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The Cournot–Bertrand profit differential: A reversal result in a differentiated duopoly with wage bargaining

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Abstract

This paper compares Cournot and Bertrand equilibria in a downstream differentiated duopoly in which the input price (wage) paid by each downstream firm is the outcome of a strategic bargain with its upstream supplier (labor union). We show that the standard result that Cournot equilibrium profits exceed those under Bertrand competition – when the differentiated duopoly game is played in imperfect substitutes – is reversible. Whether equilibrium profits are higher under Cournot or Bertrand competition is shown to depend upon the nature of the upstream agents' preferences and on the distribution of bargaining power over the input price. We find that the standard result holds unless unions are both powerful and place considerable weight on the wage argument in their utility function.

JEL classification: D43; J50; L13

Keywords: Differentiated duopoly; Wage bargaining; Cournot; Bertrand

1. Introduction

A classic result in oligopoly theory is that firms will set quantities rather than prices when goods are imperfect substitutes. This result was first formalized by Singh and Vives (1984) and has been further refined by Vives (1985), who establishes more general results on the ranking of Cournot and Bertrand outcomes, by Okuguchi (1987) and, in a geometric analysis, by Cheng (1985). The result is a cornerstone of oligopoly theory.

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Recently, the early results have attracted renewed interest. Dastidar (1997) shows that in a homogeneous product market the results are sensitive to the sharing rule and are not necessarily valid under asymmetric costs. In the standard model, costs are both symmetric and exogenous. Qiu (1997) develops a model of differentiated duopoly in which there is a two-stage game. In stage 1, each firm chooses a level of cost-reducing research and development (R&D) prior to the standard product market game played in a second stage. Qiu (1997) shows that the relative efficiency of Cournot and Bertrand competition depends upon R&D productivity, the extent of spillovers, and the degree of product market differentiation. Lambertini (1997) extends the standard analysis to the context of a repeated market game in which the firm's choice of the strategic variable is itself the outcome of a strategic (meta) game. This game is also shown to be characterized by the prisoners' dilemma.

Häckner (2000) has shown that the result concerning the dominance of Cournot over Bertrand profits is sensitive to the duopoly assumption. Häckner (2000) considers an n-firm setting with vertical product differentiation. Our paper, like that of Häckner (2000), can be thought of as testing the robustness of the standard results with respect to alternative market structures. While Häckner (2000) extends the standard model *horizontally* through increasing the number of firms within the product market, our paper extends the analysis *vertically* by examining the consequences of introducing upstream suppliers to the downstream duopolists.

In particular, we address the issue of whether the standard results on the ranking of Cournot and Bertrand equilibrium outcomes under differentiated duopoly are robust to the inclusion of a decentralized wage-bargaining game played between each firm and a firm-specific labor union. There is symmetry across the two union–firm wage bargains. Hence, in equilibrium, we retain the property of symmetric costs, typically assumed in the standard model. As in Qiu (1997) – though for very different reasons – these costs, however, are no longer exogenous in our model. Instead, in the model we develop here, they are the outcome of a strategic game played between each firm and its labor union. This can be interpreted as a particular example of a more general situation of bargaining between an upstream supplier and a downstream retailer in the context of oligopoly in the retail market. The structure of our model is similar to that of Qiu (1997), with wage bargaining rather than R& D choice in the first stage of the game. In stage 2, we consider both Cournot and Bertrand solutions to the non-cooperative product market game.

Our analysis of the Cournot solution is closely related to the model of Horn and Wolinsky (1988), which analyses the incentives to merge among upstream and downstream firms, and how these incentives depend on the degree of product differentiation. The model of Horn and Wolinsky (1988) builds on the concept of strategic substitutes and complements developed by Bulow et al. (1985). The analysis of wage determination in unionized oligopolies was first developed by Davidson (1988), who focused on a comparison of local and national bargaining and, like Horn and Wolinsky (1988), adopted the standard Cournot–Nash assumption to describe product market competition. A somewhat more generalized approach to wage setting in the context of imperfect competition in both labor and product markets is described by Dowrick (1989). Similarly, Naylor (1998, 1999) considers unionized duopoly in the context of international trade and economic integration, and again assumes Cournot behavior in the product market. Grandner (2001) examines the importance of the level of bargaining for consumer prices in vertically connected industries under oligopoly.

A further motivation for our paper stems from the observation that there is a rapidly-expanding literature based on models of unionized oligopoly. A commonplace assumption of these models is that product market competition is of the Cournot variety: Often with an appeal to the classic Singh and Vives (1984) result. But this appeal is valid only if it can be shown that the standard result – established under the assumption that input costs are given – obtains when wages are the result of bargaining. For the case of linear product demand, which is the predominant case considered in the literatures, we establish conditions under which the standard result obtains. We show that the necessary conditions are such that they are typically satisfied by common models of unionized oligopoly, but that a reversal of the Cournot–Bertrand profit ordering can occur under particular assumptions regarding union preferences and union bargaining power.

