidence that public participation and stakeholder engagement improve the acceptability of much societal decision making, relatively little evaluation has been undertaken to identify best or even good practice in these (see also Abelson et al., 2003; Abelson and Gauvin, 2006; Rowe and Frewer, 2000). To give but one example, suppose there are several distinct stakeholder groups, representing conflicting world views and values. Is it better to organise different workshops for each stakeholder group or to mix the stakeholders in multi-perspective workshops? There is little empirically tested evidence to provide guidance. Indeed it is moot what is meant by 'better' in these circumstances. Further what form should the workshops take: decision conferences, citizens' juries, or what? Bayley and French (2008) and Gilbert et al. (2016) have explored some tools that might help in the overall design of the decision process. The methods of collaboration engineering might be usefully developed to the societal context to provide more detailed planning of individual events (Briggs et al., 2003; De Vreede, 2006). The real issue, however, is that we do not have the evidence base on which to draw.

Figure 1 offers a neat separation of the issues in decision making: the analysis of uncertainty along a single pathway on the left and a single analysis of values on the right hand side. The two pathways are brought in a single maximisation of subjective expected utility. But we have argued that may be many expert views which articulate different worldviews with correspondingly different uncertainties. Moreover, some expert analyses may not use probability to encode uncertainty; indeed, some may not acknowledge any uncertainty. Statistical analyses may follow frequentist or other paradigms, different data sets may be used. Whatever views we might hold as Bayesians on the soundness of such methodologies, they will be used and their analyses will inform public and political debate on what may happen. Similarly and probably more commonly, different stakeholder groups may hold to different, quite contrary values. These too will need be brought into the debate. So the situation is much more that represented in Figure 4.

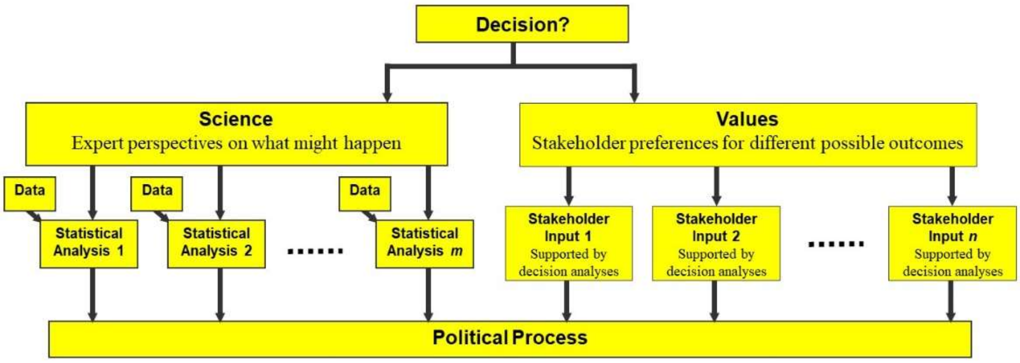

Figure 4: *Multiple analyses within societal decision making*

Decision analysts, risk analysts and statisticians have roles in supporting the parallel pathways on both the Science and Value sides. Their loyalties will be more perhaps to the expert or stakeholder group they are supporting than to the societal decision as a whole, and they will need their skills not just to support the deliberations within the group but also to support communication between groups. They may need to understand and critique other groups' analyses for their own group, as well as helping their group's analysis to others. If there is a possibility of different groups joining forces, they may need to support negotiations on the Values side (Raiffa et al., 2002) and meta-analyses on the Science side (French, 2012; Hartung et al., 2008). These are much more complex tasks than those apparent in the perspective offered by Figure 1 and they bring challenges to our community because if we see our role as providing guidance through the maximisation of subjective expected utility, we may miss many opportunities to support and inform societal deliberation and the political processes which will take lead to the ultimate decision.

