

Price Comparison websites

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Abstract

The large and growing industry of price comparison websites (PCWs) or ‘web aggregators’ is poised to benefit consumers by increasing competitive pricing pressure on firms by acquainting shoppers with more prices. However, these sites also charge firms for sales, which feeds back to raise prices. I investigate the impact of introducing PCWs to a market for a homogeneous good. I find that introducing a single PCW *increases* prices for all consumers, both shoppers and non-shoppers. Under competing PCWs, prices tend to *rise* with the number of PCWs. I use the analysis to discuss relevant industry practices and policies. (*JEL: L11, L86, D43*)

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1. Introduction

Over the past two decades a new industry of price comparison websites (PCWs) or ‘web aggregators’ has emerged. The industry has enabled consumers to check prices of many firms selling a particular service or product simultaneously in one place. This promises to be particularly helpful to consumers in a world where prices of even seemingly homogeneous items are typically dispersed. The sites are popular in many countries, and in many markets including utilities, financial services, hotels, flights and durable goods.¹ These sites command billions of dollars of revenue annually.² In the UK, PCWs for utilities and financial services have been particularly successful. There are roughly 48 such PCWs in the UK, where over 70% of Internet users have used such a site. The largest four aggregators generated approximately £800m (\$1.2bn) in revenue during 2013, with average annual profit of the group increasing by 14% that year.³

The Internet has altered search costs, allowing consumers to compare prices across firms in a matter of clicks, intensifying competitive pricing pressure between firms. While a consumer may not know of all the firms in a market, a PCW can expose the full list of market offerings, maximizing inter-firm pressure. However, underlying this increased competition are the fees paid by firms who sell their products through the websites. As an example, these are understood

¹Examples for utilities and services include Moneysupermarket.com, Google Compare and Gocompare.com; for flights Skyscanner.net and Flights.com; for hotels Hotels.com and Booking.com; and for durable goods Amazon Marketplace, Pricerunner.com and Pricegrabber.com.

²Regarding travel services, Priceline Group (which owns Booking.com and Priceline.com) and Expedia Inc. (which owns Expedia.com and Hotels.com) made approximately \$6bn in total agency revenues in 2014. Regarding durable goods, Amazon Marketplace sold 2 billion items from third-party sellers. See their 2014 Annual Reports for details.

³Number of PCWs taken from Consumer Focus (2013) report into PCWs in the UK. The ‘big four’ refers to Money Supermarket, Compare the Market, Go Compare and Confused.com. Number of PCWs Usage data from the 2013 Mintel Report on Web Aggregators. Financial information taken from the companies’ own annual reports where available, otherwise inferred from parent group reports or newspaper articles. See the Appendix for details.

to be between £44-60 (\$69-95) for a customer switching gas and electricity provider in the UK.⁴ These fees, in turn, act as a marginal cost faced by producers, affecting their pricing decisions. The industry gleans substantial profits from these fees. As such, it is not clear whether the central premise that PCWs lower prices is valid. This tension is encapsulated in a quote from the BBC (2014):

“There’s another cost in the bill. It’s hidden, it’s kept confidential, and yet it’s for a part of the industry that appears to be on the consumers’ side. This is the cut of the bill taken by price comparison websites, in return for referring customers. The recommendation to switch creates churn in the market, and it is seen by supplier companies as worth paying high fees to the websites. Whether or not customers choose to use the sites, the cost to the supplier is embedded within bills for all customers.”⁵

This article examines this “churn” and addresses the fundamental question of when consumers are better off with a PCW in the marketplace. I characterize when all consumers are made worse off following the introduction of PCWs in homogeneous-good markets.

My model features two types of consumers: shoppers, who use PCWs in equilibrium, and inactive consumers, who buy directly from a particular firm. Although shoppers are always better off than inactive consumers in equilibrium, my main result is that *both* types are made worse off by the introduction of a single PCW. I then provide conditions under which consumers are harmed by the introduction of multiple competing PCWs. This is the first article in this setting to show such results, reversing those in the existing literature, which I show can be seen as a special limiting case.⁶ My model supposes that each shopper is aware of $q > 1$ of the

⁴See BBC (2015). Fees are significant in other sectors too e.g., in the hotel-reservation sector they are reported to be 15-25% of the purchase price (see Daily Mail, 2015).

⁵The concern was also expressed by US senator Amy Klobuchar regarding mergers of hotel-reservation sites: “The whole idea of cheaper hotels is very good, but if it all starts to come under one company, you can easily foresee the situation where they can charge higher commissions that are then passed on to consumers.” (New York Times, 2015)’

⁶This is in the absence of any persuasion or direction of consumers to more expensive products by firms (e.g., Armstrong and Zhou, 2011), or biased intermediaries (e.g., de Cornière and Taylor, 2016).

n firms in the market, and in the absence of a PCW checks these firms' websites, learning q prices. After the introduction of the PCW, they visit the PCW where they learn all n prices in equilibrium. I show that the expected equilibrium prices are raised by the introduction of a single PCW, making all consumers worse off. I also show the result is broadly robust to the addition of other market features including competing PCWs, ad valorem PCW fees, price discrimination and the extensive search margin.

Regarding the model, a primary novel feature is that I obtain price dispersion in equilibrium with or without price comparison websites. Descriptively, this feature mirrors the reality that price dispersion has been a pertinent feature in markets over time, even for seemingly homogeneous products. Normatively, in a world where consumers are only informed about a subset of prices, price dispersion implies that there is some positive probability they do not all see the lowest price. This in turn provides the economic rationale for a player which can reveal the complete set of market offerings, namely a price comparison website. Furthermore, price dispersion is not an assumption in my model, it arises endogenously in equilibrium from the fact that some consumers have incomplete information.

The main result of this paper is that the equilibrium fee a single PCW charges for a sale through its site is so high that it more than negates any benefits from the increased firm competition. It may then be natural to conjecture that allowing multiple competing aggregators will undo this result, akin to textbook Bertrand competition, but at the aggregator level. Further, it has been a challenge in this class of model to analyze competing PCWs.⁷ I extend the model to allow for multiple PCWs, and for shoppers to check any number of them. My characterization shows that both the number of PCWs and the number of PCWs that shoppers check, matter. Concretely, in the most direct robustness check of the main result and perhaps the most realistic scenario, when shoppers only check one of many PCWs, they all effectively remain monopolists and so consumer welfare does not improve with any number of PCWs. At the other extreme, where shoppers check all PCWs, Bertrand-style reasoning at the aggregator level results as a special case: PCWs undercut each other's fees to reach a unique zero-profit equilibrium and shoppers benefit from their existence. When shoppers visit some, but not all

⁷Here I refer to 'clearing-house' models; see Section 2 for a review.

PCWs, consumer welfare tends to rise with the number of PCWs checked, but it falls with the number of PCWs. In particular, in this plausible scenario, there is a critical number of PCWs beyond which all consumers can be worse off than without any aggregators at all.

This article makes several suggestions to regulators, many of which are novel. Firstly, the baseline analysis indicates that for markets in which price is the primary focus, the introduction of profit-maximizing price comparison websites can harm consumers. Therefore, where the introduction of PCWs into a market is itself a policy, it may reduce consumer welfare. Furthermore, in such markets, measures including capping PCW fees or providing a publicly-run site could be considered. In a setting with multiple PCWs, the results suggest that either encouraging consumers to check more PCWs, or limiting the number of PCWs would be beneficial to consumer welfare. Secondly, I use the framework and the analysis to discuss a range of policies including price-clauses, meta-sites and fee transparency. ‘Most-favored nation’ (MFN) price clauses are imposed by PCWs and stipulate that a firm listing on the PCW cannot set a lower price elsewhere. These clauses have been studied by many recent papers. In sync with this literature, my model suggests that (wide) MFNs are detrimental to consumers. In my model this occurs because these clauses tend to soften competition between PCWs. I also use the model to examine issues that to my knowledge have not been analyzed previously. I show that ‘meta-sites’, which can aggregate prices listed on firm websites and those listed on PCWs present the same fundamental problem as in the baseline model. That is, a meta-site extracts the surplus from PCWs that in turn pass it on to firms, that in turn pass it on to consumers in higher prices. I also show that forcing PCW fees to be publicly visible can stimulate inter-PCW competition, but relies on firm-consumer coordination over the fees, which I argue is unlikely in reality.

The article proceeds as follows: Section 2 reviews the literature; Section 3 presents the model; Section 4 conducts comparative statics with a monopolist PCW; Section 5 models competing aggregators; Section 6 examines extensions of the model allowing for ad valorem fees, price discrimination and the extensive search margin; Section 7 uses the analysis to discuss policy; Section 8 concludes.

2. Literature

It is well known that some forms of intermediation can reduce welfare e.g., by double marginalization. There, an upstream firm supplies an intermediate good at a cost to a downstream firm that in turn produces the final product. The distinction here is that a PCW directly affects consumers' information and hence also competition. This capacity of PCWs to increase competition between firms means that there is scope for such an intermediary have beneficial effects, especially for consumers. In pioneering work, Baye and Morgan (2001) investigate the strategic incentives of a PCW or 'information gatekeeper'. My model builds on this conceptualization of a PCW as a provider of information but is distinct in many key respects, reflecting the changes in technology and practices in the industry over the last two decades. I emphasize the most important of these developments here to provide a contrast. In the classic Baye and Morgan setup, without the PCW each consumer is served by a *single* 'local' firm which sells at the monopoly price (it is too costly for consumers to travel to another store). This leads to the result that consumers benefit from the introduction of a PCW, because firms must compete for the business of consumers who enjoy free access to the site. In the modern online marketplace however, firms also have their own websites.⁸ In the absence of an aggregator, consumers do not need to physically travel to purchase the good; they can visit another firm's website just as easily as they could an aggregator's. This suggests that without such a clearing-house, it is unlikely that no consumer compares prices. This paper contributes by showing that when at least *some* consumers engage in *some* comparison in a setting without an aggregator, the introduction of one can cause prices to be higher for *all* consumers.

More broadly, this article contributes to the literature on 'clearing-house' models (see for example, Arnold et al., 2011; Arnold and Zhang, 2014; Baye and Morgan, 2001, 2009; Baye et al., 2004; Chioveanu, 2008; Moraga-González and Wildenbeest, 2012; Rosenthal, 1980; Salop and Stiglitz, 1977; Varian, 1980). These models rationalize price-dispersion in homogeneous goods markets.⁹ Indeed, price dispersion has persisted despite the advances of tech-

⁸Often, a PCW simply re-directs users to the selected firm's own website to complete the purchase.

⁹The search-cost literature also provides explanations of price dispersion (e.g., Stigler, 1961; Burdett and Judd, 1983; Stahl, 1989, 1996; Ellison and Ellison, 2009; Ellison and Wolitzky, 2012). Motivated by the rise of

nology such the Internet and comparison sites. Early studies documented marked dispersion in the online markets for goods (e.g., Baye et al., 2004; Brynjolfsson and Smith, 2000). A more recent study by Gorodnichenko et al. (2014) finds substantial cross-seller variation in prices, and voices support for clearing-house models that categorize consumers into loyal and shopping consumers. Congruent with the empirical work, the equilibria in my model feature price dispersion regardless of whether there is an aggregator (or ‘clearing-house’). Without a PCW, this is because *some* price comparison is undertaken by consumers. That there is price dispersion without an aggregator is a novel feature of my model which makes clear the economic rationale for such a platform: to provide price information in a market where prices are different.¹⁰ Furthermore, producing price dispersion with a PCW that employs the pricing mechanism seen in practice, is a challenge. In the aggregator industry, fees are typically charged to firms rather than consumers. Previous models have shown that a clearinghouse would set such buyer-side fees such that all buyers attend the platform e.g., Baye and Morgan’s original work and Moraga-González and Wildenbeest (2012) who show the result also holds under various forms of product differentiation. In my model, I abstract from buyer-side fees but note that all shoppers visit the PCW in equilibrium. Regarding the type of fees charged to firms, there has been a shift away from charging one-off fixed fees toward pay-per-sale fees i.e., setting the fixed component of a two-part tariff to zero. This transition is rationalized as profit-maximizing PCW behavior by Baye et al. (2011). However, without the introduction of some other exogenous fixed cost to a firm of listing on the PCW (e.g., transaction costs), price-dispersion vanishes in equilibrium. I do not deny the existence of such additional costs, but emphasize that dispersion arises in my framework without an appeal to fixed listing costs.

The larger relevant literature is that of two-sided markets, initiated by Rochet and Tirole (2003) (see also Armstrong, 2006; Caillaud and Jullien, 2003; Ellison and Fudenberg, 2003; Reisinger, 2014). These articles model platforms where buyers and sellers meet to trade, focus-

the Internet, clearing-house models abstract from a direct modeling of consumer search costs. The frameworks are to some extent isomorphic (Baye et al., 2006).

¹⁰More precisely, equilibrium price distributions have no point masses which implies that the probability of any two prices being the same is zero.

ing on platform pricing and the effect of network externalities with differentiated products and platforms. The models do not explicitly model seller-side competition, which is central to my setting. More recent contributions (e.g., Belleflamme and Peitz, 2010; Boik and Corts, 2016; Hagiu, 2009; Johnson, 2017) do, but they model the platform as the only available technology, which is not appropriate for the questions I address.¹¹ Other recent contributions allow sellers and buyers to conduct business off-platform in markets where products, and often platforms, are differentiated. There, models such as Edelman and Wright (2015) show that platforms may over-invest in buyer-side benefits such as rebates. In related work, Wang and Wright (2016) provide a model of ‘show-rooming’ where the platform is assumed to ease the comparison of products. In contrast to these works, I model a homogeneous-good market, isolating price as the determinant of consumer welfare, where the only benefit that a platform brings is purely informational: it lists available prices. The potential benefit of a PCW to consumers is that it can lower prices via the interaction of strategic, competing firms who choose to list their prices there. Any benefit offered by the platform is therefore determined endogenously via the equilibrium actions of firms and consumers. Furthermore, as opposed to models which exogenously assume a benefit for buyers from using the platform, my model endogenously produces the rationale for consumers to use it: prices in the marketplace are dispersed.

3. Model

3.1 A World Without a PCW

There are n firms and a unit-mass of consumers. Firms produce a homogeneous product at zero cost without capacity constraints. Consumers wish to buy one unit and have a common willingness to pay of $v > 0$. Each consumer is endowed with a ‘default’, ‘current’ or ‘preferred’ firm from which they are informed of the price. This assumption has many natural interpretations. In a market for services or utilities (e.g., gas and electricity tariffs, mortgages, credit cards,

¹¹Indeed, in related work Johansen and Vergé (2016) argue that some of these models rely crucially on the implicit or explicit assumption(s) that firms do not compete against each other and/or do not choose whether to sell through the platform. I place inter-firm competition at the center of the model and allow firms to choose whether to participate on the platform.

broadband, cellphone contracts, car, home and travel insurance etc.) consumers can be thought of as having a current provider for the service for which they know the price they pay and the renewal price should they remain with the same provider.¹² In the market for flights, hotels or durable goods, consumers can be thought to have a carrier, hotel (or hotel chain) or firm which they prefer or use regularly, perhaps bookmarked in their browser or for which they receive marketing emails.