The rest of this paper is organized as follows. In Section 2, we present the basic model in which two firms compete in the product market having first bargained independently over wages with a local (firm-specific) labor union. The two firms produce differentiated products. The product market is assumed to be characterized by Cournot competition. We derive sub-game perfect Nash equilibrium values for the key variables of interest. Section 3 presents the corresponding equilibrium values for the case of Bertrand competition in the product market. In Section 4, we compare Cournot and Bertrand equilibrium profits. We show that the standard result that profits are higher under Cournot than under Bertrand competition – in the case of imperfect substitutes – is reversed under certain assumptions regarding the extent of product differentiation, union preferences and bargaining power. In Section 5, we explore the underlying reasons for the reversibility of the standard result and in Section 6 we examine the welfare properties of the model. Section 7 closes the paper with conclusions and further remarks.

2. Cournot equilibrium under unionized duopoly

The model of the differentiated product market duopoly follows Singh and Vives (1984) and Qiu (1997). We analyze a non-cooperative two-stage game in which two firms produce differentiated goods. In the first stage (the labor market game), each firm independently bargains over its wage with a local labor union. The outcome of the labor market game is described by the solution to the two union–firm pairs' sub-game perfect best-reply functions in wages. In the second stage (the product market game), each firm sets its output – given pre-determined wage choices from stage 1 – to maximize profits. Preferences of the representative consumer are given by

$$U(q_i, q_j) = a(q_i + q_j) - (q_i^2 + 2cq_iq_j + q_j^2)/2,$$

where q_i, q_j denote outputs by firm i and j, respectively, $a > 0$, and $c \in (0,1)$ denotes the extent of product differentiation with goods assumed to be imperfect substitutes.[1]

[1] We discuss the case of complements later in the paper.

The derived product market demand is linear and, for firm i for example, is given by

$$p_i(q_i, q_j) = a - cq_j - q_i. \tag{1}$$

In the standard model, the two firms face the same constant marginal cost, w. Qiu (1997) considers the case in which the firm can influence its marginal cost through R&D expenditure. In the current paper, we assume that the constant marginal cost is the result of a decentralized stage 1 bargain with a local union. We assume that the two firms have the same technology and that the two firm-specific labor unions have the same preferences and the same bargaining power over wages. In symmetric equilibrium, therefore, the two firms will have identical marginal costs: although these will be the outcome of strategic play across the two union–firm pairs.

Profits of firm i can be written as

$$\pi_i = (p_i - w_i)q_i, \tag{2}$$

where w_i denotes the wage paid by firm i and is assumed to capture all short-run marginal costs. Under the assumption of a constant marginal product of labor, normalized to unity, q_i represents both output and employment of firm i.

From (1) and (2), under profit-maximization, firm i's best-reply function is

$$q_i(q_j) = \tfrac{1}{2}(a - cq_j - w_i). \tag{3}$$

As $c > 0$, by assumption, the best-reply functions are downward-sloping: Under the Cournot assumption, the product market game is played in strategic substitutes. From (3) and its equivalent for firm j, we obtain labor demand by firm i, given w_i and w_j:

$$q_i(w_i, w_j) = \frac{1}{4 - c^2}\,[(2 - c)a - 2w_i + cw_j]. \tag{4}$$

It follows that firm i's Cournot–Nash equilibrium profits, given w_i, w_j, are

$$\pi_i(w_i, w_j) = \frac{1}{(4 - c^2)^2}\,[(2 - c)a - 2w_i + cw_j]^2. \tag{5}$$

We now consider two alternative cases. In Regime 1, wages are exogenously determined and set at the reservation wage level, $\bar{w}$. In Regime 2, wages are set endogenously through decentralized bargaining in the non-cooperative Stage 1 labor market game.

2.1. Regime 1: Exogenous wages

Assume that, in the absence of labor unions, $w_i = w_j = \bar{w}$. Then symmetric equilibrium Cournot–Nash profits are given by

$$\pi^C = \frac{1}{(2 + c)^2}\,(a - \bar{w})^2. \tag{6}$$

2.2. Regime 2: Endogenous wages

Assume that, in Stage 1, firm i, for example, bargains over the wage, w_i, with a local labor union, union i, whose utility function is given by

$$u_i(w_i, q_i) = (w_i - \bar{w})^{\theta} q_i^{1-\theta}, \tag{7}$$

where θ denotes the relative strength of the union's preference for wages over employment and $0 \leqslant \theta \leqslant 1$. This functional form is quite general and encompasses common assumptions such as rent-maximization, arising when $\theta = 1/2$ and total wage bill maximization when $\theta = 1/2$ and $\bar{w} = 0$. We follow Horn and Wolinsky (1988), inter alia, in assuming that the downstream agent has the right to choose final output. We note that empirical evidence on union–firm bargaining is not definitive on the issue of the scope of bargaining, but does not lend strong support to the assumption of efficient bargaining over both wages *and* employment.

