Competing Under Financial Constraints*

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Abstract

Firms’ debt capacity affects their ability to compete in the product market, and the competitiveness of firms in the product market determines their ability to secure debt. I model the endogenous relation between product market competition and financial constraints by characterizing a trade credit transaction where a competitive retailer has incentives to not honour the debt extended by its supplier. With linear input prices, credit rationing arises endogenously in equilibrium if competitive pressure is sufficiently strong. I show that a financially constrained retailer faces lower input prices, and it can make higher profits due to its own financial constraints. With non-linear prices, the retailer might never be constrained, even though contractual frictions affect market outcomes.

Keywords: Trade credit, financial constraints, product market competition

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1 Introduction

Firms’ limited access to credit is a widespread phenomenon. Contractual frictions may generate incentives for borrowers to strategically default on their debt, and lenders often respond by restraining credit. A firm is financially constrained when it cannot secure as much credit as it wishes, even though it is willing to pay it back (Tirole 2010). It is also well known that borrowers’ competitive pressure can exacerbate financial constraints. In particular, product market competition typically reduces the profitability of their investment, thereby increasing the agency costs of a financial transaction and further limiting access to credit. The common wisdom is therefore that financial constraints harm firms’ ability to compete in the product market (e.g., Holmstrom and Tirole 1997). In this paper I challenge such view.

The idea here is that credit constraints both affect and are affected by commercial relationships, and the pricing policy of a strategic supplier may depend on the financing conditions of its customer (retailer). More specifically, input prices determine the optimal investment of a retailer and therefore whether it is financially constrained. Furthermore, the retailer’s investment capacity has an impact on market outcomes and profits, thereby affecting the surplus that can be extracted by its supplier and thus, the pricing policy. I characterize the interplay between input prices and retailers’ financial constraints. I show that a financially constrained retailer faces lower input prices, and this can provide a competitive advantage and increase its profits.

The effect of firms’ debt on their ability to compete has been largely studied in the literature.\(^1\) Notably, Brander and Lewis (1986) and Maksimovic (1988) show that debt provides an incentive to compete more aggressively in future periods, thereby acting as a commitment device that can increase profits. In contrast, Bolton and Scharfstein (1990) argue that a company’s leverage may trigger predatory strategies by competitors to induce the company to default on its debt and exit the market. Despite distinct conclusions are reached, these studies have in common that they consider leverage to be a signal of the firm’s type as a competitor that therefore affects market outcomes.

In this context, it is natural to consider that firms’ ability to compete affects their debt capacity, creating an endogenous relation between leverage and product market competition. I study a trade credit transaction between a supplier and a (potentially) financially constrained retailer to shed light on this relation. Trade credit occurs when a supplier allows a customer (retailer) to pay with delay for goods already delivered. This provides a convenient framework for my analysis because the lender is also a supplier, and the financial

\(^1\)See Cestone (1999) for a review of the literature on corporate financing and product market competition.
transaction may affect the commercial relationship. Moreover, evidence shows that trade credit is an important source of firms’ finance, also in countries with well-developed financial systems (e.g. Giannetti 2003; Rajan and Zingales 1995).

I model the relation between a retailer that competes in the product market and a supplier extending trade credit. I follow Burkart and Ellingsen (2004) in assuming that the retailer’s investment in the product market is not contractible, and limited liability creates an incentive to divert the input without honouring the debt. As a consequence, the supplier must limit the line of credit to make repayment incentive compatible and this may generate financial constraints for the retailer. Crucially, credit is the product of price and quantity of input borrowed, and in my model the input price is strategically set by the supplier. Thus, for a given level of financial constraints, a higher (lower) input price reduces (increases) the quantity of input that can be extended in credit. Moreover, both the price and the quantity of input borrowed affect the relative profitability of diverting, and hence determine whether the retailer is financially constrained. As a result, financial constraints arise endogenously in equilibrium.

In my model, leverage has no effect on market outcomes in the absence of contractual frictions. With no financial constraints, the vertical relation between the supplier and the retailer is characterized by double marginalization, and market outcomes in the retail market are those of a Cournot-Nash equilibrium. In contrast, in the equilibrium with financial constraints the retailer exhausts the line of credit and double marginalization vanishes. This triggers two key effects for my results. First, the supplier can extract all retailer’s surplus conditional on debt repayment being incentive compatible. As a consequence, the profits of the retailer equal its agency rents. Second, financial constraints act as a commitment device for the retailer’s production, and the supplier sets an input price that yields the production of a Stackelberg leader. For intermediate levels of competitive pressure, retailer’s agency rents exceed the profits that it would make in a setting with no contractual frictions —double marginalization. Then, financial constraints are profitable for the retailer.