A proportion $\alpha \in (0, 1)$ of consumers are ‘shoppers’. Casual empiricism suggests that many people enjoy browsing or looking for a bargain, preferring to see many prices before making a purchase. Shoppers are aware of $q - 1$ rival firms, $q > 1$ firms in total (including their default firm). They may know the price offered by some of these firms already, but in case they do not, these are revealed online at negligible cost, by checking these firms’ websites. A reduced form interpretation is therefore that shoppers know $q > 1$ of the n prices. A shopper sticks with his or her default firm if its price is not beaten by another price. If a rival firm’s price is cheaper, the shopper switches.

The remaining proportion of $1 - \alpha$ consumers are ‘auto-renewing’, ‘loyal’, ‘offline’ or ‘inactive’ consumers who do not shop around. In markets where firms are service providers, such consumers simply allow their contract with their existing provider to continue with their default firm. Much of the furore surrounding PCWs has been directed at those operating in the services and utilities sectors. In the presentation that follows I adopt terminology suited to that sector, of ‘current’ firms or providers with consumers referred to as ‘shoppers’ and ‘auto-renewers’.

I assume that each firm has an equal share of current consumers of each type ($\frac{\alpha}{n}$ shoppers and $\frac{1-\alpha}{n}$ auto-renewers). Firms set prices and shoppers simultaneously collect price information. I focus on equilibria in which firms adopt identical pricing strategies and shoppers’ information sets are uniformly distributed. Going forward I refer to such symmetric equilibria simply as equilibria.

This setup generalizes Varian (1980), nesting his equilibrium.¹³ He motivates the two types

¹²Providers usually provide renewal price information directly to their consumers.

¹³The only differences being that I assume zero costs and do not employ his zero-profit condition. For related

as being completely ‘informed’ about all n prices, or ‘uninformed’. The informed buy the cheapest on the market, while the uninformed buy from their default firm. In my model, shoppers are informed of q prices where $1 < q \leq n$ and I derive the unique equilibrium. When $q = n$, Varian’s equilibrium corresponds to that derived in Lemma 1 below.

Shoppers can be characterized by $\binom{n}{q}$ groups. Each group is a list of the firms checked by that consumer. For example, consider $q = 2$, $n = 4$ with firms indexed 1, 2, 3, 4. Then there are 6 possible comparisons shoppers could make: $\{12, 13, 14, 23, 24, 34\}$, where the two digits refer to which firms’ prices are checked. Each firm is involved in $\binom{n-1}{q-1} = 3$ price comparisons. Shoppers employ symmetric shopping strategies, and hence are evenly distributed across these groups: $\frac{1}{6}$ compare the prices of firms 1 and 2, $\frac{1}{6}$ compare firms 1 and 3, and so on. Equivalently, one could interpret a consumer as randomizing uniformly over which rival-firm websites she or he checks.¹⁴

Now consider the best-response of firms. Without loss of generality, let $F : p \rightarrow [0, 1]$ denote the cumulative distribution function of prices charged by firms in equilibrium. Lemma 1 describes the equilibrium without a PCW.

Lemma 1. *In the unique equilibrium firms mix according to the distribution*

$$F(p) = 1 - \left[\frac{(v-p)(1-\alpha)}{qp\alpha} \right]^{\frac{1}{q-1}} \text{ over the support } p \in \left[\frac{v(1-\alpha)}{1+\alpha(q-1)}, v \right].$$

Price dispersion is a central feature in clearing-house models and showcases how they rationalize price dispersion in homogeneous goods markets. Furthermore, the dispersion itself provides an economic rationale for the existence of price comparison websites. The limited-search assumption does not alter the intuition for why dispersion occurs. Bertrand-style reasoning of undercutting down to the marginal cost (here normalized to 0) does not play out. A firm can guarantee itself a profit of at least $\frac{v(1-\alpha)}{n} > 0$ from its auto-renewers. Therefore, any point mass in equilibrium strategies must be for some $\hat{p} > 0$. Any such mass would always be undercut by

results, see Burdett and Judd (1983, Section 3) and Janssen and Moraga-González (2004).

¹⁴Although I focus on a symmetric distribution across these pairs, this shopper behavior is not an assumption per se; it is a best response in equilibrium to symmetric firm behavior because consumers are then indifferent to which firm they check.

firms to gain a discrete number of shoppers for an arbitrary ϵ -loss in price.

Models building on Varian's assume that in a world without a clearing-house, consumers cannot check other prices, so the pure monopolistic-price equilibrium of $p = v$ results. As in Stahl (1989), were there no shoppers, I would obtain such a Diamond (1971) equilibrium with each firm charging v ; and were there only shoppers, then the Bertrand outcome of $p = 0$ would result. Unlike Stahl, shoppers do not necessarily know the prices of all firms, but they know at least two. Lemma 1 shows that with some (but non-zero) comparison, price dispersion still emerges in equilibrium.

It is also instructive to note that equilibrium pricing does not vary with the number of firms, n , as long as $q < n$. Each shopper compares q prices, regardless of the total number of firms. When making pricing decisions, a firm is not concerned about the number of other firms per se, but rather about the other prices shoppers know. As all firms price symmetrically and independently in equilibrium, it is as if each firm only faces $q - 1$ rivals. In other words, what matters is the number of comparisons shoppers make, not the number of firms in the market.

3.2 A World With a PCW

Suppose an entrepreneur creates a price comparison website.¹⁵ I add a preliminary stage to the game at which the PCW sets a 'click-through fee' $c \geq 0$ that a firm must pay to the aggregator per sale made via the site. This is the type of fee often used in practice, for example as reported by the BBC, the PCW receives "a flat fee every time a customer buys a policy, and that these fees are "completely independent" of policy price".¹⁶ Consumers do not learn c , although they will have correct expectations in equilibrium.¹⁷ Each firm sees the fee c , chooses a price, and whether or not to post it on the PCW. Shoppers are aware of the PCW, in addition to q firms.

¹⁵In equilibrium, PCWs will make positive profits so an entrepreneur is incentivized to create a PCW, so long as any fixed costs are not too large.

¹⁶In a clearing-house setting, Baye et al. (2011) allow for any two-part tariff (consisting of fixed and per-unit components) to be set and show that a profit-maximizing PCW would optimally set the fixed component to zero. In some markets, the per-unit fee is ad valorem. In Section 6.1 I show that the main result of this article continues to hold under ad valorem fees.

¹⁷As the BBC quote in the Introduction notes, the exact fee is "kept confidential", not publicly announced.

As before, if their current provider is revealed to be the cheapest, they stick with them.

I assume that such auto-renewals occur with firms off the PCW, a distinction that did not arise without a PCW as consumers could only buy directly from a firm. This ‘tie-breaking’ rule carries natural motivations in the relevant markets when a shopper discovers their default firm is the cheapest: in services and utilities markets, these shoppers need take no further action for their contract to renew; in the travel reservations market, these shoppers may then take no further action, instead allowing a secretary, colleague or spouse to book directly with a default firm. It is in this way that firms get some surplus from shopping consumers in equilibrium which will allow there to be price-dispersion in equilibrium.¹⁸

Throughout the paper, I focus on symmetric equilibria in which shoppers only check PCWs. In equilibrium, firms list on the PCW with probability one and prices are dispersed. This means that there are n distinct prices on the PCW, which provides shoppers a strict incentive to check the PCW to learn all n prices. Once they see all n prices on the PCW, if their current provider is not the cheapest, I assume they switch provider there and then from the cheapest firm.¹⁹

With a monopoly PCW, I find there is a unique equilibrium.²⁰ To find the equilibrium, I use the following lemma which takes c as fixed and characterizes the ensuing mutual best-responses of firms. Define $G(p; c)$ to be the cumulative distribution function of prices charged by firms for a given click through fee c :

$$G(p; c) = 1 - \left[\frac{(v - p)(1 - \alpha)}{\alpha(np - (n - 1)c)} \right]^{\frac{1}{n-1}} \text{ over the support } p \in \left[\frac{v(1 - \alpha) + c\alpha(n - 1)}{1 + \alpha(n - 1)}, v \right].$$

Lemma 2. *The mutual best-responses of firms as a function of c :*

¹⁸Note that relaxing this tie-breaking rule would only strengthen the qualitative results of this article: If there were some probability $\gamma > 0$ that shoppers whose default firm is the cheapest instead buy through the PCW, this increases the traffic through the PCW, decreasing the outside option of the firms, allowing it to raise its fee higher.

¹⁹Notice there is no incentive to go additionally to a provider’s website directly, as the PCW revealed all price information.

²⁰More generally there exist other equilibria e.g., trivial equilibria in which no firms or shoppers attend the PCW. The set-up is then identical to that of the previous section and the equilibrium is given by Lemma 1 with the vacuous addition that the PCW can charge any c . There also exist asymmetric equilibria where not all firms list on the PCW (see Footnote 21).

1. If $c \in [0, v(1 - \alpha))$, firm best-responses are described by $G(p; c)$, and have no point masses.
2. If $c = v(1 - \alpha)$, there are two classes of responses, one with no point masses described by $G(p, c)$, and those in which all firms charge the same price, $p = v(1 - \alpha)$.
3. If $c \in (v(1 - \alpha), v]$, all firms charge the same price, p , where $p \in [v(1 - \alpha), c]$.

Where pricing is described by $G(p; c)$ each firm always lists its price on the PCW.

When c is in the lower interval described by Lemma 2, firms undercut each other until the point at which they would be better off charging the maximum price v and only selling to their auto-renewers. This trade-off between exploiting loyal consumers and trying to entice shoppers leads to price dispersion in equilibrium, as it did in the world without a PCW. However, if c exceeds $v(1 - \alpha)$, undercutting does not reach this point, so no firm would jump to v . Rather, all firms charge the same price, selling to all their consumers directly, bypassing the PCW's (high) fee. To illustrate in more detail why this is an equilibrium when $c \geq v(1 - \alpha)$, consider two potential unilateral deviations from an equilibrium price $p \in [v(1 - \alpha), c]$: (i) undercutting rival firms' prices and (ii) charging v . Undercutting others to some $\hat{p} < p$ and listing on the PCW would win all rival firms' shoppers, but would do so at a loss because $\hat{p} < c$. Undercutting others and not listing \hat{p} would result in sales only to their own consumers, netting $\frac{\hat{p}}{n}$, which is lower than the equilibrium profit $\frac{c}{n}$. Jumping from the equilibrium price to v results in sales only to their $1 - \alpha$ auto-renewers, netting $\frac{v(1-\alpha)}{n}$, but this is less than the $\frac{p}{n}$ made in equilibrium by selling to their auto-renewers and shoppers at the lower price p . Finally, if $c = v(1 - \alpha)$ then both mixed and pure equilibria obtain as threshold cases.

To derive the equilibrium fee set by the PCW, consider its incentives. The PCW will make zero profit if firms all charge the same price, as then no shoppers switch. In contrast, the PCW earns positive profit in any mixing equilibrium wherever $c > 0$. The PCW thus has a strong incentive to induce price dispersion because shoppers switch when they can obtain a strictly lower price with a new firm. A feature of distributions with no point masses is that the probability of a tie in price is zero. As a result, shoppers at $n - 1$ of the n firms will switch. Given that firms mix in this way, the PCW will raise c as high as possible before reversion to a pure equilibrium, which here happens for $c > v(1 - \alpha)$. Lemma 3 characterizes the

equilibrium.²¹

Lemma 3. *In the unique equilibrium, the PCW sets a click-through fee of $c = v(1 - \alpha)$, firms list on the PCW and mix over prices according to $G(p; v(1 - \alpha))$ over the support $p \in [v(1 - \alpha), v]$.*

That only pure equilibria exist for $c > v(1 - \alpha)$ is what limits the PCW fee and allows there to be price dispersion at equilibrium without fixed costs for firms. It is the outside option of current shoppers for firms that limits the fee that the PCW charges. Shoppers know their current provider's price and stay with their current firm if there are no lower prices to be found on the PCW. Hence, a firm can always guarantee itself its own shoppers and avoid the aggregator's fee by charging a price low enough to undercut the market and not list on the PCW. It is this threat that discourages the PCW from charging fees that are even higher in equilibrium. If firms did not have this outside option, the PCW could raise its fee to $c = v$, firms would charge $p = v$, price dispersion would be lost and the PCW would be able to extract all the surplus.

4. Comparative Statics

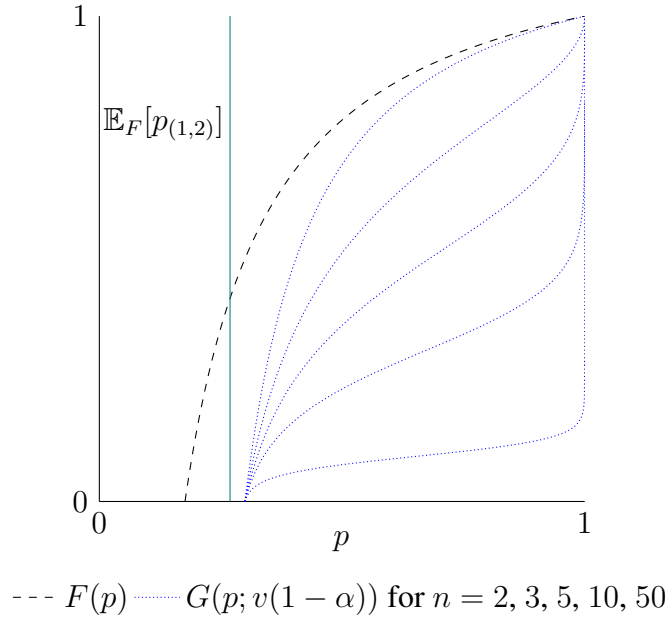
Comparing the equilibria of Lemmas 1 and 3 reveals how the PCW affects consumer welfare.

Proposition 1. *Both types of consumer are worse off with the PCW than without.*

The key to the proof is to show that the expected shopper-price under F is *less* than the lower bound of the support of G (see Figure 1). It immediately follows that shoppers expect to pay more under G , as the expected lowest price is even higher. That G first-order stochastically dominates F shows that auto-renewers expect to pay more under G , as the expected price is higher.

²¹ Note that in the symmetric equilibrium, all firms list on the PCW. In practice, some firms do not always list on PCWs. Note that here there exist asymmetric equilibria where $m \geq 2$ firms list on the PCW, mixing over prices $[v(1 - \alpha), v]$ in a similar way to the CDF of Lemma 3, where the other $n - m$ do not list, charging v and selling only to their auto-renewers, with all shoppers uniformly spread over the mixing firms. The price-rising result of this article also holds in any of these asymmetric equilibria.

Figure 1: Equilibrium price distributions calibrated with $q = 2, v = 1, \alpha = .7$



With the introduction of a PCW, the two effects stated in Corollaries 1 and 2 give rise to Proposition 1:

Corollary 1. *Within the mixed-price equilibrium firm responses of Lemma 2, as $c \in [0, v(1 - \alpha)]$ increases, the expected price paid by both types of consumer increases.*

Corollary 2. *As the number of firms increases, the expected price paid by shoppers falls, but the expected price paid by auto-renewers rises.*

Corollary 1 shows that when the PCW sets a higher fee, the expected price paid by both shoppers and auto-renewers rise. The fee is passed on by firms to consumers through a first-order stochastic shift in the prices set in equilibrium. Upon winning, a firm must pay the PCW for all of the $(\frac{n-1}{n})$ shoppers who purchase through the site. Because the amount paid rises with the fee, the price charged in equilibrium also rises with the fee.