The general asymmetric Nash bargain over wages between union–firm pair i solves:

$$w_i = \arg\max\{B_i = u_i^{\beta}\pi_i^{1-\beta}\}, \tag{8}$$

where β is the union's Nash bargaining parameter and $0 \leqslant \beta \leqslant 1$. In the two-stage sequential game, the union and firm bargain over wages only: the firm is assumed to have the right-to-manage autonomy over employment. We rule out the special case in which $\beta = \theta = 1$.[2]

Substituting (4), (5) and (7) into (8) yields

$$B_i = \left[\frac{1}{4-c^2}\right]^{2-\beta(1+\theta)} (w_i - \bar{w})^{\beta\theta}[(2-c)a - 2w_i + cw_j]^{2-\beta(1+\theta)}, \tag{9}$$

where disagreement payoffs are assumed to be zero. From (8) and (9), the first-order condition yields

$$w_i^C = \bar{w} + \left[\frac{1}{2(2-\beta)}\right][\beta\theta(2-c)(a-\bar{w}) + c\beta\theta(w_j - \bar{w})], \tag{10}$$

which defines the sub-game perfect best-reply function in wages of union–firm pair i under the assumption of a non-cooperative Cournot–Nash equilibrium in the product market. From (10), the slope of union–firm pair i's best-reply function is given by

$$\frac{\partial w_i^C}{\partial w_j} = \frac{c\beta\theta}{2(2-\beta)}. \tag{11}$$

The slope of the best-reply wage function is positive for $c > 0$, $\theta > 0$, $\beta > 0$, confirming that the labor market game is played in strategic complements. In symmetric sub-game perfect equilibrium, $w_i = w_j$ and hence, from (10), equilibrium wages are given by

$$w_i^C = w_j^C = \bar{w} + \left[\frac{(2-c)\beta\theta}{2(2-\beta) - c\beta\theta}\right](a - \bar{w}) = w^C. \tag{12}$$

Note that $w^C = \bar{w}$ if either $\theta = 0$ or $\beta = 0$. From substitution of (12) in (5), we conclude that sub-game perfect equilibrium profits under Cournot competition are given by

$$\pi^C = \left[\frac{2}{2(2-\beta) - c\beta\theta}\right]^2 \left(\frac{a-\bar{w}}{2+c}\right)^2 [2 - \beta(1+\theta)]^2. \tag{13}$$

[2] By ruling out $\beta = \theta = 1$, we avoid the problem of the 'Cheshire cat' monopoly union which sets such a high wage that employment collapses to zero.

3. Bertrand equilibrium under unionized duopoly

In this section of the paper, we suppose that the product market game in stage 2 is characterized by price-setting behavior by firms. From (1) and its counterpart for firm j, we can write product demand facing firm i as

$$q_i(p_i, p_j) = \frac{1}{1-c^2}\,[a(1-c) - p_i + cp_j]. \tag{14}$$

Profits of firm i are then given by

$$\pi_i(p_i, p_j) = \frac{1}{1-c^2}\,[a(1-c) - p_i + cp_j](p_i - w_i). \tag{15}$$

From (15), the first-order condition for profit-maximization gives

$$p_i(p_j) = \tfrac{1}{2}[a(1-c) + cp_j + w_i], \tag{16}$$

and hence, for $c > 0$, the Bertrand product market game is played in strategic complements. Hence, substituting (16) in (14) yields

$$q_i(w_i, w_j) = \frac{1}{(4-c^2)(1-c^2)}\,[(2+c)(1-c)a + cw_j - (2-c^2)w_i], \tag{17}$$

which is the sub-game perfect labor demand function facing union i in the stage 1 wage-bargaining game and is the Bertrand equivalent to (4). Substitution yields

$$\pi_i(w_i, w_j) = \frac{1}{[4-c^2]^2[1-c^2]}\,[(2+c)(1-c)a + cw_j - (2-c^2)w_i]^2, \tag{18}$$

which is the Bertrand equivalent to (5) for the case of Cournot competition.

As in Section 2, we now distinguish between the 2 possible labor market regimes.

3.1. Regime 1: Exogenous wages

Assume that, in the absence of labor unions, $w_i = w_j = \bar{w}$. Then symmetric equilibrium Bertrand–Nash profits are given by

$$\pi^B = \frac{1-c}{(2-c)^2(1+c)}\,(a-\bar{w})^2. \tag{19}$$

In the standard model of differentiated duopoly, with marginal costs (wages) determined exogenously, the relation between Cournot and Bertrand profits is based on a comparison of (19) and (6). It is easily demonstrated that in this non-union case, the sign of $(\pi^C - \pi^B)$ is equal to the sign of c. Hence, if firms produce imperfect substitutes, $c > 0$, Cournot profits will exceed Bertrand profits in equilibrium. Accordingly, firms would prefer Cournot to Bertrand competition. We now derive the expression for sub-game perfect Bertrand equilibrium profits when wages are subject to bargaining.

3.2. Regime 2: Endogenous wages

As in Section 2, we assume that there is an independent wage bargain between each firm and its labor union. Union preferences are given by (7) and the Nash maximand is represented by (8). Substituting (17), (18) and (7) in (8) and solving produces a first-order condition for the Nash maximand that is satisfied when

$$w_i^B = \bar{w} + \left[\frac{1}{(2-c^2)(2-\beta)}\right][\beta\theta(2+c)(1-c)(a-\bar{w}) + c\beta\theta(w_j - \bar{w})], \tag{20}$$

which defines the sub-game perfect best-reply function in wages of union–firm pair i under the assumption of a non-cooperative Bertrand–Nash equilibrium in the product market. Eq. (20) is the Bertrand counterpart of (10) for the case of Cournot competition in the product market. Differentiating (20) with respect to w_j gives the slope of union–firm pair i's best-reply function in wage-space as

$$\frac{\partial w_i^B}{\partial w_j} = \frac{c\beta\theta}{(2-c^2)(2-\beta)}. \tag{21}$$

The slope of the best-reply wage function is again positive for $c>0$, $\theta>0$, $\beta>0$, confirming that the labor market game is played in strategic complements, independent of the type of product market competition. Again, we note that the slope of the best-reply function in wages is increasing both in θ and in β.