The separation of science and values that remains in Figure 4 is itself a questionable assumption. Whether Science, i.e. knowledge, is value-laden is itself a question (Chalmers, 2013; Cortner, 2000), but it is certainly true that this neat separation is not always found in practice. Presentations of 'objective scientific facts' are often conveyed in value-laden frameworks; while arguments in support of particular values can embody rather specific worldviews. Perhaps one of the more significant benefits of helping individuals move from holistic System 1 ways of thought to explicit decision analytic System 2 thinking is that is encourages this separation, clarifying those aspects of arguments. For instance, debates in the UK on whether to legalise cannabis have arguably confused science and values, and the limitations of scientific advice in the policy process (Berridge, 2012; Nutt et al., 2010; Stevens and Ritter, 2013). We maintain the conceptual separation of Science and Values in the following, though we do recognise that it will be challenging to do so in practice.

7.2 The importance of facilitating communication

This view of societal decision analysis challenges us to recognise even more the importance of supporting communication between groups. Moreover, if we subscribe to the broad imperatives of democracy, then we surely cannot disenfranchise groups because they adopt different ways of thinking. We must recognise that many of the parallel statistical analyses on the left of Figure 4 will be supported by non-Bayesian analyses. The data used may have a dubious experimental design. Experts with very different views to conventional Science may have input their views. Over the past decades there have been many occasions when such views have held sway. The MMR vaccine issue that led to many children being unprotected arose because of a dubious publication on supposed side effects. But the parents accepted its conclusion despite apparently overwhelming evidence to the contrary (Bellaby, 2003; McMurray et al., 2004). Similarly on the Values side of the figure, we must recognise that some groups may reject Bayesian multi-attribute value or utility analyses and, indeed, *any* MCDA methodology, arguing that values are holistic, non-decomposable and that intangibles are not subject to any quantitative analysis. As statisticians, risk and decision analysts, we need to plan communication strategies alongside our developing analyses, recognising that we need both to help our clients explain their perspectives on the issues and also to understand the perspectives of other groups. Without attention to engagement and communication, the entire political process can be fraught with conflict, fail to build any consensus and may lead to very poor overall outcomes (Beierle and Cayford, 2002; Bennett et al., 2010; McDaniels and Small, 2004; Pidgeon and Rogers-Hayden, 2007; Renn, 1998; 2008).

7.3 Potential ways forward

Against this background what should decision analysts be doing to develop methodologies to support societal decision making? We have a few suggestions, but do not believe that our list is more than a beginning.

Problem structuring

Firstly, issue structuring and problem formulation has always been part of decision analysis, but it has not the emphasis that it needs – save for the literature on attribute tree structuring and value-focused thinking (Keeney, 1992; Keeney and Gregory, 2005). The soft OR literature offers many catalytic tools and techniques that can be used to help build decision models (Belton et al., 1997; French et al., 2009; Losa and Belton, 2006; Marttunen et al., 2017; Rosenhead and Mingers, 2001; Shaw et al., 2006; 2007). Moreover, such tools not only help develop models, but also set provide a framework for communication (French et al., 2005). Cognitive maps, rich picture diagrams, even checklists such as PESTLE, all provide succinct perspectives on the uncertainties, constraints, values, assumptions, etc. that are considered important to the group.

Scenario-focused decision analysis

Over the past decade, there have has been a growing convergence between quantitative and more qualitative approaches to analysis. We note particularly the growing interest in scenario-focused decision analysis (French, 2015; Montibeller et al., 2006; Ram et al., 2011; Schroeder and Lambert, 2011; Stewart et al., 2013; Wright and Goodwin, 1999). This combines ideas from scenario planning in which qualitative future scenarios are constructed to form backdrops for strategic debates (Schoemaker, 1995; van der Heijden, 1996) with conventional quantitative decision analysis. The aim is to generate several interesting scenarios and then build a decision analysis within each scenario based on the assumptions within that scenario. How 'interesting' is defined is moot. There are many possibilities. A scenario may:

- assume some particular event happens or does not, which can help address 'deep' uncertainties for which no agreed probabilities are available, different experts and models suggesting a wide range of plausible models;
- represent a worst case of some form, which can help bound possibilities;
- represent a best case of some form, again which can help bound possibilities;
- represent a most likely case in some sense, which is useful for maintaining a balanced perspective;
- develop a future that some of the parties to the discussion wish to explore, even if its likelihood is infinitesimally small, which can be help compare stakeholder values and differences.