The second effect is that the PCW increases competitive pressure among firms to fight for all $n - 1$ rival firms' shoppers. In contrast, without the PCW, firms effectively competed against only $q - 1$ rivals. Different models in the clearing house literature offer different predictions about the effect of n on equilibrium prices. Some derive distributions for which an increase in n raises prices for both types of consumer (e.g., Rosenthal, 1980); while in other models it

lowers prices for shoppers and raises prices for captive consumers (e.g., Morgan et al., 2006; Varian, 1980).²² My model belongs to this second category. An increase in n has two effects on equilibrium prices. First, increased competition pushes probability mass to the high-price extreme of the distribution, as Figure 1 shows. This results in a first-order stochastic ordering in n : expected price thus increases in n and auto-renewers pay more. Second, shoppers now pay the lowest of $n + 1$ prices rather than n , which reduces the expected lowest price. Corollary 2 reveals that this second effect more than offsets the first, implying that shoppers pay less in expectation when there are more firms.²³

In order to relate these two effects of a PCW to Proposition 1, I make the following remark.

Remark 1. *If $q = n$, $G(p; 0) = F(p)$.*

The entrance of a PCW increases both the fee firms pay, from 0 to $v(1 - \alpha)$, and the number of prices that shoppers compare, from q to n . The first effect raises the expected price auto-renewers pay, which is compounded further by $n > q$, so they are unambiguously worse off with a PCW. The effects pull shopper welfare in opposite directions, but Proposition 1 shows that no matter how large n is, it fails to undo the effect of the optimally chosen PCW fee c . Hence, shoppers are also worse off in expectation with a PCW.

Due to the constant-sum nature of the game, welfare necessarily sums to v in equilibrium. As firms make the same expected profit in both worlds, there is a one-to-one relation between the decrease in consumer welfare and the profits of the PCW. That the PCW does not reduce firm profits comes from the fact that (with or without the PCW) firms have $(\frac{1-\alpha}{n})$ auto-renewers. Firms can therefore guarantee themselves $v(\frac{1-\alpha}{n})$ in both worlds. Although this article focuses on consumer rather than producer welfare, it is important to point out that the incentives of the PCW and firms are not aligned. This is because there is exactly one cheapest price and hence

²²Also see Janssen and Moraga-González (2004, Footnote 11) and Baye et al. (2006) for a discussion on this point.

²³Although the result of Corollary 2 is common to clearing-house models, it may seem a rather nuanced prediction of equilibrium pricing. Morgan et al. (2006) conduct an experiment with participants playing the role of firms against computerized buyers and found that when n was increased, prices paid by inelastic consumers indeed increased whereas those paid by shoppers decreased.

$\alpha \left(\frac{n-1}{n}\right)$ shoppers who switch from non-cheapest prices, which increases with n . An increase in n however, would of course, squeeze per firm profit. Therefore, a PCW would always encourage market entry if it could.

One channel through which all consumers would gain is by more auto-renewers becoming shoppers:²⁴

Corollary 3. *As the proportion of shoppers α increases, expected prices paid by shoppers and auto-renewers both decrease.*

One may also conjecture that a PCW would want to maximize α (the number of shoppers) in order to obtain more referral fees. However, this logic is incomplete. Expanding the PCW's action set to include the determination of α (one can think of the PCW determining α through advertising) yields the following result:²⁵

Corollary 4. *If the PCW can determine α as well as c in the preliminary stage, then the PCW sets $\alpha = \frac{1}{2}$.*

PCW revenue is hump-shaped in α . As $\alpha \rightarrow 0$ it receives less and less traffic, and hence vanishing revenue. As $\alpha \rightarrow 1$, firms have fewer auto-renewers to exploit, which pushes $v(1 - \alpha)$, the maximum fee for which firms are willing to mix, to zero. Indeed, when $\alpha = 1$ all consumers are shoppers and the PCW removes any incentive for firms to increase prices as there are no auto-renewers to exploit. As a result, all firms charge the same price and no shoppers switch providers, leaving the PCW with zero profit. Thus, even if the PCW could bring all consumers online, it has a strict incentive to ensure that only some do so.

5. Competing Aggregators

In this section I extend the model to allow for multiple PCWs, and for shoppers to check any number of them. First, I present a direct robustness check of the preceding results and perhaps

²⁴This is a prediction common to clearing-house models, for which Morgan et al. (2006) find experimental support.

²⁵ α is determined costlessly for the PCW. If this advertising costs were a convex function of α , it would not change the results qualitatively.

the most realistic scenario, when shoppers only check one of many PCWs. In this case, PCWs effectively remain monopolists and so consumer welfare does not improve with any number of PCWs. At the other extreme where shoppers check all PCWs, Bertrand-style reasoning at the aggregator level results as a special case: PCWs undercut each other's fees to reach a unique zero-profit equilibrium and shoppers benefit from their existence. When shoppers visit some, but not all PCWs, consumer welfare tends to rise with the number of PCWs checked, but it falls with the number of PCWs. In particular, in this plausible scenario, there is a critical number of PCWs beyond which all consumers can be worse off than without any aggregators at all.

Suppose there are $K > 1$ PCWs indexed by $k = 1, \dots, K$. Each PCW moves simultaneously in the first period with PCW _{k} setting a fee c_k . A crucial measure of competitive pressure is the number, r , of aggregators shoppers check where $1 \leq r \leq K$. One can interpret $r \leq K$ as the number of aggregators that shoppers are aware of. Although shoppers will be indifferent to which set of PCWs they check, PCW fees and firm-pricing strategies depend on r . Hence, there are different sets of equilibria for each r . This leads me to index equilibria by r . I first consider the case where shoppers check just one of the PCWs.

Proposition 2. *With $K > 1$ competing PCWs, if shoppers check $r = 1$ PCW then both types of consumer are worse off with the PCWs than without.*

Proposition 2 obtains because the high-fee, single-PCW equilibrium of Lemma 3 with $c = v(1 - \alpha)$ remains the unique equilibrium fee.²⁶ Introducing competing PCWs exerts no downward pressure on fees. There are no equilibria at lower fee levels because shoppers only check one PCW, which means there is no incentive for PCWs to undercut each other's fees. If they did, they would not increase their volume of sales, but they would receive lower fees. In contrast, for any candidate equilibrium with $c < v(1 - \alpha)$, there is an incentive to *raise* the fee because PCWs can maintain the same volume of sales and earn a higher fee.²⁷

Proposition 2 is the primary robustness check of the price-rising result Proposition 1. In the

²⁶As in the monopolist case, firms' outside option gives rise to pure-pricing firm responses at higher fee levels, which bounds equilibrium fee levels from above at $c = v(1 - \alpha)$; see the Appendix for details.

²⁷One can interpret this result as being caused by a 'competitive bottleneck' arising in equilibrium: one side of the market 'single-homes' ($r = 1$), and the other 'multi-homes' (Armstrong, 2006; Armstrong and Wright, 2007).

equilibria I derive, firms list on all PCWs so shoppers are indifferent to which set of PCWs they check. There is no strict incentive for consumers to check more than one PCW, so one could arguably go no further. However, this would ignore some potentially relevant descriptive and theoretical issues in the industry. Firstly, it is relevant to know the equilibrium effects were regulators to consider increasing inter-PCW competition by e.g., increasing the number of PCWs consumer are aware of; encouraging or incentivizing consumers to visit more PCWs; or offering a meta-site displaying the results from multiple PCWs. Secondly, shoppers commonly visit multiple PCWs (Mintel Group Ltd., 2009), which may reflect shopper uncertainty or mistrust (BBC, 2012). I now investigate such equilibria where shoppers check $r > 1$ PCWs.

Theoretically, in the simplest textbook Bertrand result, there is an immediate fall in the equilibrium price between that of a monopoly firm and the marginal-cost pricing of two firms. At the aggregator level however, Proposition 2 shows that this logic is incomplete: with $K > 1$ such undercutting does not even necessarily begin. The persistence of the consumer-welfare-decreasing equilibrium in Proposition 2 is an artifact of each shopper only checking one PCW which effectively makes each PCW a monopoly, facing no competitive pressure. In this respect, the equilibrium is reminiscent of Diamond (1971), but at the aggregation level, not the firm level. One might think that the Bertrand remedy for shoppers would be to require them to check at least two PCWs, causing PCWs to undercut each other until an equilibrium with all PCWs charging $c = 0$ is reached. I now explain how this logic is incomplete. Suppose now that there are $K > 1$ aggregators, and shoppers check $r > 1$ PCWs. First, I examine the special case of $r = K > 1$:

Lemma 4. *With $K > 1$ competing PCWs, shoppers are guaranteed to be better off than before the introduction of PCWs if $r = K > 1$. Further, the unique equilibrium PCW fee-level is $c = 0$ if and only if $r = K > 1$.*

The spirit of this result resembles that of Bertrand. To see why there cannot be some other equilibrium with $c > 0$ when $r = K > 1$, suppose so and consider an undercutting deviation by PCW_1 to some $\hat{c}_1 = c - \epsilon$. Shoppers do not detect the deviation and so do not change their behavior. As for firms, notice that for any p they strictly prefer to list exclusively on PCW_1 : When a firm is the cheapest, it will sell to *all* shoppers so long as it lists on *some* PCW. This is

precisely because $r = K$. Hence by listing on PCW_1 only, there is no reduction in the number of shoppers switching to them when they are cheapest, but there is a reduction in the fee the firm pays as $\hat{c}_1 < c$. The PCW finds this deviation strictly profitable because it receives a discrete gain in the number of shoppers switching through it, for an arbitrarily small loss in price.

Where shoppers check more than one PCW, but not all PCWs, we have:

Lemma 5. *When $K > r > 1$, there exists an equilibrium in which PCWs charge $\bar{c} > 0$, and firms list on all PCWs, mixing over prices by $G(p; \bar{c})$ where*

$$\bar{c} = \frac{v(1 - \alpha)Kr(K - r)}{K(1 + r(K - 2)) + \alpha r(K - 1)(r - 1)(n - 1)}.$$

More generally, there exist equilibria in which all PCWs charge $c \in [0, \bar{c}]$. To understand how much consumer welfare can be reduced, I analyze the highest-fee equilibrium from this set. Because PCWs have a strong incentive to coordinate on this equilibrium, it may be especially relevant in practice.

Notice that substituting $r = K$ in Lemma 5 yields $\bar{c} = 0$, and one obtains Lemma 4. One can see now that the Bertrand-style reasoning underlying Lemma 4 was a special case. To understand why the principle does not apply more generally, consider a fee level $c > 0$ and an undercutting deviation by PCW_1 to some $c_1 = c - \epsilon > 0$. Unlike when $K = r > 1$, when $K > r > 1$ it is not necessarily better for a firm to only list on the cheaper PCW_1 . By listing a price exclusively on PCW_1 , there are now $\frac{K-r}{K} > 0$ shoppers who do not see the firm's price. These shoppers will not buy from it even if it is the cheapest. Firms now face a trade-off: Exclusively listing on PCW_1 means that any sales incur only the lower fee \hat{c}_1 upon a sale, but there will be a reduction in sales volume because not all shoppers check PCW_1 . Which force is stronger in this trade-off depends on the size of the undercut ϵ . If PCW_1 undercuts by enough, firms will deviate to list exclusively on PCW_1 , breaking the symmetric equilibrium. Unlike the simpler logic of Lemma 4, it is no longer true that any $\epsilon > 0$ undercut will attract firms to exclusively list on the cheapest PCW. Hence, PCWs do not always have an incentive to undercut each other and higher-price equilibria are sustained.

I now discuss how the set of equilibria varies with how many PCWs shoppers check (r) and the number of aggregators (K). Firstly, as shoppers check more PCWs in equilibrium, the

incentive for a PCW to undercut the fees of other PCWs increases so that only lower fee-levels can be sustained in equilibrium. That is, \bar{c} is limited by a higher r : $\frac{d\bar{c}}{dr} < 0$

However, as the number of aggregators increases, the incentive is reversed. The number of shoppers checking a given PCW ($\frac{r}{K}$) falls. Accordingly, in equilibrium each firm receives less of its expected revenue from any single PCW. It then requires a more severe undercut from a PCW to get firms to exclusively list on it and forgo the business available from the other aggregators. For undercuts that are too severe, it is unprofitable for a PCW to deviate, even if it were to win exclusive arrangements with all firms as a result. Thus, as K increases, higher equilibrium fees can be sustained in equilibrium: $\frac{d\bar{c}}{dK} > 0$. This allows for the result that a higher number of aggregators can lead to higher fees, and hence higher prices. Furthermore:

Proposition 3. *For any r , there exists a \tilde{K} such that as long as there are more than \tilde{K} aggregators both types of consumer are worse off than before the introduction of PCWs.*

In the limit, the proportion of firm income that comes from sales on any one aggregator becomes vanishingly small. As this happens, $\bar{c} \rightarrow v(1 - \alpha)$ i.e., sustainable equilibrium fee levels approach the monopoly-PCW level, again making both types of consumer worse off than before the introduction of the sites.

6. Extensions and Robustness

6.1 Ad Valorem Fees

In some markets PCWs charge firms a percentage of the purchase price for each sale, rather than a fixed per-unit amount. Here I show that the main price-raising result of Proposition 1 continues to hold under such fees. Furthermore, price dispersion is maintained in equilibrium.

Proposition 4. *When the PCW charges ad valorem fees, both types of consumer are worse off with the PCW than without.*

In equilibrium, the PCW is able to extract the full price charged by a firm for a sale made through it.²⁸ This pushes prices up so that they have the same support as in Lemma 3, hence

²⁸The equilibrium is specified in Proposition A1 in the Appendix.

shoppers are worse off than without the PCW. The distribution of prices is distinct from that with a constant per-sale fee (G). However, it also first-order stochastically dominates the distribution of prices in the world without a PCW (F), hence auto-renewers are also worse off.

Although the PCW extracts the full sale price from firms, price dispersion emerges in equilibrium. This is because the PCW cannot extract all the surplus from all shoppers. Specifically, firms' own-shoppers renew directly, which are sales that PCWs cannot monetize. Indeed in equilibrium under ad valorem fees, when a firm is cheapest it makes nothing on rival firms' shoppers as $p = c$, but p on its own shoppers. Therefore firms still have some positive surplus to compete for on the PCW, which produces price dispersion in equilibrium.

6.2 Price Discrimination

So far, I have considered the impact of web services that list or 'aggregate' the available information (prices) charged by firms offering a product or service. In practice, this is often the case in the markets for gas, electricity, financial products such as mortgages, and durable goods.²⁹ However, in other markets, a firm may set distinct prices for a direct purchase and for purchases made through PCWs. In this section, I show that allowing for price discrimination does not undo the price-raising result of Proposition 1. I also show that the case of discrimination is nested within the case without, when markets are 'large'. This provides an analytic bridge between the settings with and without discrimination, grounding this extension within the initial framework.

Suppose a firm can set a price p_0 for a direct purchase, p_k different prices for each PCW $_k$ that it lists on and that $K \geq r = 1$.

Proposition 5. *With price discrimination, both types of consumer are worse off with the PCW than without.*

Proposition 5 follows because in equilibrium PCWs set $c = v(1 - \alpha)$, firms list on all PCWs, $p_0 = v$ and $p_1 = \dots = p_K = v(1 - \alpha)$. The ability of firms to price discriminate does not prevent PCWs from setting fees at the same high level as in Proposition 2 because

²⁹For UK gas and electricity markets for example, regulation has limited each energy company to offering a maximum of four tariffs in total.