In symmetric equilibrium, $w_i = w_j$ and hence, from (20), sub-game perfect equilibrium wages are given by

$$w_i^B = w_j^B = \bar{w} + \left[\frac{(2+c)(1-c)\beta\theta}{(2-c^2)(2-\beta) - c\beta\theta}\right](a-\bar{w}) = w^B, \tag{22}$$

where w^B signifies that the product market game in stage 2 is described by the non-cooperative Bertrand–Nash outcome. Note that $w^B = \bar{w}$ if either $\theta = 0$ or $\beta = 0$.

Substituting (22) into the expression for Bertrand equilibrium profits, given in (18), we derive sub-game perfect equilibrium profits as

$$\pi^B = \frac{1-c}{(2-c)^2(1+c)}\left[\frac{(2-c^2)[2-\beta(1+\theta)]}{(2-c^2)(2-\beta) - c\beta\theta}\right]^2 (a-\bar{w})^2. \tag{23}$$

We note that (23) replicates (19) for the non-union benchmark case in which $\beta = 0$. We are now in a position to compare Cournot and Bertrand profits for the case in which wages are determined by decentralized bargaining in stage 1.

4. The Cournot–Bertrand profit differential

Comparison of (13) and (23) yields the expression for the Cournot–Bertrand profit differential, F:

$$F = \left[\frac{1}{(2+c)^2}\left[\frac{2}{2(2-\beta) - c\beta\theta}\right]^2 - \frac{1-c}{(2-c)^2(1+c)}\left[\frac{2-c^2}{(2-c^2)(2-\beta) - c\beta\theta}\right]^2\right] A, \tag{24}$$

where $A = [2-\beta(1+\theta)]^2(a-\bar{w})^2 > 0$.

From expression (24), we formulate the question to be answered next: *is* $F > 0$ *for every* $c \in (0,1)$ *or are there certain values of the parameters* $\{\beta, c, \theta\}$ *that render* $F \leqslant 0$ *even if* $c \in (0,1)$? In other words, we evaluate whether there is a range of parameter values over which firms prefer the Bertrand-type of product market competition to the Cournot-type in the presence of unions and imperfect substitutes. From the standard result, we know that this is not the case in the absence of unions.

In (24), $F = 0$ defines the surface, in (β, c, θ)-space, along which the profit differential is zero. For the firms, this can be thought of as an iso-profit or 'indifference surface'. In order to examine the properties of this surface, we consider cross-sections of the surface in (β, θ)-space produced at given values along the c dimension. This yields 'indifference curves' in (β, θ)-space, each drawn for given c.

When $F = 0$, it follows that the term in brackets acting on A in (24) must also be equal to zero, as $A > 0$ under the assumptions of the model. It is then easily shown that $F = 0$ implies that

$$\theta = \frac{2(2-\beta)}{c\beta}\hat{A}, \tag{25}$$

where $0 < \hat{A} < 1, \forall c$: $0 < c < 1$ (see Appendix A for proof). Furthermore, given that $A > 0$, it follows that the term in brackets, on which A acts, in (24) determines the sign of the profit differential. That is, from (24), it is straightforward to prove algebraically that

$$\text{sign}[F] = -\text{sign}\left[\theta - \frac{2(2-\beta)}{c\beta}\hat{A}\right]. \tag{26}$$

Eq. (25) defines the firms' indifference curve in (β, θ)-space, for given c. Differentiation of (25) gives the slope of the indifference curve as

$$\frac{\partial\theta}{\partial\beta} = -\frac{4}{c\beta^2}\hat{A} < 0. \tag{27}$$

As $\hat{A} > 0$, it follows from (27) that the indifference curve has a negative slope in (β, θ)-space. The key question is whether this indifference curve cuts through the unit-square in (β, θ)-space, for $0 < c < 1$, dividing the unit-square into two segments: One representing combinations of β and θ associated with $F > 0$ (as in the standard result) and the other with $F < 0$. Our strategy for addressing this question is to show that, for some c such that $0 < c < 1$, $\exists\ 0 < \theta \leqslant 1$ such that $F = 0$ holds for some β satisfying $0 < \beta \leqslant 1$.

For simplicity, consider the case in which $c = 0.5$. It follows that Eq. (25) can be simplified to

$$\theta = \frac{0.84(2-\beta)}{\beta}, \tag{28}$$

where $\hat{A} = 0.21$. Eq. (28) represents the indifference curve for the case in which $c = 0.5$, as depicted in Fig. 1.[3]

[3] Fig. 1 is a direct plot of Eq. (25) using Mathematica (Wolfram, 1999).