Whether financial constraints arise in equilibrium depends on market fundamentals. I focus on the intensity of the retailer’s competitive pressure. With low competition, production is relatively profitable and agency costs are small. Then, debt repayment is incentive compatible in a setting with double marginalization and the retailer is not financially constrained. Conversely, high competition increases incentives to divert and, under double marginalization, repayment is not incentive compatible, leading to financial constraints. Several papers have studied the effects of supplier’s (lender’s) product market competition on the extension of trade credit (e.g., Petersen and Rajan 1997; McMillan and Woodruff 1999; Fisman and Raturi 2004; Fabbri and Klapper 2016). However, little attention has been paid to the ef-
fects of the retailer’s competitive pressure. Unsurprisingly, in my model higher competition reduces the value of credit offered by the supplier because it lowers the profitability of producing and increases agency costs. This, nonetheless, need not imply that less input can be borrowed, as the reduction might be driven by a lower input price.

When contractual frictions lead to financial constraints, they affect market outcomes through both the retailer’s marginal cost (input price) and its production capacity (quantity of input extended in credit). The literature has studied the role of these two on a firm’s own, and rivals’, production choices. A representative work is Dixit (1980), which shows that a firm can invest in capacity to lower the relevant marginal cost and increase production, potentially deterring the entrance of competitors.\(^2\) Here, the operating mechanism is totally different because both marginal cost and capacity are determined by a strategic supplier, which needs to offer a line of credit so that repayment is incentive compatible. This setting is close to Fershtman and Judd (1987), who study the incentive contracts that owners (principals) choose for their managers (agents) in an oligopolistic context. I analyse the product market effects of an agency problem where a supplier effectively delegates production to a retailer because it cannot access the market itself.

The strategic role of input prices drives the main results of the paper. Input price determines both the retailer’s optimal production and the profitability of participating in the retail market. Thus, it also regulates the incentives to divert and therefore the quantity of input that the supplier can extended in credit while making repayment incentive compatible. Other work in trade credit has studied the strategic role of prices in a different context. For example, Smith (1987) and Brennan et al. (1988) show that price discrimination can be used to reveal the creditworthiness of different retailers. Daripa and Nilsen (2011) demonstrate that inter-firm credit may subsidise inventory holding costs, and evaluate input price adjustment as an alternative to the extension of credit. However, as noted in Giannetti et al. (2011), it is still not clear whether a supplier would reduce input prices to a credit-constrained retailer. I answer this question.

I assume that input prices are linear to characterize the endogenous relation between leverage and product market competition. This yields two different types of equilibria (constrained and unconstrained) that depend on market fundamentals. I conduct comparative statics on the levels of competitive pressure to study how these determine equilibrium outcomes. Then, I study the same model when the supplier can set non-linear prices. I show that market outcomes are equal to those of the equilibrium with financial constraints in the

\(^2\)The work of Dixit (1980) is followed by a number of papers studying the effects of capacity on the product market. These include Kirman and Masson (1986), Kulatilaka and Perotti (1998), Reynolds (1991). Notably, Leach et al. (2013) study the interaction of debt and capacity commitments.
main setting (linear prices), but the retailer need not be constrained. More specifically, the supplier can always extract all the retailer’s surplus conditional on repayment being incentive compatible, and the line of credit acts as a commitment device because the retailer exhausts it. However, the retailer may never be constrained because optimal production is determined by the marginal price of input, and with a non-linear tariff this is not relevant for the supplier’s profit maximization.

The remainder of the paper is organized as follows. Section 2 describes the model and Section 3 characterizes the equilibrium. Section 4 conducts comparative statics on the retailer’s competitive pressure and interprets the results. Section 5 considers a setting with non-linear prices. Section 6 concludes.

2 The Model

I consider the vertical relation between a penniless retailer and an input supplier who can extend trade credit and study how this relationship is shaped by competition in the retail market. The retailer needs to borrow input in order to enter the product market, which is operated by \( N \) other firms that I call incumbents. Contractual frictions in the vertical relation allow the retailer to divert the input borrowed without honouring his debt. As a consequence, the supplier must limit the credit extended to keep repayment incentive compatible. There are three dates \( t = 0, 1, 2 \). There is no discounting and all agents are risk neutral.

At \( t = 0 \) the supplier sets a price \( \omega \) per each unit of input. The price is observable to all market participants, which know that the supplier must incur a cost \( c_s \) to produce each unit of input. In Section 5 I relax the assumption that prices are linear and discuss its implications.