$r = 1$. As before, there is effectively no competitive pressure between PCWs. However, price discrimination does lead to firms listing a common price on PCWs in equilibrium because PCWs no longer have the ability nor the incentive to keep prices posted on it dispersed. This is because firms can set a high direct purchase price $p_0 = v$ and a lower price through the PCWs. Auto-renewers are worse off than without PCWs, as firms charge them the monopoly price v . Shopper welfare is lower because they now face $p_k = v(1 - \alpha)$ for sure. This price was the minimum of the support of equilibrium prices in the world without discrimination which in turn I showed to be above the expected price paid by shoppers in the world without PCWs. Therefore Proposition 1 continues to hold: all consumers are worse off than in a world with no PCWs. The ability of firms to discriminate allows them to fully extract surplus from their captive auto-renewers; but PCWs can now extract the revenue from sales to all shoppers through their sites.³⁰ In markets for services and utilities, this corresponds to the general understanding that those who allow their contract to roll over and do not engage in search get charged higher premiums than those who shop around. In the travel-reservation industry, this could be interpreted as corresponding to high walk-in or last-minute prices.

6.2.1 Aggregation and Discrimination in Large Markets

In the setting with a PCW and price discrimination, the equilibrium of Proposition 5 shows the price paid by the two types are as maximally separated: Shopper price is competed down to firms' marginal cost c , and auto-renewers pay v . In the setting with an aggregator and no price discrimination, the equilibrium is given by Lemma 3 where Corollary 2 explains that as the number of firms increases, the expected prices paid by shoppers and that paid by auto-renewers, diverge. This occurs because as the number of firms increases, so does the competitive pressure on pricing to win shoppers. As a result, more probability mass is placed on lower prices. Firms compensate for this by also increasing the mass placed on higher prices, increasing their expected profit from auto-renewers. In fact, for arbitrarily large markets, I show that these two

³⁰As before, in the equilibrium of Proposition 5, PCWs cannot raise fees further, to say $c' > v(1 - \alpha)$ because of firms' outside option. Following such a unilateral PCW deviation, firms would set $p_0 = v(1 - \alpha)$ so that their shoppers purchase directly from them, reducing PCW profit to zero.

settings are equivalent.

Proposition 6. *As $n \rightarrow \infty$, following the introduction of a PCW, the expected prices faced by both types of consumer in a setting without price discrimination approach those in a setting with price discrimination.*

The result highlights the connection between the two market structures. Indeed, because the earlier results are independent of n , I found that all consumers can be worse off with a PCW with or without the possibility of price discrimination.

6.3 The Extensive Search Margin

This paper utilizes a clearing-house framework, where auto-renewers are inactive and can also be interpreted as being offline, loyal, uninformed or as having high search costs. Here, I focus on a search-cost rationalization, better applied to markets where obtaining a quote requires more information from the consumer e.g., home insurance. In an environment without a PCW where auto-renewers find it too costly to enter these details into a firm's website to retrieve *one* extra price, the introduction of a PCW offers to expose *all* prices to them, for the same single search cost. Depending on their search cost, the introduction of a PCW may then cause an auto-renewer to engage in comparison via the PCW. Some empirical studies have offered a similar argument to explain observed increases in market competitiveness (e.g., Brown and Goolsbee, 2002; Byrne et al., 2014). Their arguments are distinct from mine because they contrast a world with web-based aggregators relative to a world without the Internet, rather than a world with the Internet and firm websites. This engagement of new customers is commonly referred to as the 'extensive search margin' (for a recent discussion see Moraga-González et al., 2017).

The benefit of an additional search for a consumer in the world without a PCW is the difference between the expected price and the expected lowest of two prices drawn (from F). With a PCW, the benefit of a search on the PCW is the difference between the expected price and the expected lowest of n draws (from G).³¹ I denote these search benefits with and without

³¹These terms are analogous to the 'value of information' in Varian (1980).

an aggregator respectively as,

$$\mathcal{B}_1 = \mathbb{E}_G[p] - \mathbb{E}_G[p_{(1,n)}], \quad \mathcal{B}_0 = \mathbb{E}_F[p] - \mathbb{E}_F[p_{(1,2)}].$$

The model is as before save that each auto-renewer faces a search cost s . I assume these costs are heterogeneous, distributed by S over $s \in [\underline{s}, \infty)$.³² I assume $\underline{s} > \mathcal{B}_0$ which means that without a PCW, no auto-renewers shop, preserving the equilibrium of Lemma 1. After the introduction of a PCW, the benefit of a search (\mathcal{B}_1) may outweigh the cost (s) for some auto-renewers, who then choose to use the site. I use the term ‘converts’ and ‘non-converts’ to distinguish between auto-renewers who decide to shop or not in equilibrium with a PCW.

The total number of consumers using the PCW (shoppers and converts) is endogenously determined in equilibrium and is denoted $\tilde{\alpha} = \alpha + (1 - \alpha)S(\mathcal{B}_1)$. Given $\tilde{\alpha}$, the PCW sets its profit-maximizing fee $c = v(1 - \tilde{\alpha})$. As c and $\tilde{\alpha}$ are exogenous to firms, equilibrium pricing is as in Lemma 3 with $\tilde{\alpha}$ replacing α . In turn, pricing determines \mathcal{B}_1 . There is an equilibrium when this value of $\tilde{\alpha}$ satisfies $S(\mathcal{B}_1) = \frac{\tilde{\alpha} - \alpha}{1 - \alpha}$. When there exists such an S , $\tilde{\alpha}$ is said to be ‘rationalized’.

Corollary 3 showed that a higher α increases the welfare of all consumers. Corollary 1 showed that a lower c has the same effect. As the equilibrium fee level is $v(1 - \tilde{\alpha})$, both forces work to benefit all types of consumer. However, whether consumers actually gain depends on how many auto-renewers are converted. In fact, relative to the world without a PCW, the presence of converts is not sufficient to guarantee lower prices for any consumer, not even converts themselves:

Proposition 7. *Shoppers, converts and non-converts can all be worse off with a PCW than without.*

The proof gives an example of such an equilibrium, along with a distribution S that rationalizes it. More generally, there can be many equilibria, each with a different $\tilde{\alpha}$. If the benefit (\mathcal{B}_1) is small or there are no types with low search costs so that $S(\mathcal{B}_1) = 0$, there are no converts ($\tilde{\alpha} = \alpha$) and the equilibrium of Lemma 3 applies. Proposition 7 shows that when

³²That there is no upper bound ensures that there are always some auto-renewers, and hence price dispersion, in equilibrium.

some auto-renewers convert, all consumers can be worse off with a PCW. However, there may also exist equilibria where $\tilde{\alpha}$ is high enough such that some consumers benefit. Whether these equilibria exist depends on the distribution of search costs S . For a given PCW search benefit \mathcal{B}_1 , when more auto-renewers have low search costs (higher $S(\mathcal{B}_1)$), $\tilde{\alpha}$ is higher, c is lower and total consumer welfare is higher.

The number of firms also determines the size of the benefit the PCW offers (\mathcal{B}_1), and hence the number of converts. I now investigate which consumers benefit from an aggregator when the potential benefit it offers to consumers ($\mathcal{B}_1 - \mathcal{B}_0$) is as large as possible. For a given $\tilde{\alpha}$, a higher number of firms increases the equilibrium search benefit \mathcal{B}_1 (see Corollary 2). Specifically, as $n \rightarrow \infty$, $\mathcal{B}_1 \rightarrow v - c$ (see Proposition 6), which is as large as possible. To maximize $B_1 - B_0$, I let $q = 2$ in the world without a PCW, which makes \mathcal{B}_0 as low as possible, as \mathcal{B}_0 is increasing in q .³³ Accordingly, I define,

Definition. *A market has ‘maximum potential’ when $q = 2$ and $n \rightarrow \infty$.*

Proposition 8. *If the market has maximum potential: converts are better off; shoppers can be worse off; and non-converts are worse off with a PCW than without.*

When the market has maximum potential, converts are guaranteed to be better off but shoppers still may not be.³⁴ Shoppers may not be better off because even with maximum potential, there may still not be sufficiently many auto-renewers converting to PCW use.

7. Policy

This study shows that the introduction of a profit-maximizing PCW into a market can raise prices and harm consumers. The analysis also shows that competition between aggregators does not necessarily undo this effect and that a higher number of aggregators can be worse than fewer. Therefore, where the introduction of profit-maximizing PCWs into a market is itself a policy, it may harm consumers. Furthermore, in markets where price is the predominant

³³See the end of the proof of Lemma 4.

³⁴By Proposition 6, one can also use Proposition 8 to consider the effect of introducing a PCW into a market with search costs and price discrimination under the equilibrium behind Proposition 5.

dimension of concern, measures such as limiting the number of PCWs or capping fees could be considered. In this section, I use and build on the preceding analysis to discuss the effects of various other policies.

7.1 Price-Discrimination Clauses

The issue of price discrimination in markets with platforms is complex. In relation to markets where discrimination occurs, recent studies have considered the implications of interactions between price discrimination and various market features (e.g., Boik and Corts, 2016; Johnson, 2017; Edelman and Wright, 2015; Johansen and Vergé, 2016; Wang and Wright, 2016). Regulator and academic attention has focused on price-parity and ‘most-favored-nation’ clauses (MFNs).³⁵ MFNs are imposed by the PCW and state that a firm listing on the PCW cannot set a lower price elsewhere. Where there is more than one PCW in a market, a further distinction is introduced by whether the MFN is ‘wide’ or ‘narrow’. A wide MFN states that a firm listing on the PCW cannot set a lower price anywhere else. A narrow MFN states that a firm listing on the PCW can only not set a lower price on its own firm-website, with no restriction on prices at other PCWs. In this section, I first show that a PCW prefers an MFN to a price-parity clause, rationalizing why MFNs are the clauses commonly used by PCWs in reality.³⁶ Secondly, I argue that wide, rather than narrow, MFNs are likely to be damaging for consumers.

A price-parity clause is a stronger restriction than an MFN because it imposes that the price listed should be the same anywhere else it is listed. In reality, it is MFNs that are more commonly found in markets with PCWs. Where a PCW can choose to impose either clause, my model predicts that the MFN is strictly preferred. This can be seen by comparing the equilibrium with a PCW and price-parity (Lemma 3) against the equilibrium with a PCW and an MFN (the equilibrium behind Proposition 5). When an MFN is adopted, some price discrimination is allowed and firms do discriminate in equilibrium. They set the monopoly price for

³⁵For a detailed report of many cases receiving regulatory investigation or intervention, see Hviid (2015).

³⁶Although the terms price-parity and MFN are often used interchangeably in the literature, I define ‘price-parity’ to mean a clause that specifies prices to be the same across the locations where they are listed. In Edelman and Wright (2015) this is termed ‘price coherence’.

a direct purchase to exploit their captive auto-renewers, while competing for shoppers on the PCW. This is desirable from the perspective of a PCW. The PCW only makes money from sales to shoppers. With price-parity, it cannot quite extract all the revenue from shoppers because those already with the lowest-priced firm do not switch. However, the adoption of an MFN prompts shoppers buy through the PCW because firms set high direct-prices to exploit their captive auto-renewers.

In a market with one PCW, there is no distinction between a wide and a narrow MFN. In markets where there are multiple PCWs ($K > 1$), the effect of wide relative to narrow MFNs depends on the number of PCWs consumers check (r). If $r = 1$, the equilibrium of Proposition 5 applies regardless of the type of MFN applied and consumers are worse off with the PCWs than without. This follows because when $r = 1$ there is no reason for a PCW to lower its fee because it cannot attract business away from rival PCWs. However, when $K > r > 1$ a wide MFN softens inter-PCW competition. To see this, suppose a PCW undercuts rivals by a small amount in order to induce firms to post lower prices on it. In the presence of wide-MFNs firms face a trade-off: in order to list a lower price and increase sales through the cheaper PCW, firms would have to de-list from other PCWs, forgoing the associated profit. Without wide MFNs, this trade-off does not present itself. Firms can simultaneously list different prices on low-fee and high-fee PCWs. In turn, this gives an incentive for PCWs to undercut one another by arbitrarily small amounts. In other words, in markets with price discrimination and without wide MFNs, there is the potential to have Bertrand competition at the aggregator level.³⁷ In sum, in markets with competing PCWs, wide, rather than narrow MFNs can be damaging for consumers by impeding competition in PCW fees.

7.2 Meta-Sites and Publicly-Run PCWs

In recent years, ‘meta-sites’ have also entered the marketplace. These sites typically list all prices, whether they are listed on PCWs or not.³⁸ However, with shoppers incentivized to check the meta-site, the same logic as presented in Section 3.2 regarding an aggregator of firms applies

³⁷See Corollary A1 and Lemma A17 in the Appendix.

³⁸Examples of such meta-sites include www.kayak.com and www.trivago.com.

to this aggregator of firms and aggregators: in equilibrium, the meta-site extracts all the surplus from PCWs, that in turn pass it on to firms, that in turn pass it on to consumers in higher prices. Specifically, the introduction of the meta-site selects the $K = r$ equilibrium of the preceding analysis in Section 5 i.e., it shows shoppers the prices from all PCWs simultaneously. This leads to Bertrand competition at the aggregator level. However, the meta-site is able to charge the monopoly fee of $v(1 - \alpha)$ to PCWs, who pass it on to firms, who pass it on to consumers, who again all end up worse off than with no aggregators or meta-sites.³⁹ One can then see how this logic would extend to ‘meta-meta-sites’ and beyond.

In my model, either introducing a publicly-run PCW into a market without PCWs, or introducing a publicly-run meta-site into a market with PCWs would benefit shoppers by the same amount.⁴⁰ The difference is solely semantic: so long as the proposed site displays all the prices on the market be they listed on a firm’s site, PCW or meta-site, it will maximize the benefit to shoppers. Furthermore, as discussed in Section 6.3, if there are sufficiently many inactive shoppers with ‘low’ search costs at the extensive search margin, then such a site is well-poised to raise total consumer welfare relative to a world without aggregators.

7.3 Fee Transparency

In reality, PCW fees are generally not publicized directly. One may hypothesize that a policy of making PCW fees known to consumers could help drive them down in equilibrium. Efforts towards making fees known could include requiring PCWs to display their fees or publishing a publicly-available list of all fees. Here, I consider the effect of introducing such fee-transparency into the model.⁴¹

One reason that competing aggregators do not drive fees to zero in the model is that shoppers do not detect changes in the fees set by PCWs in equilibrium. This precludes a coordinated response between firms and shoppers that could punish a PCW that charges higher fees. If fees

³⁹See Proposition A2 in the Appendix.

⁴⁰For an example of a publicly-run aggregator see www.fuelwatch.wa.gov.au, a government-run PCW for gasoline prices in Western Australia (discussed in Byrne et al., 2014).

⁴¹For an analysis of unobserved wholesaler prices in traditional vertical supply chains, see Janssen and Shelegia (2015).

were publicly announced so that shoppers were aware of them, then credible subgame equilibria follow the fee-setting decision in which the PCW charging the lowest fee is attended by all firms and shoppers. Any higher-fee equilibrium is then undercut until $c = 0$, leading to lower shopper-prices.⁴² However, there are multiple equilibria because there are many subgame equilibria that can follow any vector of PCW fees, including less intuitive ones where coordination occurs at more expensive PCWs. If one adopts the refinement that firms and consumers only patronize the lowest-fee PCWs, then one obtains the zero-fee equilibrium as the unique equilibrium. However, the prediction that consumers will coordinate in this way seems questionable in reality. Even if shoppers could all be informed about fees, they do not care about them *per se*, paid as they are between firms and PCWs.⁴³ Furthermore, even with sufficient refinement criteria to implement the competitive-fee equilibrium, in some markets one may question a policy of fee-announcements on a more fundamental level. That is, if fees can be publicly announced, then surely so can firm prices, which would extinguish the role of a PCW in the first place.