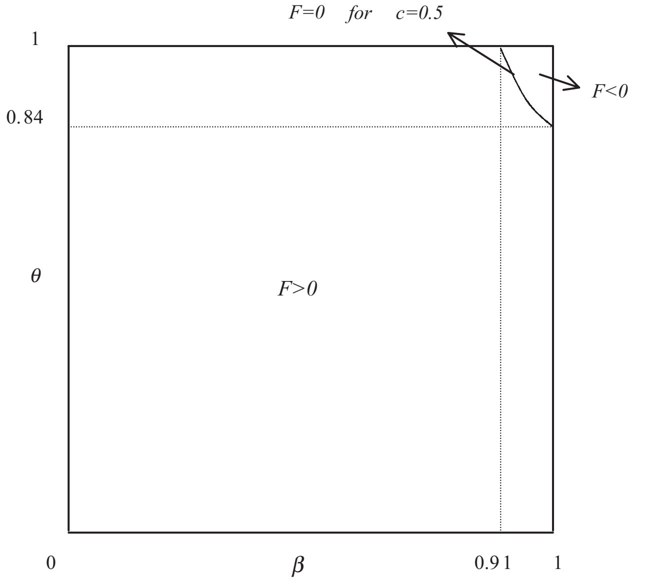

Fig. 1. Plot of the firms' indifference curve in $\{\beta, \theta\}$-space: for $c = 0.5$.

For $\beta = 1$, Eq. (28) yields $\theta = 0.84$. Similarly, for $\theta = 1$, Eq. (28) yields $\beta = 0.91$. Thus, we have established that, for $c = 0.5$, $\exists\ 0 < \theta \leqslant 1$ such that $F = 0$ holds for some β satisfying $0 < \beta \leqslant 1$. Furthermore, in the light of (26), we conclude from (28) that for $\theta < 0.84$, $F > 0$ for all possible values of β. Similarly, $\beta < 0.91$ implies that $F > 0$ for all possible values of θ. On the other hand, (26) and (28) imply that, for $c = 0.5$, $F < 0$ for combinations of β and θ in which both $\beta < 0.91$ and $\theta < 0.84$. The result that $F < 0$ for some combinations of β and θ when $c = 0.5$ can be generalized for $\forall c$.[4] Our analysis establishes Proposition 1.

Proposition 1. *In sub-game perfect equilibrium, Bertrand profits exceed Cournot profits in the case of imperfect substitutes, for sufficiently high values of β and θ. In other words, the standard result on the ranking of Cournot and Bertrand profits is reversed when upstream suppliers (labor unions) have sufficient bargaining power and place sufficient weight on wages in their utility functions.*

As stated in Proposition 1, the standard result concerning the ranking of Cournot and Bertrand profits under duopoly, when products are imperfect substitutes, is reversed only when unions are both relatively powerful in the wage bargain and attach relatively high importance to wages in their objective functions. It follows that under symmetric Nash bargaining, for example, the reversal result does not obtain. Similarly, the standard

[4] For reasons of space, the full proof for the general case is not reproduced here but can be found in a longer working paper version of this paper (see Correa-López and Naylor, 2002), where we also show that the critical values of β and θ vary with c.

profit-ranking will not be reversed if unions are simple rent-maximizers, attaching equal weight to wages and employment. A corollary of this is that if upstream agents are profit-maximizing firms, then the standard result will obtain: rent-maximizing by the union is formally equivalent to profit-maximizing by an upstream firm.

5. Wages under Cournot and Bertrand competition

In this section of the paper, we examine the reasons for the reversal result. We establish two key analytical results. First, we show in Proposition 2 that SPNE bargained wages are indeed higher under Cournot than under Bertrand product market competition: Unions influence wages more aggressively in the case of Cournot competition.

Proposition 2. *Sub-game perfect Nash equilibrium wages are higher under Cournot than under Bertrand product market competition*, $\forall c > 0$.

Proof. See Appendix A. □

Proposition 3 provides the essential intuition underlying Proposition 2.

Proposition 3. *The sub-game perfect labor demand schedule derived under Bertrand competition in the product market is more elastic than the sub-game perfect labor demand schedule derived under Cournot competition.*

Proof. See Appendix A. □

Proposition 3 implies that a percentage increase in the wage rate will induce a higher percentage reduction in employment under Bertrand than under Cournot competition. The union perceives this difference as a higher proportional marginal cost for a given wage increase when bargaining with a Bertrand-type firm. Thus, union i has a stronger incentive to settle for a lower bargained wage rate when facing a Bertrand-type competitor in the product market.

We now show that Cournot equilibrium profits decrease more steeply in wages than do Bertrand profits. Under Cournot competition, the change in firm i's profitability induced by a wage increase can be decomposed into two effects:

$$\begin{aligned} \frac{\partial \Pi_i^C}{\partial w_i} &= \left(\frac{\partial p_i}{\partial q_j}\frac{\partial q_j}{\partial q_i}\frac{\partial q_i}{\partial w_i}\right) q_i \quad - \quad q_i. \\ &\qquad\quad (-) \qquad\qquad\quad - \quad (+) \\ &= \text{strategic effect} \quad - \text{ size effect.} \end{aligned} \tag{29}$$

Hence, under Cournot competition, the strategic effect on profits induced by a wage increase is strictly negative. The underlying intuition behind the negative strategic effect lies in the nature of the adjustment of the Cournot firm to an exogenous change in marginal costs. A unit increase in w_i expands firm j's output which in turn induces a reduction in price and hence in firm i's revenue. The size effect captures the negative effect on profits associated with the higher costs of producing q_i after a unit increase in the wage rate.