While the decision analyses within each scenario should be coherent and rigorous, the analyses are not commensurate and directly comparable across scenarios (French, 2014; Stewart et al., 2010). Moreover, the scenarios do not form a spanning partition of the future; indeed, not only may some futures be missed, but some scenarios may overlap others. What they do is focus discussion and explore issues, assumptions and contingencies. Looking back to Figure 4, the fit of scenario-focused decision analysis with societal decision making is clear. The disparate experts and stakeholder groups effectively frame and explore different scenarios in their reports. Thus the nascent techniques of scenario-focused decision analysis can be drawn upon to inform and support societal deliberation and the general political process.

Further development of sensitivity techniques

Debates between different parties to a decision can be inflamed by disagreements over aspects of a decision model that are turn out to have little or no influence in determining the ultimate direction of the analysis. The decision analysis community has long realised how sensitivity analysis can inform deliberations and steer them towards the issues that matter (French, 2003; Keeney and Raiffa, 1976; Rios Insua, 1990). Thus in societal decision making we would expect increased emphasis on sensitivity techniques, and particularly intuitive graphical presentations of these. However we may go further. Eliciting a decision model and its judgemental inputs creates the possibility for disagreement. In many cases, disagreement can stimulate valuable discussion; but some disagreements will, as just noted, be irrelevant. If the modelling and elicitation process can be constructed in a way that reduces the number of judgements required of the participants while focusing those judgements on the more important issues, there will be less chance of being side-tracked into sterile debate. This was the motivation behind the development of interactive decision making algorithms several decades ago (French, 1984; Zionts and Wallenius, 1976). These methods concentrated on eliciting weights and parameter values within models, but more recently attention has switched to interactively eliciting the shape and functional behaviour of the model itself (Argyris et al., 2014; Greco et al., 2012). We believe that such approaches will have a role to play in avoiding and defusing some of the tensions between the different, parallel inputs to the political process, focusing attention on where agreements lie.

Communicating the import of analyses

We have noted that one of the implications of Figure 4 is that decision analysts will need to pay increased attention to communication between groups and the political process. Since the 1990s there has been much research into many aspects of risk communication, particularly to lay stakeholders (see, e.g., Bennett et al., 2010; Campbell, 2011; Fischhoff, 2008; Maule, 2008; Palenchar and Heath, 2007; Spiegelhalter and Riesch, 2011). Moreover, much of the accumulated knowledge has been encoded into advice on effective means of communication (see, e.g., Cabinet Office, 2011; EFSA, 2012; Risk and Regulation Advisory Council, 2009; US DHHS, 2002). However, informative though this literature is, it focuses more in the communication of risk and uncertainty than on decision analyses. Moreover, risk communication tends to deal with issues that are confounded with dread and other emotions driven by the risk context. We need a parallel development of communication tools to convey the implications of decision analyses to lay stakeholders and the public who have not been party to particular deliberations. The tools need convey the reasoning behind the recommendations in a measured way. Decision analysis still has a long way to go in communicating its results within the context of a wider public debate.

8 Concluding Remarks

It is our contention that the Bayesian paradigm of decision analysis, which has worked so well for supporting individuals, groups and organisations, will not extend fully to supporting major societal decisions. It will certainly help government agencies, stakeholder groups and others develop their views and perspectives, but the overall decision will be the result of a political process. We have argued that to support this process, we may need to deconstruct our paradigm, recognising that many different, often partial analyses relating to different scientific and stakeholder perspectives will feed into this process. Rather than chasing an impossible goal of bringing these analyses into a coherent whole, we should develop many more means of helping all parties to this political process understand and compare the import of each of these analyses, in relation its assumptions, data, and underpinning beliefs and values. We have suggested a few elements of a broad research agenda, but our main intention is to stimulate discussion.

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