8. Conclusions

The analysis presented shows that the introduction of PCWs may not in fact benefit consumers by reducing expected prices. The introduction of an aggregator facilitates comparison of the whole marketplace for shoppers, exerting competitive pressure on firm pricing. However, the aggregator charges a fee which, in turn, places upward pressure on prices. The net effect is that prices increase for all consumers, who would be better off without the site. This result is robust to allowing ad valorem fees, price discrimination, an extensive search margin and meta-sites.

⁴²This intuition is formalized in Proposition A3 in the Appendix.

⁴³Although fee information is rarely visible, some PCWs do advertise summary statistics of the price information of firms that list on them. PCWs sometimes advertise the average savings a consumer using their site is expected to make, which could direct shoppers to the cheapest PCW. By Proposition A3, this could lead to a shopper-welfare-improving equilibrium. However, many PCWs do not advertise based on purchase-relevant information (e.g., the campaign of Comparethemarket.com, based on a story about meerkats). If such ‘persuasive’ advertising caused all shoppers to loyally visit one site each i.e., $r = 1$, Lemma 3 applies and all consumers would have been better off without the PCW industry.

As such, for markets where price is the variable of primary concern, regulators may wish to consider capping fees or providing a publicly-run site.

Competition at the aggregator level need not lead to a reduction in fees. More aggregators only guarantee to benefit shoppers if they check all of them. If shoppers check an intermediate number, increasing the number of aggregators can lead to higher prices; for a sufficiently high number, all consumers can again be better off without the industry. As a result, regulatory bodies may wish to consider limiting the number of aggregators alongside encouraging consumers to check more.

In online markets with non-negligible search costs, even those consumers who rationally start engaging in price comparison may be worse off following the introduction of a PCW. However, if there are sufficiently many consumers with low search costs, consumer welfare can rise with the introduction of a PCW. Therefore, helpful policies would help erode search costs where possible and encourage more inactive consumers to engage in comparison. In markets with price discrimination where shoppers check multiple PCWs, Bertrand competition in aggregator fees can ensue. As such, regulators could consider whether it is possible to facilitate the introduction of price discrimination into markets in which it is not currently present. In markets with price discrimination, regulators could oppose price clauses that soften inter-PCWs competition e.g., wide most-favored-nation clauses.

Appendix

A1 Summary Statistics of UK PCW Industry

The figures quoted for turnover and profit in the introduction for the UK utilities and services industry are estimates for 2013 for the largest four such companies. The estimate for each site was taken from the following sources:

- Money Supermarket: Their own 2013 annual report.
- Confused: The Admiral Group's Annual Reports 2012 and 2013.
- Go Compare: Newspaper 'The Guardian' 2014 article.
- Compare the Market: Parent BGL Group's Annual Report 2013 and BBC article 2015.

For the first three, the figures were taken directly from the cited sources. For Compare the Market, estimated to be the largest of the four sites, the estimate is particularly rough as BGL Group offer no breakdown of their accounts. I assumed that the proportion of BGL's total revenue and profit due to Compare the Market was the same where the estimate for annual profit due to Compare the Market is taken from the BBC article. Even if the estimated £800m of total turnover and 14% for average annual profit growth for these sites is off by a margin, the industry can still be considered large and growing.

A2 A World Without a PCW

The domain of prices is \mathbb{R} . The equilibrium pricing strategy can always be described by its CDF (denoted F in this section, and with other letters later on). In what follows, either equilibrium pricing distributions will be pure (so that F is flat, with one jump discontinuity at this price); or will have no point masses so that F is continuous, which implies the density f exists and $f = F'$, wherever F' exists. Lemmas A1-A3 are variants of Varian (1980)'s Propositions 1,3 and 7 respectively.

Lemma A1. *In any equilibrium, there are no prices p charged s.t. $p \leq 0$ or $p > v$.*

Proof: Any $p \leq 0$ generates firm profits $\pi(p) \leq 0$ which is dominated by $p = v$ which gives profit of at least $v \frac{(1-\alpha)}{n} > 0$ because firms always sell to their auto-renewers. For $p > v$, $\pi(p) = 0$ because no one will buy at such a high price, hence again $p = v$ dominates. ■

Lemma A2. *In any equilibrium, there are no point masses.*

Proof: Suppose not. Then there is a point mass in equilibrium, \dot{p} s.t. $pr(p = \dot{p}) > 0$. Note that $\dot{p} \in (0, v]$ from Lemma A1. Because the number of point masses must be countable, there exists an $\varepsilon > 0$ small such that $\dot{p} - \varepsilon > 0$ and is charged with probability zero. Consider a deviation of a firm from the equilibrium F to a distribution over prices where the only difference is that the new distribution charges $\dot{p} - \varepsilon$ with probability $pr(p = \dot{p})$ and \dot{p} with probability zero. Note that a firm appears in $\binom{n-1}{q-1}$ of the groups of shoppers. Index these groups $z = 1, \dots, \binom{n-1}{q-1}$ (this is without loss as F is symmetric). Let $pr(\dot{p}; t, z)$ be the probability under F that a firm is cheapest in group z along with $t = 0, \dots, q - 1$ others. Call the difference in profit due to the deviation d and note that:

$$\lim_{\varepsilon \rightarrow 0} d = \sum_{z=1}^{\binom{n-1}{q-1}} \sum_{t=0}^{q-1} pr(\dot{p}; t, z) \frac{\alpha}{\binom{n}{q}} \left(1 - \left(\frac{1}{q} + \frac{q - (t+1)}{q(t+1)} \right) \right) \dot{p}$$

The term in parentheses is the difference in the amount of shoppers won under the deviation and under F , in given group, when the firm along with t others charge \dot{p} , and \dot{p} is the lowest price in that group. Due to symmetry of the groups, $pr(\dot{p}; t, z)$ is the same for all groups, so let this more simply be termed $pr(\dot{p}; t)$. Also given $pr(\dot{p}; t) > 0$, this simplifies to:

$$\frac{\binom{n-1}{q-1}}{\binom{n}{q}} \sum_{t=1}^{q-1} pr(\dot{p}; t) \alpha \left(\frac{t}{t+1} \right) \dot{p} > 0$$

hence for some $\varepsilon > 0$ there exists a profitable deviation, so F could not have been an equilibrium. ■

Lemma A3. *In any equilibrium, the maximum of the support of f must be v .*

Proof: Suppose not. Define \bar{p} as the maximum element of the support and note that by Lemma A2 the probability of a tie at any price is zero. By Lemma A1, $\bar{p} \in (0, v)$. Consider a firm called upon to play $\bar{p} < v$ in equilibrium. They only sell to their auto-renewers, making $\bar{p} \frac{(1-\alpha)}{n}$ but would strictly prefer to deviate to v and make $v \frac{(1-\alpha)}{n}$, a contradiction. ■

Lemma 1. *In the unique equilibrium firms mix according to the distribution*

$$F(p) = 1 - \left[\frac{(v-p)(1-\alpha)}{qp\alpha} \right]^{\frac{1}{q-1}} \text{ over the support } p \in \left[\frac{v(1-\alpha)}{1+\alpha(q-1)}, v \right].$$

Proof: By Lemma A2, there is more than one element of the equilibrium support, and by Lemma A3 v is the maximal element. In equilibrium, a firm must be indifferent between all elements of the support p , hence profit must equal $v \left(\frac{1-\alpha}{n} \right)$ for all p , that is:

$$(1) \quad v \left(\frac{1-\alpha}{n} \right) = p \left[\frac{1-\alpha}{n} + \frac{\alpha}{\binom{n}{q}} X(p) \right]$$

$$X(p) \equiv \binom{n-1}{q-1} \binom{n-1}{n-1} F(p)^0 (1-F(p))^{n-1} + \dots + \binom{q-1}{q-1} \binom{n-1}{q-1} F(p)^{n-q} (1-F(p))^{q-1}$$

The first term on the RHS of (1) is the profit from ARs, who always purchase at $p \leq v$. The second term is the expected proportion of shoppers that a firm will win, charging price p . Shoppers can be characterised by $\binom{n}{q}$ groups, where the set of groups is given by $\{1, \dots, n\}^q$. $X(p)$ describes the expected number of groups a firm expects to win given it charges p . By Lemma A2 there are no ties in price, so label prices by $p_1 < \dots < p_n$. p_1 will be the cheapest in every group in which it appears, and it appears in $\binom{n-1}{q-1}$ of the groups. The probability of being the lowest price is given by $\binom{n-1}{n-1} F(p)^0 (1-F(p))^{n-1}$, which accounts for the first term in $X(p)$. The observation that p_i is the cheapest in $\binom{n-i}{q-1}$ groups if $i \leq n - (q-1)$, zero groups otherwise, accounts for the remaining terms of $X(p)$. The following manipulations to simplify the second term on RHS of (1) this make use of the binomial theorem:

$$= p \frac{\alpha}{\binom{n}{q}} \sum_{j=q-1}^{n-1} \binom{j}{q-1} \binom{n-1}{j} F(p)^{n-j-1} (1-F(p))^j$$

which after some manipulations can be shown to be:

$$= p\alpha \frac{q}{n} (1-F(p))^{q-1}$$

rearranging for $F(p)$ gives:

$$F(p) = 1 - \left[\frac{(v-p)(1-\alpha)}{qp\alpha} \right]^{\frac{1}{q-1}}$$

Notice that this is a well-defined c.d.f. over:

$$p \in \left[\frac{v(1-\alpha)}{1+\alpha(q-1)}, v \right]$$

Notice that v is strictly preferred to any $p \in \left[0, \frac{v(1-\alpha)}{1+\alpha(q-1)} \right)$. ■

A3 A World With a PCW

I look for symmetric equilibria where PCWs charge some fee level $c \geq 0$ and shoppers check PCWs in equilibrium. I do not look at equilibria where firms never list on PCWs, where the setting without a PCW applies. To derive equilibria, one needs to know the mutual best-responses of firms to unilateral deviations of PCWs. To do so, take \mathbf{c} and equilibrium shopper strategy as given, and consider the firm responses.

Let $K \geq 1$ denote the number of PCWs and $r : K \geq r \geq 1$ the number of PCWs checked by shoppers. In symmetric equilibrium, a proportion $\frac{r}{K}$ of each firm's shoppers check any given PCW. Define a vector of PCW fees as $\mathbf{c} = (c_1, \dots, c_K) \in \mathbb{R}_+^K$ labelled such that $c_1 \leq \dots \leq c_K$. Let $\beta_k \in [0, 1]$ be the probability with which a firm enters PCW $_k$ and define the event E: "all PCWs are empty". Denote $(a_1, a_2) = (p, \mathcal{K})$ as a firm's action, where p is the price charged and $2^{\{1, \dots, K\}}$ is set of all combinations of PCWs they could choose to list in where \mathcal{K} a typical element, and \emptyset denotes not listing on any PCW. Define the following CDF, which is used throughout:

$$(2) \quad G(p; \mathbf{c}) = 1 - \left[\frac{(v-p)(1-\alpha)}{\alpha \left(np - (n-1) \frac{1}{K} (c_1 + \dots + c_K) \right)} \right]^{\frac{1}{n-1}}$$

which is well-defined over the support $[p(\mathbf{c}), v]$ where $p(\mathbf{c}) = \frac{v(1-\alpha) + \frac{1}{K}(c_1 + \dots + c_K)\alpha(n-1)}{1+\alpha(n-1)}$. When $c_1 = \dots = c_K \equiv c$, let $G(p; \mathbf{c})$ and $p(\mathbf{c})$ also be written $G(p; c)$ and $p(c)$.

Lemma A4. *In any equilibrium, there are no prices p charged s.t. $p \leq 0$ or $p > v$.*

Proof: No $p \leq 0$ or $p > v$ because they yield negative and zero profit respectively, whereas (v, \emptyset) yields $\frac{v(1-\alpha)}{n} > 0$. ■

Lemma A5. *If $c_1 \in [0, v(1-\alpha))$, $pr(E) = 0$.*

Proof: Suppose $pr(E) > 0$. Denote the infimum of prices charged when no PCW is listed on and that of prices ever listed on a PCW as p_0 and \underline{p} respectively. [Note the infima exist because

prices are bounded from below by Lemma A4]. Note that $p_0 \geq \underline{p}$ because $(\underline{p}, \emptyset)$ is strictly preferred to any lower price as a firm faces no competition for prices below \underline{p} off the PCWs. Consider when the firm is called upon to play (p_0, \emptyset) (or a price arbitrarily closely above p_0):

If $p_0 > c_1$, a deviation to $(p_0 - \epsilon, 1)$ is strictly profitable. This is because with probability $pr(E)^{n-1} > 0$ PCWs are empty with other firms charging at least p_0 . By listing the firm then has a positive probability of winning $\alpha \frac{r}{K} \frac{n-1}{n}$ new shoppers. For a sufficiently small $\epsilon > 0$, this will offset the arbitrary loss in revenue from its own consumers.

If $p_0 \leq c_1$, firm profit must be at least $\frac{p}{n}$ which can be guaranteed by $(\underline{p}, 1)$, because $p_0 \geq \underline{p}$. In turn, this must be at least as much as $\frac{v(1-\alpha)}{n}$ which the firm can guarantee by playing (v, \emptyset) . Putting these together, $p_0 \geq \underline{p} \geq v(1-\alpha) > c_1$ which contradicts $p_0 \leq c_1$. ■

Lemma A6. *If $c_1 \in [0, v(1-\alpha))$, firm strategies have no point masses.*

Proof: Define $\underline{p}_1(c_1)$ as the price at which a firm is indifferent between selling to all shoppers exclusively through the cheapest PCW(s) and charging v , only sell to auto-renewers:

$$\underline{p}_1(c_1) = \frac{v(1-\alpha) + \alpha c_1(n-1)}{1 + \alpha(n-1)}$$

By Lemma A5, some (p, \mathcal{K}) is played. Note that firms would by construction not play (p, \mathcal{K}) where $p < \underline{p}_1(c_1)$, strictly preferring (v, \emptyset) . To see that there are no point masses:

If there were a point mass at $(\underline{p}_1(c_1), \mathcal{K})$ then there is a positive probability of being tied for the lowest price at $(\underline{p}_1(c_1), \mathcal{K})$. By definition of $\underline{p}_1(c_1)$ firms would strictly prefer to deviate to (v, \emptyset) .