Correspondingly, under Bertrand competition, firm i's marginal profit from a wage increase can be also decomposed into two effects:

$$\begin{aligned} \frac{\partial \Pi_i^B}{\partial w_i} &= \left(\frac{\partial q_i}{\partial p_j}\frac{\partial p_j}{\partial p_i}\frac{\partial p_i}{\partial w_i}\right)(p_i - w_i) - \quad q_i. \\ &\qquad\qquad (+) \qquad\qquad\qquad\quad - \quad (+) \\ &= \text{strategic effect} \qquad\qquad - \text{ size effect.} \end{aligned} \tag{30}$$

Under Bertrand competition, the strategic effect is strictly positive. This is due to the way in which firm i reacts to the wage increase: a unit increase in w_i leads firm i to increase its price, after which firm j follows by increasing its own. The latter is transmitted to an expansion of firm i's output. The increased output multiplied by the price mark-up raises total revenue. This is an important feature of the Bertrand competitor. Thus, the strategic effects are of opposite sign if we compare Cournot and Bertrand perceptions. Qiu (1997) also found an opposite sign in terms of R&D activity. As in the Cournot case, the size effect is negative.

The most important and distinctive feature arising from the comparison of (29) and (30) is that firm i perceives a proportional marginal benefit from a wage increase when product market competition is Bertrand whereas it perceives a proportional marginal cost when competition is Cournot. This arises from the fact that the strategic effects are of opposite sign. We now establish Proposition 4.

Proposition 4. *A unit increase in the wage rate reduces equilibrium profits for both types of product market competition. From any given initial level of wages, Cournot equilibrium profits decrease more steeply in wages than do Bertrand equilibrium profits.*

Proof. See Appendix A. □

The first part of Proposition 4 confirms that the negative size effect dominates the positive strategic effect in determining the sign of the marginal profitability from a wage increase of the Bertrand competitor. The second part demonstrates that Cournot profits are the more adversely affected by wage increases. This is due to the negative strategic effect coupled with the negative size effect induced by a wage increase. As an anonymous referee has observed, the greater sensitivity of profits to wages under Cournot will induce firms to be more resistant to union wage demands than under Bertrand competition. Thus, the fact that equilibrium bargained wages are higher under Cournot, as demonstrated in Proposition 2, arises from the dominating effect of greater wage moderation under Bertrand, associated with its greater labor demand elasticity as described in Proposition 3.[5]

In summary, we have established that the possibility of the reversal result rests on the fact that: (i) Under Cournot competition unions bargain a higher wage level than under Bertrand competition and (ii) equilibrium Cournot profits are more sensitive to the level of the bargained wage than are Bertrand profits. The force of these arguments is strong enough to overturn the standard result – that profits are higher under Cournot – only if unions have sufficient influence over wages and are sufficiently wage-oriented. If unions do not exert a strong influence on wages, then the standard result obtains. In the limit, of course, if either θ or β is zero, then wages will be equal to the non-union level in both Cournot and Bertrand cases and the arguments identified above will have no force.

5.1. The case of complements

Throughout the analysis, we have considered the case of imperfect substitutes. Given that the standard Singh and Vives (1984) result is symmetric over substitutes and complements, an interesting question concerns whether the reversal result which we have established holds also for the case of imperfect complements. From the proof of Proposition 4, it follows that when the wage-setting game is played in strategic substitutes, an increase in the bargained wage does not damage profits as much as in the case of strategic complements. It can be demonstrated that this prevents Bertrand profits from falling below Cournot profits as θ and β increase in the case of imperfect complements. Thus, the unionized oligopoly is not symmetric with respect to the effects of product differentiation. This itself is an interesting finding.

6. Welfare comparison

The traditional welfare results in the standard model of differentiated duopoly with exogenous marginal costs establish that Bertrand competition yields higher welfare at equilibrium. In this section, we investigate if the conventional wisdom still holds when labor unions, and their interaction with firms, influence welfare.

Under Cournot competition equilibrium consumer surplus ($CS = U(q_i, q_j) - p_i q_i - p_j q_j$), total duopoly profits ($\pi = \pi_i + \pi_j$), and total union utility ($u = u_i + u_j$) are given by

$$CS^C = 4(1+c)\left(\frac{a-\bar{w}}{2+c}\right)^2 \left[\frac{2-\beta(1+\theta)}{2(2-\beta)-c\beta\theta}\right]^2,$$

$$\pi^C = 8\left[\frac{2-\beta(1+\theta)}{2(2-\beta)-c\beta\theta}\right]^2 \left(\frac{a-\bar{w}}{2+c}\right)^2,$$

$$u^C = [(2-c)\beta\theta]^\theta \left(\frac{2(a-\bar{w})}{2(2-\beta)-c\beta\theta}\right) \left[\frac{2(2-\beta(1+\theta))}{2+c}\right]^{1-\theta}.$$

[5] For a complete proof, together with derivations of the first-order conditions for both the Cournot and Bertrand cases under the right-to-manage model, see Correa-López and Naylor (2002).