If there were a point mass at (\dot{p}, \mathcal{K}) s.t. $\dot{p} > \underline{p}_1(c_1)$ then there is a positive probability of being tied for the lowest price at (\dot{p}, \mathcal{K}) . A firm would strictly prefer to shift that probability mass to $(\dot{p} - \epsilon, \mathcal{K}')$ where $\mathcal{K}' = \mathcal{K} \setminus \{k : c_k \geq \dot{p}\}$. Here, the firm would sell to $\alpha \frac{r}{K} \frac{n-1}{n}$ other firms' shoppers at an arbitrary loss in revenue from its own consumers. There is always an $\epsilon > 0$ small enough to ensure this is profitable because $\underline{p}_1(c_1) > c_1 \iff c_1 < v(1-\alpha)$. ■

Lemma A7. *If there are no point masses, the maximum of the support f must be v .*

Proof: This is a variant of Varian (1980) Proposition 7. ■

Lemma A8. *If $c_1 \in [0, v(1-\alpha))$ and $c_1 < c_2$, $\beta_1 = 1$.*

Proof: By Lemma A5 it is never the case that all PCWs are empty. Suppose $\beta_1 < 1$. Lemma A6 implies there is more than one price, \hat{p} , that is listed on some other PCWs. By Lemma A7 there is one such that $\hat{p} < v$. Consider a firm being called upon to play this $(\hat{p}, \mathcal{K}_{-1})$ where $1 \notin \mathcal{K}_{-1}$ and $\bar{m} = \max\{\mathcal{K}_{-1}\}$. As this price has a positive probability of being the lowest of all firms, it will generate sales through the PCWs in \mathcal{K}_{-1} . But as PCW₁ is the unique cheapest PCW, there is a strictly profitable deviation to $(\hat{p}, \mathcal{K}_{-1} \cup 1 \setminus \bar{m})$. ■

Lemma A9. *If $c_1 = \dots, c_K \equiv c \in (v(1 - \alpha), v]$, firm pricing strategies are pure where $p \in [v(1 - \alpha), c]$. Any $(\beta_1, \dots, \beta_K) \in (0, 1]^K$ can be supported.*

Proof: Either there is a point mass or there is not.

1. Suppose there is a point mass at price \dot{p} . If $\dot{p} > c$, firms have a strict incentive to shift this mass to $(\dot{p} - \epsilon, \{1, \dots, K\})$. If $\dot{p} < v(1 - \alpha)$, firms prefer to shift this mass to (v, \emptyset) . These leaves $\dot{p} \in [v(1 - \alpha), c]$ as the only points that can be point masses. There can be at most one point mass: If not, then there a second point mass $\ddot{p} < c$, which if played with $\mathcal{K} \neq \emptyset$ would generate negative profit, so (\ddot{p}, \emptyset) is preferred; if $\mathcal{K} = \emptyset$, then $(\ddot{p} + \epsilon, \emptyset)$ for some sufficiently small $\epsilon > 0$ is preferred. To see that this pure pricing at $p \in [v(1 - \alpha), c]$ can be part of firm strategies, note that firm profit is $\pi = \frac{p}{n} \geq \frac{v(1-\alpha)}{n}$, so there is no strict incentive to sell only to auto-renewers instead. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1, \dots, \beta_K) \in (0, 1]^K$.
2. Suppose there is no point mass. By Lemma A7 the maximum of the support is v , where v is not the only element of the support, else it would be a point mass. There is therefore a positive probability of a firm being the cheapest at some p . There can be no (p, \mathcal{K}) s.t. $p < c$ charged: If $\mathcal{K} \neq \emptyset$ played profit from these sales is negative, so (p, \emptyset) is preferred; if $\mathcal{K} = \emptyset$, then $(p + \epsilon, \emptyset)$ for some sufficiently small $\epsilon > 0$ is preferred. Given $p \in [c, v]$, $pr(E) = 0$. This follows because firms strictly prefer $(p, \{1, \dots, K\})$ to (p, \emptyset) for all $p \in (c, v)$. For any $1 \leq r \leq K$ firms are content to list prices on at least as many PCWs as is necessary to make sure every shopper sees their price e.g., for $r = K = 1$ all of them; for $r = K$, just one of them. There can therefore, be different configurations of β_K 's depending on r, K so long as $pr(E) = 0$. To determine firm pricing strategy it must be that firms are indifferent between

every p they are called upon to play:

$$v \left(\frac{1 - \alpha}{n} \right) = p \left(\frac{1 - \alpha}{n} \right) + (1 - G(p; c))^{n-1} \left[\frac{\alpha}{n} p + \frac{\alpha(n-1)}{n} (p - c) \right]$$

which can be re-arranged to give $G(p; c)$ from (2). However, $\underline{p}(c) < c$ because $c > v(1 - \alpha)$, so firms would make strictly negative profits at prices $p \in (\underline{p}(c), c)$, preferring not to list. This provides a contradiction, so there do not exist strategic firm responses with no point masses.

■

A4 Results for $K = 1$ or $r = 1$

Lemma A10. *If $r = 1$, $c_1 = \dots = c_{K-1} \in [0, v(1 - \alpha))$ and $c_K \in [c_1, \underline{p}(c)]$: $\beta_k = 1$ for all k where firms mix by CDF $G(p; \mathbf{c})$.*

Proof: By Lemma A5 there is always some price posted on some PCW(s). By Lemma A6 these prices are not point masses and by Lemma A7 the maximum of the support is v , where only auto-renewers are sold to. There is therefore a positive probability of a sale through some PCW(s) at some (p, \mathcal{K}) . If $k \in \mathcal{K}$ where $k < K$ and sales there are profitable (when $p > c_1$), then $\{1, \dots, K - 1\} \in \mathcal{K}$; if $K \in \mathcal{K}$ and sales there are profitable $p > c_K$ (whether or not $p > c_K$ is the only consideration because $r = 1$), it follows that $\{1, \dots, K\} \in \mathcal{K}$. To be part of firm strategy, it must also be that firms prefer to play (p, \mathcal{K}) than to charge v and sell only to their auto-renewers. Note that $\underline{p}(c)$ from (2) is the price at which firms are indifferent between selling through $\{1, \dots, K\}$ with certainty and selling only to their auto-renewers. Similarly, denote $\underline{p}_{-K}(c)$ as the indifference point between selling for sure on $\{1, \dots, K - 1\}$ and charging v , and note $\underline{p}_{-K}(c) < \underline{p}(c)$. There will then be no $(p, \{1, \dots, K - 1\})$ played s.t. $p < \underline{p}_{-K}(c)$ and no $(p, \{1, \dots, K\})$ s.t. $p < \underline{p}(c)$. For prices close to v , $\mathcal{K} = \{1, \dots, K\}$. To determine firm pricing strategy, it must be that firms are indifferent between every $(p, \{1, \dots, K\})$ they are called upon to play:

$$v \left(\frac{1 - \alpha}{n} \right) = p \left(\frac{1 - \alpha}{n} \right) + (1 - G(p; \mathbf{c}))^{n-1} \left[\frac{\alpha}{n} p + \frac{\alpha(n-1)}{n} \left(p - \left(c_1 \frac{K-1}{K} + c_K \frac{1}{K} \right) \right) \right]$$

which can be re-arranged for $G(p; \mathbf{c})$ to give the CDF from (2). Because the minimum of the support is $\underline{p}(c) > c_K \geq c_1$ (the first relation follows because $c_1 < v(1 - \alpha)$), all prices

in the support generate profitable sales through all the PCWs, there is no price charged s.t. $p \in [\underline{p}_{-K}(\mathbf{c}), \underline{p}(\mathbf{c})]$. It follows that $\beta_k = 1$ for all k . For $K = 1$, let $c_1 \in [0, v(1 - \alpha))$ and set $K = 1$ in the expressions of the Lemma. ■

Lemma A11. *If $r = 1$ and $c = v(1 - \alpha)$ there are the following firm responses:*

1. *Pure-price strategies where $p = v(1 - \alpha)$ is the only price ever charged. Here, any $(\beta_1, \dots, \beta_K) \in (0, 1]^K$ can be supported.*
2. *Mixed-price strategies where $\beta_k = 1$ for all k and prices are distributed according to the CDF $G(p; v(1 - \alpha))$ where $\underline{p}(v(1 - \alpha)) = v(1 - \alpha)$.*

Proof: Either there is a point mass or there is not.

1. Suppose there is a point mass at price \hat{p} . If $\hat{p} > c$, firms have a strict incentive to shift this mass to $(\hat{p} - \epsilon, \{1, \dots, K\})$. If $\hat{p} < c$, firms have a strict incentive to shift this mass to (v, \emptyset) . These leaves $\hat{p} = c$ as the only point that can be a point mass. To see that this pure pricing can be part of firm strategies, note that firm profit is $\pi = \frac{c_1}{n} = \frac{v(1-\alpha)}{n}$, so there is no strict incentive to sell only to auto-renewers instead. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1, \dots, \beta_K) \in (0, 1]^K$.
2. Suppose there is no point mass. By Lemma A6, v is the maximum of the support of prices. No prices $p < c$ are charged because (v, \emptyset) is strictly preferred. For $p > c$, sales through all PCWs are profitable so $\beta_k = 1$ for all k . When v is played, firm profit is $\pi(v) = \frac{v(1-\alpha)}{n}$. To determine firm pricing strategy, it must be that firms are indifferent between every p they are called upon to play:

$$(3) \quad \pi(v) = p \left(\frac{1-\alpha}{n} \right) + \frac{\alpha}{n} (1 - G(p; v(1 - \alpha)))^{n-1} [np - (n - 1)v(1 - \alpha)].$$

which can be re-arranged to give the CDF from (2).

■

Lemma A12. *If $r = 1$, $c_1 = \dots = c_{K-1} = v(1 - \alpha)$ and $c_K \in (c_1, v]$ there exist firm responses in pure-price strategies where $v(1 - \alpha)$ is the only price ever charged. Here, any $(\beta_1, \dots, \beta_K) \in (0, 1]^K$ can be supported.*

Proof: In such an equilibrium, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. Consider a deviation to p . If $p \in (v(1-\alpha), v]$, deviation profit is $\hat{\pi} = \frac{p(1-\alpha)}{n} \leq \pi$. If $p < v(1-\alpha)$, firms have a strict incentive to shift this mass to (v, \emptyset) . Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1, \dots, \beta_K) \in (0, 1]^K$. ■

Lemma 2. *The mutual best-responses of firms as a function of c :*

1. *If $c \in [0, v(1-\alpha))$, firm best-responses are described by $G(p; c)$, and have no point masses.*
2. *If $c = v(1-\alpha)$, there are two classes of responses, one with no point masses described by $G(p, c)$, and those in which all firms charge the same price, $p = v(1-\alpha)$.*
3. *If $c \in (v(1-\alpha), v]$, all firms charge the same price, p , where $p \in [v(1-\alpha), c]$.*

Where pricing is described by $G(p; c)$ each firm always lists its price on the PCW.

Proof: See Lemmas A10, A11 and A9 respectively. ■

Lemma 3. *In the unique equilibrium, the PCW sets a click-through fee of $c = v(1-\alpha)$, firms list on the PCW and mix over prices according to $G(p; v(1-\alpha))$ over the support $p \in [v(1-\alpha), v]$.*

Proof:

If $c \in [0, v(1-\alpha))$, Lemma A10 shows that there is a profitable upward deviation to $c \in (c_1, \underline{p}(c))$. It is profitable because there is an increase in fee-level but no reduction in the quantity of sales.

If $c \in (v(1-\alpha), v]$, Lemma A9 shows that firms will play pure-price strategies and so $u = 0$. If $c = v(1-\alpha)$ and firms play pure-pricing strategies, then $u = 0$ again. In these cases, Lemma A10 shows that a deviation to $c \in (0, v(1-\alpha))$ will generate $u_1 > 0$, so there are no equilibria where $c \in (v(1-\alpha), v]$ or for $c = v(1-\alpha)$ when firms respond with pure-pricing strategies.

If $c = v(1-\alpha)$ and firms mix over prices by $G(p; v(1-\alpha))$ as in Lemma A11, equilibrium PCW profit is $u^* = v(1-\alpha) \frac{\alpha}{K} \frac{n-1}{n} > 0$. Lemma A9 shows that any upward deviation would yield $u = 0$. There can be no profitable deviation downwards because, as Lemma A10 shows, the fee would be reduced for no gain in the quantity of sales. ■

Proposition 1. *Both types of consumer are worse off with the PCW than without.*

Proof of Proposition 1: First I show that auto-renewers are worse off under $G(p; v(1 - \alpha))$ than $F(p)$ (referred to as G and F here). Auto-renewers pay the price quoted by their current firm. To show they are worse off with the PCW, I show that $\mathbb{E}_F[p] < \mathbb{E}_G[p]$ by showing that G first-order stochastically dominates (FOSDs) F . The distributions share the same upper bound on their supports, with F having a lower lower bound. Hence, G FOSDs F if $G \leq F$ for $p \in [v(1 - \alpha), v]$, which can be re-arranged as

$$\left[\frac{(v-p)(1-\alpha)}{\alpha qp} \right]^{\frac{1}{q-1}} \leq \left[\frac{(v-p)(1-\alpha)}{\alpha np - \alpha v(1-\alpha)(n-1)} \right]^{\frac{1}{n-1}}.$$

To see that this holds, I first note that when $n = q$ this simplifies to $\alpha v(1 - \alpha)(n - 1) \geq 0$, which is always true. Secondly, I show that G is FOSD-ordered in n :

$$\frac{dG}{dn} \leq 0 \iff \log \left(\frac{(v-p)(1-\alpha)}{\alpha(np - v(1-\alpha)(n-1))} \right) + \frac{(n-1)(p - v(1-\alpha))}{np - v(1-\alpha)(n-1)} \equiv X(p) \leq 0.$$

Note that $X(v(1 - \alpha)) = 0$, and $\frac{dX}{dp} < 0$:

$$\frac{dX(p)}{dp} \leq 0 \iff p \geq v(1 - \alpha) \left[\frac{(2n - 2 + \alpha(n - 1)^2)}{(2n - 1 + \alpha(n - 1)^2)} \right].$$

The term on RHS in square brackets is below 1. Recall that $p \geq v(1 - \alpha)$ as this is the lower bound of the support, hence this is satisfied.

To show shoppers are worse off under G than F , first show that $\mathbb{E}_F[p_{(1,2)}]$ is lower than the lower bound of the support of g in the case of $q = 2$. Then, I use Proposition 3 of Morgan et al. (2006) which corresponds to my setup (the only difference is that they have $v = 1$), which states that $\mathbb{E}_F[p_{(1,2)}]$ is decreasing in q . Hence I show the first step here to prove that $\mathbb{E}_F[p_{(1,2)}]$ is below $v(1 - \alpha)$ for all q . For $q = 2$,

$$\mathbb{E}_F [p_{(1,2)}] = \int_{v[\frac{1-\alpha}{1+\alpha}]}^v f(p_{(1,2)}) p dp$$

where $f(p_{(1,2)}) = 2(1 - F(p))f(p)$ is the density function of the lower of the two draws shoppers receive from F . Computing yields

$$\mathbb{E}_F [p_{(1,2)}] = \left(\frac{1-\alpha}{\alpha} \right)^2 \frac{v}{2} \left[\log \left(\frac{1-\alpha}{1+\alpha} \right) + \frac{2\alpha}{1-\alpha} \right].$$

Then $\mathbb{E}_F [p_{(1,2)}] < v(1 - \alpha)$ can be rearranged to obtain

$$\log \left(\frac{1 + \alpha}{1 - \alpha} \right) > 2\alpha$$

which holds for $\alpha \in (0, 1)$. ■

Corollary 1. *Within the mixed-price equilibrium firm responses of Lemma 2, as $c \in [0, v(1 - \alpha)]$ increases, the expected price paid by both types of consumer increases.*

Proof: From Lemma 2 the pricing strategy for $c \in [0, v(1 - \alpha)]$ is given by, $G(p; c)$ over $[p(c), v]$. Differentiating,

$$\frac{dG(p; c)}{dc} = \frac{1}{c(n-1) - np} \left(\frac{(v-p)(1-\alpha)}{\alpha np - \alpha(n-1)c} \right)^{\frac{1}{n-1}}.$$

The second term is ≥ 0 else $G(p; c) > 1$. The first term is $\leq 0 \iff c^{\frac{n-1}{n}} < p$ which is ensured because $p \geq c > c^{\frac{n-1}{n}}$ when $c \leq v(1 - \alpha)$ (this follows because $p(c) \geq c \iff v(1 - \alpha) \geq c$). Then for any $c, c' \in [0, v(1 - \alpha)]$, if $c > c'$ then the equilibrium pricing distribution under c first order stochastically dominates that under c' . Hence the expected price (paid by auto-renewers) and the expected lowest price from n draws (paid by shoppers) are higher under c . ■

Corollary 2. *As the number of firms increases, the expected price paid by shoppers falls, but the expected price paid by auto-renewers rises.*

Proof: Using an observation from Morgan et al. (2006), industry profit of firms is given by $\alpha \mathbb{E}_G [p_{(1,n)}] + (1 - \alpha) \mathbb{E}_G [p] = v(1 - \alpha)$, where $\mathbb{E}_G [p_{(1,n)}]$ denotes the lowest price of n draws from $G(p; v(1 - \alpha))$ and $\mathbb{E}_G [p]$ denotes the expected price from $G(p; v(1 - \alpha))$. The RHS is the industry profit of firms if it charged v and only sold to auto-renewers. Differentiating and rearranging,

$$\frac{d\mathbb{E}_G [p_{(1,n)}]}{dn} = - \left(\frac{1 - \alpha}{\alpha} \right) \frac{d\mathbb{E}_G [p]}{dn}$$

so the derivatives have opposite signs. Finally, recall that $\frac{d\mathbb{E}_G [p]}{dn} \geq 0$ because $G(p, v(1 - \alpha))$ is stochastically ordered in n (which is shown in the proof of Proposition 1). ■

Proposition 2. *With $K > 1$ competing PCWs, if shoppers check $r = 1$ PCW then both types of consumer are worse off with the PCWs than without.*

Proof: There are no equilibria where $c \in [0, v(1 - \alpha))$: Lemma A10 shows that there is a profitable upward deviation to $c_K \in (c_1, \underline{p}(c))$. It is profitable because there is an increase in fee-level but no reduction in the quantity of sales. Note that there can be no profitable deviation downwards given $r = 1$ because the fee would be reduced for no gain in the quantity of sales.

There are no equilibria where $c \in (v(1 - \alpha), v]$: Lemma A9 shows $u_k = 0$ for all k . Lemmas A6 and A8 show that a deviation to $c_1 \in (0, v(1 - \alpha))$ will generate $u_1 > 0$.

The only remaining option is $c_1 = \dots = c_K \equiv c = v(1 - \alpha)$, where firm responses are described by Lemma A11. If firms play a pure-pricing strategy, then as in the previous case, PCWs make zero profit and there is a profitable deviation to $c_1 \in (0, v(1 - \alpha))$. If however, firms respond with the mixed-price strategy, this fee-level is an equilibrium: There can be no profitable deviation downwards for PCWs because the fee would be reduced for no gain in the quantity of sales as $r = 1$. Following an upward deviation from PCW $_K$ to $c_K \in (v(1 - \alpha), v]$, when firms respond with a pure-pricing strategy as detailed in Lemma A11, PCW $_K$'s profit falls to zero. There is then an equilibrium at this fee level, and it is the unique such level where there exists an equilibrium.

At this fee-level, firms play just as they did when $K = 1$ with the same CDF over prices and all PCWs list all n firm prices. Both types of consumer are therefore left with the same level of surplus they had under $K = 1$. ■

Corollary 3. *As the proportion of shoppers α increases, expected prices paid by shoppers and auto-renewers both decrease.*

Proof: Differentiating $G(p; c)$ by α ,

$$\frac{dG(p; c)}{d\alpha} = \frac{1}{\alpha(n-1)(1-\alpha)} \left(\frac{(v-p)(1-\alpha)}{\alpha n p - \alpha(n-1)c} \right)^{\frac{1}{n-1}}$$

The first term is > 0 . The second term is always ≥ 0 or $G(p; c) > 1$. Then for any $\alpha, \alpha' \in (0, 1)$, if $\alpha > \alpha'$ then the equilibrium pricing distribution under α' first order stochastically dominates that under α . Hence the expected price (paid by auto-renewers) and the expected lowest price from n draws (paid by shoppers) are lower under α . ■

Corollary 4. *If the PCW can determine α as well as c in the preliminary stage, then the PCW sets $\alpha = \frac{1}{2}$.*

Proof: I expand the PCW's action set to include $\alpha \in (0, 1)$. Notice that for any choice of $\alpha \in (0, 1)$, by the reasoning as in the proof of Lemma 3, the PCW will avoid the pure equilibria of Lemma 2 so $c \in [0, v(1 - \alpha)]$, firms mix and the PCW is given by $c\alpha \left(\frac{n-1}{n}\right)$. The PCWs optimisation problem can hence be solved by,

$$\max_{c, \alpha} c\alpha \left(\frac{n-1}{n}\right) \text{ s.t. } c \in [0, v(1 - \alpha)] \text{ and } \alpha \in (0, 1)$$

where the solution is $c = v(1 - \alpha)$, $\alpha = \frac{1}{2}$. ■

A5 Results for $K > 1$ and $r > 1$

Lemma A13. *If $r > 1$, $c_1 = \dots = c_{K-1} \in [0, v)$, $c_K \in (c_1, v]$, $u_K = 0$.*

Proof: This Lemma says that there is no profitable upwards deviation for a PCW from any equilibrium. Consider such a unilateral deviation by PCW_K . Firm response can either be pure or mixed pricing. If pure, then $u_k = 0$ for all k . If mixed, then at any price p that has positive probability of sales through PCWs where $K \in \mathcal{K}$, firms have a strict preference to play $(p, \{1, \dots, K - 1\})$ instead. This is because $r > 1$: Every shopper who sees the prices on PCW_K also sees the prices on another PCW. Firms can therefore avoid PCW_K 's higher fee by not listing there while facing no reduction in the quantity of sales. ■

Lemma A14. *If $r = K$, $c_2 = \dots = c_K \in (0, v(1 - \alpha)]$ and $c_1 \in [0, c_2)$: $\beta_1 = 1$, $\beta_k = 0$ for $k = 2, \dots, K$ and prices are distributed according to the CDF $G(p, c_1)$.*

Proof: There are no point masses by Lemma A6. By Lemma A8 $\beta_1 = 1$. $\beta_k = 0$ for $k = 2, \dots, K$ follows because all shoppers check every PCW ($r = K$). Therefore, at any price p that has positive probability of generating sales through PCWs, firms have a strict preference only to list on PCW_1 without any reduction in the quantity of sales. By Lemma A7 the maximum of the support is v . Firms must be indifferent to all $(p, 1)$ they are called upon to play, hence:

$$v \left(\frac{1-\alpha}{n}\right) = p \left(\frac{1-\alpha}{n}\right) + \frac{\alpha}{n}(1 - G(p; c_1))^{n-1} [np - (n - 1)c_1].$$

which can be re-arranged to give $G(p, c_1)$ from (2). ■

Lemma 4. *With $K > 1$ competing PCWs, shoppers are guaranteed to be better off than before the introduction of PCWs if $r = K > 1$. Further, the unique equilibrium PCW fee-level is*

$c = 0$ if and only if $r = K > 1$.

Proof: Sufficiency: Suppose not. Then there exists an equilibrium with $c > 0$. By Lemma A13, there is no profitable upward deviation. Now consider a downward deviation. If $c \in [v(1-a), v]$ and firms respond with a pure-pricing equilibrium, as detailed in Lemma A9, then $u_k = 0$. A deviation by PCW₁ to $c_1 \in (0, v(1-\alpha))$ would lead to the responses detailed in Lemma A14 and deviation profit of $u_1 = c\alpha \frac{n-1}{n} > 0$. If $c \in (0, v(1-a)]$ and firms respond with by mixing by $G(p, c)$, as in Lemma A14, then $u_k = c \frac{\alpha}{K} \frac{n-1}{n} > 0$ for all k . But a deviation by PCW₁ to $c_1 < c$ exists s.t. $u_1 = c_1 \alpha \frac{n-1}{n} > c \frac{\alpha}{K} \frac{n-1}{n} = u_k$. This deviation is strictly profitable, so c could not have been an equilibrium. To see that $c = 0$ is an equilibrium, recall that by Lemma A13 there is no profitable upward deviation.

Necessity: Lemma A11 shows that for $K \geq r = 1$ the unique equilibrium fee level is $c = v(1-\alpha)$. Lemma 5 shows that for $1 < r < K$ there are multiple equilibria. Hence $K = r > 1$ is the only case where the unique equilibrium of $c = 0$ obtains.

Consumer welfare: As noted in the text, as firms make the same expected profit in both worlds, there is a one-to-one relation between consumer welfare and PCW profit. To see the difference in the changes to shopper and auto-renewer welfare from a move to a world with a PCW but $c = 0$, Proposition 3 of Morgan et al. (2006) (the only difference is that they have $v = 1$) shows that the increase from q to n results in a reduction in the expected price paid by shoppers, and an increase for auto-renewers. ■

Lemma 5. *When $K > r > 1$, there exists an equilibrium in which PCWs charge $\bar{c} > 0$, and firms list on all PCWs, mixing over prices by $G(p; \bar{c})$ where*

$$\bar{c} = \frac{v(1-\alpha)Kr(K-r)}{K(1+r(K-2)) + \alpha r(K-1)(r-1)(n-1)}.$$

Proof: I show that there exist equilibria such that $c \in [0, \bar{c}]$. Note that $\bar{c} \in [0, v(1-\alpha))$ hence any $c \in [0, v(1-\alpha))$. Take such a c as a candidate equilibrium fee level. There are no point masses by Lemma A6. By Lemma A7 the maximum of the support is v . By Lemma A5 $pr(E) = 0$ and as all PCWs charge the same fee, firms are content to list in all of them i.e., $\beta_k = 1$ for all k . Firms must be indifferent to all $(p, \{1, \dots, K\})$ they are called upon to play,

hence

$$v \left(\frac{1-\alpha}{n} \right) = p \left(\frac{1-\alpha}{n} \right) + \frac{\alpha}{n} (1 - G(p; c))^{n-1} [np - (n-1)c]$$

which can be re-arranged to give $G(p; c)$ in (2).

To confirm c is an equilibrium fee level, ensure there is no profitable PCW deviation. By Lemma A13, there is no profitable upward deviation. However, unlike Lemma A14, when $1 < r < K$ it is no longer true that any undercut by PCW₁ to $c_1 < c$ will result in all firms listing on it exclusively. This is because consumers see $r > 1$ PCWs: when a firm sells having played $(p, \{1, \dots, K\})$ it pays $c_1 \frac{1}{K} + c \frac{K-1}{K}$, but were it to have played $(p, 1)$ it would have paid $c_1 \frac{r}{K}$ i.e., the firm faces a trade-off between lower fees and higher sales volume. By Lemma A8 $\beta_1 = 1$. PCW₁'s deviation profit is therefore determined by $\beta_k, k > 1$. For small-enough undercuts of c , firms will still be content to list on all PCWs. Suppose however, that PCW₁ undercuts by just enough such that such that firms are no longer content to list all their prices on the other PCWs. In the best case for PCW₁, $\beta_k = 0$ for all $k > 1$ so that all shoppers checking PCW₁ buy only through PCW₁. If in this case, PCW₁ still does not make more than its equilibrium profit $u^* = c \frac{\alpha}{K} \frac{n-1}{n}$, then the undercut is never profitable and c constitutes an equilibrium fee level. I now carry out this logic.

Denote $c_1 < c$ as the undercut of PCW₁ and \tilde{c}_1 as the threshold level required for c_1 to entice firms to de-list some of their prices from PCW_k s.t. $k > 1$. Given $\mathbf{c} = (c_1, c, \dots, c)$, similarly to the derivation above, one can show firms will respond by $G(p; \mathbf{c})$ over $[\underline{p}(\mathbf{c}), v]$ as in (2). Firm deviation profit from this response to $(p, 1)$ is given by

$$\pi(p) = p \left(\frac{1-\alpha}{n} \right) + \frac{(v-p)(1-\alpha) [Kp - r(p-c_1)(n-1)]}{n(Knp - (n-1)(c_1 + c(K-1)))}$$

Note that this is valid for $p \in [\underline{p}(\mathbf{c}), v]$, but $p < \underline{p}_1$ give strictly less profit than $p = \underline{p}(\mathbf{c})$. One can show that $\pi(p)$ is convex in p . Together with the observation that $(v, 1)$ gives the equilibrium profit $\frac{v(1-\alpha)}{n}$, this says that the optimal deviation is to $(\underline{p}(\mathbf{c}), 1)$ and is profitable if and only if $\pi(\underline{p}(\mathbf{c})) > \frac{v(1-\alpha)}{n}$, which can be rearranged to give

$$c_1 < \frac{c(K-1)(K+r\alpha(n-1)) - Kv(K-r)(1-\alpha)}{r\alpha(K-1)(n-1) + K(r-1)} \equiv \tilde{c}_1.$$

Suppose that the firm response following the undercut was such that $\beta_k = 0$ for all $k > 1$ when

PCW₁ sets the highest such undercut just below \tilde{c}_1 . PCW₁ prefers not to make the deviation whenever

$$u^* \geq \tilde{c}_1 \frac{\alpha r n - 1}{K n} \iff \frac{c}{r} \geq \tilde{c}_1$$

which can be rearranged to give

$$c \leq \frac{v(1 - \alpha)Kr(K - r)}{K(1 + r(K - 2)) + \alpha r(K - 1)(r - 1)(n - 1)} \equiv \bar{c}.$$

Therefore at fee levels $c \in [0, \bar{c}]$ there exist equilibria where $\beta_k = 1$ for all k and firms mix by the CDF given in the Lemma. ■

Proposition 3. *For any r , there exists a \tilde{K} such that as long as there are more than \tilde{K} aggregators both types of consumer are worse off than before the introduction of PCWs.*

Proof: To show for shoppers, let the equilibrium be given as in Lemma 5 with $c = \bar{c}$ and see that

$$(4) \quad \lim_{K \rightarrow \infty} \bar{c} = v(1 - \alpha).$$

For shopper welfare notice that for $K = r$, $\bar{c} = 0$ and

$$(5) \quad \mathbb{E}_F[p_{(1,q)}] \geq \mathbb{E}_{G(0)}[p_{(1,n)}]$$

where $G(c)$ and F denote $G(p; c)$ and $F(p)$. This is because the PCWs effectively increase the number of firms competing for each shoppers from q to n , and from the last point of the proof of Lemma 4, this lowers the price paid by shoppers. Proposition 1 shows that both types are worse off with PCWs when $c = v(1 - \alpha)$ for $K \geq r = 1$, but notice that if $c = v(1 - \alpha)$ in equilibrium with $K > r \geq 1$, then this will be true a fortiori, because it can be shown that $G(v(1 - \alpha))$ first-order stochastically dominates F for all q . By (4) then as $K \rightarrow \infty$, $\mathbb{E}_F[p_{(1,q)}] < \mathbb{E}_{G(v(1-\alpha))}[p_{(1,n)}]$. Because \bar{c} is continuous in K , $\frac{d\bar{c}}{dK} > 0$ and $\frac{d\mathbb{E}_{G(\bar{c})}[p_{(1,n)}]}{dK} > 0$ by Corollary 1, there exists \tilde{K} s.t. for $K > \tilde{K}$ there is always an equilibrium fee level that makes shoppers worse off relative to a world without PCWs.