Welfare (W) under Cournot is given by

$$W^C = CS^C + \pi^C + u^C$$
$$= \left(\frac{2(a-\bar{w})}{2(2-\beta)-c\beta\theta}\right)\left[\frac{(2-\beta(1+\theta))}{2+c}\right]^{1-\theta}$$
$$\times\left[\frac{2(3+c)(a-\bar{w})}{(2(2-\beta)-c\beta\theta)}\left(\frac{(2-\beta(1+\theta))}{2+c}\right)^{1+\theta}+2^{1-\theta}[(2-c)\beta\theta]^{\theta}\right].$$

Correspondingly, under Bertrand competition, consumer surplus, duopoly profits and union utility are given by

$$CS^B = \frac{1}{(2-c)^2(1+c)}\left[\frac{(2-c^2)(2-\beta(1+\theta))}{(2-c^2)(2-\beta)-c\beta\theta}\right]^2(a-\bar{w})^2,$$

$$\pi^B = \frac{2(1-c)}{(2-c)^2(1+c)}\left[\frac{(2-c^2)(2-\beta(1+\theta))}{(2-c^2)(2-\beta)-c\beta\theta}\right]^2(a-\bar{w})^2,$$

$$u^C = [(1-c)(2+c)\beta\theta]^{\theta}\left(\frac{2(a-\bar{w})}{(2-c^2)(2-\beta)-c\beta\theta}\right)\left[\frac{(2-c^2)(2-\beta(1+\theta))}{(1+c)(2-c)}\right]^{1-\theta}.$$

Hence, welfare under Bertrand is given by the following expression:

$$W^B = \left(\frac{(a-\bar{w})}{(2-c^2)(2-\beta)-c\beta\theta}\right)\Psi^{1-\theta}$$
$$\times\left[\frac{(3-2c)(a-\bar{w})}{((2-c^2)(2-\beta)-c\beta\theta)(1+c)^{\theta}}\Psi^{1+\theta}+2[(1-c)(2+c)\beta\theta]^{\theta}\right],$$

where $\Psi = ((2-c^2)(2-\beta(1+\theta)))/(1+c)(2-c)$. We define the welfare differential (Δ) as

$$\Delta \equiv W^B - W^C = (CS^B - CS^C) + (\pi^B - \pi^C) + (u^B - u^C). \tag{31}$$

After substitution, numerical analysis of (31) shows that, as in the standard model with exogenous labor prices, welfare is greater under Bertrand competition: there is no welfare reversal.[6] In the standard model with exogenous costs, profits are higher under Cournot, but this is not enough to offset the excess of consumer surplus under Bertrand competition. With wage bargaining, even profits can be higher under Bertrand than under Cournot competition militating against the possibility of a welfare reversal result. Against this, however, union utility can be higher under Cournot – depending on the union's relative preference for wages – as unions bargain higher wages than in the Bertrand case. However, for the combination of parameter values $\{c,\beta,\theta\}$ where

[6] The nonlinearity of the resulting expression for the welfare differential Δ prevents us from using algebraic methods to show this.

$u^C > u^B$ *and* $\pi^C > \pi^B$ we still find $\Delta > 0$. In other words, in the context of imperfect substitutes, higher union utility under Cournot together with higher Cournot profits do not dominate the positive effect on welfare brought about by $CS^B > CS^C$.

7. Conclusions and further remarks

In this paper, we have considered the standard model of differentiated duopoly in which it is well-known that Cournot equilibrium profits are higher than those associated with Bertrand equilibrium when firms produce imperfect substitutes. In the standard model, costs are assumed to be determined exogenously. We have examined the situation in which costs (wages) are determined through a process of decentralized bargaining between each firm and its upstream supplier (labor union). We have found that, under certain conditions, the relative magnitude of Cournot and Bertrand profits is reversed when we allow for bargaining over costs. Specifically, if unions are sufficiently powerful and care enough about wages in their utility function, then Bertrand profits exceed Cournot profits in sub-game perfect Nash equilibrium when goods are (imperfect) substitutes: otherwise, the standard result holds. We note in particular that if the upstream agents are profit-maximizing firms, the traditional result is obtained.

There are a number of obvious directions for further work. First, we have followed standard assumptions in our specification of the basic unionized duopoly model. It would be interesting to see to how sensitive the results are to alternative assumptions – such as efficient bargaining – or to more general functional forms. Second, we have found that if firms can choose cooperatively the strategic variable (price or quantity) with which to play the game, then their choice will depend on the values of the bargaining parameters. However, we have not considered how these parameters influence the outcome of the non-cooperative choice of strategic variable. We leave this for further work.

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Appendix A

Proof that $0 < \hat{A} < 1$, $\forall c$: $0 < c < 1$.
$\hat{A}$ can be defined by

$$\hat{A} = \frac{y - x}{ey - x},$$

where

$$y = (2 - c)(1 + c)^{1/2}; \quad x = (2 + c)(1 - c)^{1/2}$$

and

$$e = \frac{2}{2 - c^2}. \tag{A.1}$$

Eq. (A.1) implies that $e > 1$, $\forall c$: $0 < c < 1$. From this it follows that for $\hat{A}$ to be strictly positive, it is sufficient that $y - x > 0$. It is easily shown that $y > x$ $\forall c > 0$. Hence, from $e > 1$ and $y > x$ it follows that $0 < \hat{A} < 1$, $\forall c : 0 < c < 1$.