The proof for auto-renewers is similar but simpler to the above steps for shoppers and has p replacing $p_{(1,q)}$ and $p_{(1,n)}$: From Proposition 3 of Morgan et al. (2006), the effect of increasing the number of firms competing for each shopper increases prices for auto-renewers. Then by

Corollary 1, their prices only rise further under $G(c)$. Hence auto-renewers are worse off for any c and hence any K . ■

A6 Results under Ad Valorem Fees

Here, I present the equilibrium where the PCW to charge a proportion of the sale price, $\lambda \in [0, 1]$, rather than fixed per-sale fees. The rest of the model setup is as before with $K = 1$.

Proposition A1. *In the unique equilibrium with ad valorem fees, the PCW sets a fee equal to the sale price i.e., $\lambda = 1$, firms list on the PCW and mix over prices according to:*

$$H(p) = 1 - \left[\frac{(v-p)(1-\alpha)}{p\alpha} \right]^{\frac{1}{n-1}} \text{ over the support } p \in [v(1-\alpha), v].$$

Proof: Many parts of the proof are similar to those with fixed per-sale fees. As such, a brief proof is provided. Firstly, notice that in any equilibrium $\lambda = 1$. This is because for any $\lambda \in [0, 1)$ all firms list their prices and mix over an interval of prices with no point masses. Furthermore, these distributions are first-order stochastically ordered in λ where a higher λ corresponds to higher prices. Therefore, the PCW would prefer to slightly raise λ . The only remaining candidate is $\lambda = 1$. When a firm sells, it is the cheapest, it sells to other firms' shoppers through the PCW, and all the rent is extracted by the PCW because $\lambda = 1$. However, when it is cheapest it also sells to its own shoppers, for whom the sale is completed directly. In equilibrium, the firms therefore only compete for their own shoppers. This results in the familiar equilibrium indifference relation for a firm between charging v and only selling to its loyal, or charging another price $p < v$ in the equilibrium support which generates sales to loyal and when it is the lowest price charges, also to shoppers:

$$v \left(\frac{1-\alpha}{n} \right) = p \left(\frac{1-\alpha}{n} \right) + p(1-H(p))^{n-1} \frac{\alpha}{n}$$

which rearranges to give the expression in the Proposition. Note that H is a well-defined CDF over the support $p \in [v(1-\alpha), v]$. ■

Proposition 4. *When the PCW charges ad valorem fees, both types of consumer are worse off with the PCW than without.*

Proof: Shoppers are worse off because the lower bound of the equilibrium distribution of prices in Proposition A1 is the same as in Lemma 3. Hence we can follow the same proof as in Proposition 1 to show that the price shoppers face is guaranteed to be above that in the world without a PCW.

To see that auto-renewers are worse off, one can compare H of Proposition A1 with the distribution of prices in the world without a PCW, F of Lemma 1. Because $q > 1$ and $q \leq n$, it is clear that H first-order stochastically dominates F , which means that expected price is higher under H . ■

A7 Results under Price Discrimination

Now assume that different prices can be charged directly and through the PCW. Denote such prices p_0 and p_k respectively, where $k = 1, \dots, K$ indexes the PCW as before.

Lemma A15. *If $r = 1$, $c_1 = \dots = c_{K-1} = v(1 - \alpha)$ and $c_K \in [c_1, v]$ then there exist firm responses s.t. $p_0 = p_1 = \dots = p_K = v(1 - \alpha)$ and $\beta_k = 1$ for all k .*

Proof: Under these strategies, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_k for any k : lower would prompt sales at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving p_0 : lower would still sell to auto-renewers and own-shoppers but at a lower price, higher would only sell to auto-renewers for which the optimal such deviation is to v generating profit $\frac{v(1-\alpha)}{n} = \pi$. ■

Proposition 5. *With price discrimination, both types of consumer are worse off with the PCW than without.*

Proof: I show there is an equilibrium in which PCWs set $c = v(1 - \alpha)$, firms list on all PCWs, $p_0 = v$ and $p_1 = \dots = p_K = v(1 - \alpha)$. Given $c_1 = \dots = c_K \equiv c = v(1 - \alpha)$, let us confirm there are no profitable deviations for firms. Equilibrium firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_k for any k : lower would prompt sales at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving p_0 : lower would either sell only to auto-renewers at a lower price, or to both auto-renewers and own-shoppers but only for prices at least as low as $v(1 - \alpha)$, generating profit no greater than $\frac{v(1-\alpha)}{n}$; higher would be

above v and hence make zero profits. Now consider a PCW deviation. Equilibrium PCW profit is $u_k = \frac{\alpha v(1-\alpha)}{K} > 0$. There is no profitable deviation to a lower fee: as $r = 1$ doing so would at best sell to the same proportion of shoppers ($\frac{\alpha}{K}$) but at a lower fee. As for a deviation to a higher fee: assume that firms' mutual best responses are given by those in Lemma A15 where PCW profit is zero. Auto-renewers are worse off than in the world without PCWs because they now pay v . Shoppers are worse off because they pay $v(1-\alpha)$ which the proof of Proposition 1 shows is an upper bound of the expected price they pay in the world without PCWs. ■

Corollary A1. *When firms price discriminate and $K \geq r > 1$, following $c_1 \in (0, c_2)$ and $c_2 = c_3 = \dots = c_K \equiv c \in (0, v(1-\alpha)]$ there exist mutual best-responses of firms such that they list on all PCWs, setting $p_0 = v$ and $p_k = c_k$ for all k .*

Proof: Given c as in the Lemma, let us confirm there are no profitable deviations for firms. Firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_k for any k : lower could prompt sales, but only at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving p_0 : lower would either sell only to auto-renewers at a lower price, or to both auto-renewers and own-shoppers but only for prices at least as low as c_2 , generating profit no greater than $\frac{c_2}{n} < \pi$; higher would be above v and hence make zero profits. ■

Lemma A16. *If $r > 1$, $c_1 = \dots = c_{K-1} = 0$ and $c_K \in [0, v]$ then there exist firm responses s.t. $p_0 = v$, $p_1 = \dots = p_{K-1} = 0$, $\beta_k = 1$ for $k < K$ and $\beta_K = 0$.*

Proof: Under these strategies, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_K : $p_K < 0$ would give sell at a loss whereas $p_K > 0$ would never attract any shoppers as $r > 1$. There is no profitable deviation involving p_k for any $k < K$: Lower would only create sales at a loss, higher would never attract any shoppers. There is no profitable deviation involving p_0 : lower would still sell to auto-renewers and (for $p_0 \leq 0$) own-shoppers but at a lower price, higher would sell to no-one. ■

Lemma A17. *If $r > 1$, there exists an equilibrium where $c = 0$, firms list on all PCWs, $p_0 = v$ and $p_1 = \dots = p_K = 0$.*

Proof: One can follow the proof of Proposition 5 to confirm there are no profitable deviations for firms. For PCWs, one only need consider an upward deviation in fee level in which case, assume firms respond as in Lemma A16, which yields zero profit for the deviating PCW. ■

Proposition 6. *As $n \rightarrow \infty$, following the introduction of a PCW, the expected prices faced by both types of consumer in a setting without price discrimination approach those in a setting with price discrimination.*

Proof: Taking limits,

$$\lim_{n \rightarrow \infty} G(p; c) = 0$$

which shows $\lim_{n \rightarrow \infty} \mathbb{E}_G[p] = v$. The CDF for the lowest of n draws is given by

$$H(p; c) = 1 - (1 - G(p; c))^n = 1 - \left[\frac{(v - p)(1 - \alpha)}{\alpha(np - (n - 1)c)} \right]^{\frac{n}{n-1}}$$

for $p \in [p(c), v]$. Taking limits,

$$\lim_{n \rightarrow \infty} H(p; c) = 1 \text{ and } \lim_{n \rightarrow \infty} p(c) = c$$

which shows $\lim_{n \rightarrow \infty} \mathbb{E}_G[p_{(1,n)}] = c$. These prices are the same as those in the equilibrium of Proposition 5. ■

A8 Results with Search Costs

In the world without a PCW under $q = 2$, one can compute

$$\begin{aligned} \mathbb{E}_F[p] &= \frac{v}{2} \left[\frac{1 - \alpha}{\alpha} \right] \left[\log \left(\frac{1 + \alpha}{1 - \alpha} \right) \right] \\ \mathbb{E}_F[p_{(1,2)}] &= \frac{v}{2} \left[\frac{1 - \alpha}{\alpha} \right]^2 \left[\log \left(\frac{1 - \alpha}{1 + \alpha} \right) + \frac{2\alpha}{1 - \alpha} \right] \\ \mathcal{B}_0 &= \frac{v}{2} \left[\frac{1 - \alpha}{\alpha^2} \right] \left[\log \left(\frac{1 + \alpha}{1 - \alpha} \right) - 2\alpha \right]. \end{aligned}$$

In the world with a PCW under $n \rightarrow \infty$, Proposition 6 shows that,

$$\mathbb{E}_G[p] = v, \quad \mathbb{E}_G[p_{(1,\infty)}] = c, \quad \mathcal{B}_1 = v - c$$

where $c = v(1 - \tilde{\alpha}) \leq v(1 - \alpha)$ in equilibrium.

Proposition 7. *Shoppers, converts and non-converts can all be worse off with a PCW than without.*

Proof: This is a proof by example. Let $v = 1$, $\alpha = 0.7$, $n = 3$ and $q = 2$. Then, in the world

without a PCW, it can be computed that,

$$\mathbb{E}_F[p] = 0.3717, \quad \mathbb{E}_F[p_{(1,2)}] = 0.2693, \quad \mathcal{B}_0 = 0.1024.$$

In the world with a PCW, assume $\tilde{\alpha} = 0.71$ (i.e., the PCW attracts 0.01 more consumers i.e., $\frac{1}{30}$ auto-renewers, to compare prices), so $c = v(1 - \alpha) = 0.29$ and

$$\mathbb{E}_G[p] = 0.5557, \quad \mathbb{E}_G[p_{(1,3)}] = 0.3748, \quad \mathcal{B}_1 = 0.1808.$$

Comparing, one can see that if there is an S such that this can be rationalised as an equilibrium, then all types of consumer will be worse off with a PCW than without. To rationalise, I construct an S such that:

$$S(\mathcal{B}_1) = \begin{cases} 0 & \text{if } \mathcal{B}_1 < \underline{s} \\ \frac{\mathcal{B}_1 - \underline{s}}{\mathcal{B}_1} & \text{else} \end{cases}$$

where \underline{s} is determined by

$$S(0.1808) = \frac{1}{30} \iff \underline{s} = 0.1748.$$

Finally, notice that $\underline{s} > \mathcal{B}_0$. ■

Lemma A18. *Under maximum potential, any $\tilde{\alpha}$ can be rationalised by an S .*

Proof: Note that any $\tilde{\alpha}$ can be rationalised with an S (as in the proof of Proposition 7) because $\mathcal{B}_0 < \mathcal{B}_1$. To see this holds, one can show that $\mathcal{B}_0 < v\alpha$ holds for any $\alpha \in (0, 1)$ and hence so does $\mathcal{B}_0 < v\tilde{\alpha}$. ■

Proposition 8. *If the market has maximum potential: converts are better off; shoppers can be worse off; and non-converts are worse off with a PCW than without.*

Proof: As $v(1 - \alpha) > v(1 - \tilde{\alpha})$ when there are converts, to show converts are better off with a PCW under maximum potential one can show,

$$\mathbb{E}_F[p] > v(1 - \alpha) \iff \log\left(\frac{1 + \alpha}{1 - \alpha}\right) > 2\alpha$$

which is satisfied for $\alpha \in (0, 1)$. Note that $\tilde{\alpha}$ is rationalisable by some S (Lemma A18).

Shoppers are worse off under maximum potential wherever $v(1 - \tilde{\alpha}) > \mathbb{E}_F[p_{(1,2)}]$. To show this can occur, note that $v(1 - \alpha) > \mathbb{E}_F[p_{(1,2)}]$ holds for all $\alpha \in (0, 1)$ (see Proposition 1). Hence for any α there exists some $\tilde{\alpha} > \alpha$ small enough such that shoppers are worse off. Note that such a $\tilde{\alpha}$ is rationalisable by some S (Lemma A18).

Non-converts are worse off under maximum potential because $\mathbb{E}_F[p] < \mathbb{E}_G[p] = v$. ■

A9 Result with a Meta-Site

Suppose there are $K \geq 1$ PCWs and a meta-site which charges a click-through fee where shoppers only check the meta-site.

Proposition A2. *There is an equilibrium where the meta-site sets a click-through fee of $v(1 - \alpha)$, the PCWs set a click-through fee of $c_k = v(1 - \alpha)$ for $k = 1, \dots, K$, firms list either on the PCW or the meta-site, or both, and mix over prices according to $G(p; v(1 - \alpha))$ over the support $p \in [v(1 - \alpha), v]$.*

Proof: The meta-site (or any PCW) has no incentive to raise its fee: If it did, then by similar reasoning to Lemma 2, pure pricing on the meta-site (or PCW) would ensue and no consumers switch. No PCW would de-list from the meta-site because shoppers do not check PCWs. Firms face the same incentives as in Lemma 3 because no matter which channel they sell to shoppers through, they incur a per-unit cost of $v(1 - \alpha)$: If they sell through a direct listing on the meta-site, they pay the fee to the meta-site; if they sell through a PCW listed on the meta-site, they pay the fee to the PCW. Therefore, i) firms have no incentive to de-list from any PCW or meta-site; and ii) the pricing strategy is the same as in Lemma 3. ■

A10 Results under Fees-Transparency

Definition (Coordinated Subgame). *Given c , a ‘coordinated subgame’ is when shoppers attend and firms list in only the cheapest PCWs i.e., shoppers in PCW_k if $c_k = c_1$, and for firms $\beta_k = 1$ if $c_k = c_1$ else $\beta_k = 0$. Firms mix by $G(p, c_1)$.*

Lemma A19. *When fees are observed by shoppers, and all subgames are ‘coordinated subgames’ there exists a subgame perfect equilibrium where $c = 0$, $\beta_k = 1$ for all k and prices are distributed according to the CDF $G(p, 0)$.*

Proof: There are no profitable deviations for firms: $G(p, 0)$ ensures they are indifferent between listing all prices in the support $[\underline{p}(0), v]$, listing any price less than $\underline{p}(0)$ is less profitable than listing v , and any (p, \emptyset) generates at most $\frac{v(1-\alpha)}{n}$ when $p = v$. To see there is no profitable upward deviation for PCWs, note that due to the ‘coordinated subgame’ shoppers and firms would not attend this PCW following such a deviation, so the PCW receives zero profit. To see that these coordination subgames are Nash equilibria: for firms one can conduct the same round of checks as at the beginning of this proof; for consumers notice that as the non-cheapest PCWs are all empty, there is no incentive to check them. ■

Proposition A3. *When $K > 1$ and PCW fees are observed by shoppers, there exists an equilibrium with $c = 0$.*

Proof: That there exists an equilibrium with $c = 0$ follows directly from Lemma A19. For shopper welfare at $c = 0$, see the last point of the proof of Lemma 4. ■

Proposition A3 excludes the monopoly-PCW case ($K = 1$), where the unique equilibrium is still that of Lemma 3 because no coordination between firms and consumers over which PCW to attend is possible when there is only one PCW.

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