Proof of Proposition 2. From comparison of (12) and (22), it follows that

$$\begin{aligned} \Delta^W = w^C - w^B = [(2-c)[(2-c^2)(2-\beta) - c\beta\theta] \\ - (2+c)(1-c)[2(2-\beta) - c\beta\theta]]\tilde{A}, \end{aligned} \tag{A.2}$$

where

$$\tilde{A} = \frac{\beta\theta(a - \bar{w})}{[2(2-\beta) - c\beta\theta][(2-c^2)(2-\beta) - c\beta\theta]} > 0, \ \forall \ w > \bar{w}.$$

From (A.2), it follows that sign $[\Delta^W] = \text{sign}[c^3(2 - \beta(1+\theta))]$. The latter is positive $\forall c > 0$ and hence $w^C > w^B$, which establishes the proposition. □

Proof of Proposition 3. Evaluating $|\eta_i^B|$ and $|\eta_i^C|$ from (17) and (4), respectively, yields:

$$\begin{aligned} &|\eta_i^B| - |\eta_i^C| > 0 \\ &\Leftrightarrow \frac{(2-c^2)}{(1-c^2)(4-c^2)} \frac{w_i(1-c^2)(4-c^2)}{[(1-c)(2+c)a - (2-c^2)w_i + cw_j]} \\ &\quad - \frac{2}{(4-c^2)} \frac{w_i(4-c^2)}{[(2-c)a + cw_j - 2w_i]} > 0 \\ &\Leftrightarrow (2-c^2)[(2-c)a + cw_j - 2w_i] > 2[(1-c)(2+c)a - (2-c^2)w_i + cw_j] \\ &\Leftrightarrow c^3(a - w_j) > 0 \ \forall c > 0. \end{aligned}$$

Since $c^3(a - w_j)$ is strictly positive $\forall c > 0$, it follows that $|\eta_i^B| - |\eta_i^C| > 0$. □

Proof of Proposition 4. It follows from (6) and (19) that for any given level of wages

$$\pi^C = \frac{1}{(2+c)^2}(a-w)^2; \quad \pi^B = \frac{1-c}{(2-c)^2(1+c)}(a-w)^2.$$

Differentiating with respect to the wage rate yields

$$\frac{\partial \pi^C}{\partial w} = -\frac{2}{(2+c)^2}(a-w) < 0 \ \forall c, \tag{A.3}$$

$$\frac{\partial \pi^B}{\partial w} = -\frac{2(1-c)}{(2-c)^2(1+c)}(a-w) < 0 \ \forall c. \tag{A.4}$$

The comparison of (A.3) and (A.4) establishes Proposition 4:

$$\frac{\partial \pi^C}{\partial w} - \frac{\partial \pi^B}{\partial w} < 0 \Leftrightarrow -\frac{2(a-w)}{(2+c)^2} + \frac{2(1-c)(a-w)}{(2-c)^2(1+c)} < 0$$

$$\Leftrightarrow \frac{2(a-w)\{-(2-c)^2(1+c)+(1-c)(2+c)^2\}}{(2+c)^2(2-c)^2(1+c)} < 0 \ \forall c > 0.$$

It is straightforward to demonstrate that $\{-(2-c)^2(1+c)+(1-c)(2+c)^2\} < 0 \ \forall c > 0$, from which it follows that $\frac{\partial \pi^C}{\partial w} - \frac{\partial \pi^B}{\partial w} < 0$. $\square$

References

Bulow, J.I., Geanakoplos, J.D., Klemperer, P.D., 1985. Multimarket oligopoly: Strategic substitutes and complements. Journal of Political Economy 93, 488–511.

Cheng, L., 1985. Comparing Bertrand and Cournot equilibira: A geometric approach. Rand Journal of Economics 16, 146–152.

Correa-López, M., Naylor, R.A., 2002. On the reversal of the Cournot–Bertrand profit ranking under unionised oligopoly. Mimeo., University of Warwick.

Dastidar, K.G., 1997. Comparing Cournot and Bertrand in a homogenous product market. Journal of Economic Theory 75, 205–212.

Davidson, C., 1988. Multiunit bargaining in oligopolistic industries. Journal of Labour Economics 6, 397–422.

Dowrick, S.J., 1989. Union–oligopoly bargaining. Economic Journal 99, 1123–1142.

Grandner, T., 2001. Unions in oligopolistic, vertically connected industries. European Economic Review 45, 1723–1740.

Häckner, J., 2000. A note on price and quantity competition in differentiated oligopolies. Journal of Economic Theory 93, 233–239.

Horn, H., Wolinsky, A., 1988. Bilateral monopolies and incentives for merger. Rand Journal of Economics 19, 408–419.

Lambertini, L., 1997. Prisioners' dilemma in duopoly (super) games. Journal of Economic Theory 77, 181–191.

Naylor, R.A., 1998. International trade and economic integration when labour markets are generally unionised. European Economic Review 42, 1251–1267.

Naylor, R.A., 1999. Union wage strategies and international trade. Economic Journal 109, 102–125.

Okuguchi, K., 1987. Equilibrium prices in the Bertrand and Cournot oligopolies. Journal of Economic Theory 42, 128–139.

Qiu, L.D., 1997. On the dynamic efficiency of Bertrand and Cournot equilibria. Journal of Economic Theory 75, 213–229.

Singh, N., Vives, X., 1984. Price and quantity competition in a differentiated duopoly. Rand Journal of Economics 15, 546–554.

Vives, X., 1985. On the efficiency of Bertrand and Cournot equilibria with product differentiation. Journal of Economic Theory 36, 166–175.

Wolfram, S., 1999. The Mathematica Book, 4th Edition. Cambridge University Press, Cambridge.