At \( t = 1 \) the supplier extends trade credit of up to \( L \) units of input to the retailer. This can be paid for at the end of the game. For simplicity, I assume that there is no interest on the loan. Moreover, I ease presentation by assuming that (i) the retailer cannot borrow money from a financial institution, hence the supplier is the only potential source of credit; (ii) the retailer has zero funds, being forced to borrow all the input that it needs for production. Neither of these assumptions affect the results qualitatively.\(^3\) After observing

\(^3\) (i) In line with Burkart and Ellingsen (2004), money is more divertible than input, so agency problems are stronger in standard credit transactions than in trade credit transactions. As a result, suppliers can extend credit in situations where financial institutions are not able to do so. Introducing standard credit would diminish, but not eliminate, the role of the supplier as a lender. (ii) With a positive amount of funds, the retailer would have to borrow a smaller quantity of input, if any, and thus financial constraints would arise for a smaller range of parameters.
the offer $L$, the retailer borrows $I \leq L$ units of input and incurs a debt of $\omega I$. Neither the offer $L$ nor the transaction $I$ are observable to other parties. The retailer can transform each unit of input into a unit of output costlessly.

At $t = 2$ retailer and incumbents simultaneously decide their production. The retail market is characterized by an inverse demand function $P(Q) = M - Q$, where $Q$ captures the total quantity of homogeneous product that is sold in it. In particular, $Q = \sum_{i=1}^{N} q_i + q_e$, where $q_i$ represents the quantity produced by incumbent $i \in \{0, 1, 2, ..., N\}$ and $q_e \leq I$ the quantity produced by the retailer. All incumbents have the same production cost $c$ per unit of output.

Crucially, the retailer may divert the input borrowed rather than investing it in the retail market and honouring the debt. I assume that while both output and sales revenues are verifiable and can therefore be pledged to the supplier, neither the input purchase nor the investment decision are contractible. The retailer has limited liability, so the debt is honoured only to the extent of market revenues. These, in turn, can only be enjoyed by the retailer after honouring repayment obligations. Each unit of input diverted generates a private benefit $\beta < \min\{c, c_s\}$, where the relatively low revenue reflects the inefficiency of diverting. Furthermore, I assume that $\beta + c_u > c$, so the supplier is not “too efficient.” This leads the agency problem to affect equilibrium outcomes.

From the previous assumptions it follows that the retailer’s net profits read

$$\pi_e = \max \left\{ \left( M - \sum_{i=1}^{N} q_i - q_e \right) q_e - \omega I, 0 \right\} + \beta(I - q_e).$$ (1)

Here, the first term within the maximum operator captures net market profits, and zero is the lower bound guaranteed by limited liability. The last term in (1) represents the revenues from diversion: $\beta$ for each unit of input borrowed and not invested in the retail market, i.e., for $I - q_e$ units. Note that if the supplier does not limit the credit line to $L$, the retailer’s best response is to borrow an unlimited amount of input and divert it.

I consider the following definition of financial constraints:

**Definition 1** Let $q^u_e(\omega)$ denote the optimal retailer’s production for a given input price $\omega$ in the absence of incentives to divert. The retailer is financially constrained when it cannot borrow enough input to produce optimally, i.e., if $L < q^u_e(\omega)$.

The definition captures the essence of credit rationing, which Bester and Hellwig (1987) describe as “a would-be borrower is said to be rationed if he cannot obtain the loan that he wants even though he is willing to pay the interest that the lenders are asking, perhaps even a higher interest.”

4 Notably, Definition 1 shows that whether the retailer is financially

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4 This quote opens the discussion of the chapter 'Outside Financing Capacity' in Tirole (2010)
constrained does not only depend on the line of credit $L$, but also on the optimal investment, and hence on the input price $\omega$.

Parameter $\beta$ captures input liquidity. This is, the profitability of allocating input to alternative uses for private benefit. In the presence of contractual frictions, large $\beta$ increases agency costs and may lead to tighter financial constraints. Evidence by Cunat (2007) and Giannetti et al. (2011) suggests that input liquidity is related to product characteristics. In particular, generic inputs are easy to divert and thus highly liquid, whereas more tailored inputs have low value in alternative markets, thereby alleviating agency costs.

The agency problem here is akin to Burkart and Ellingsen (2004), who provide a rationale for trade credit. I extend their setting in two key dimensions that allow me to study the interaction of financial constraints and product market competition. First, I endogenize input price. By setting $\omega$, the supplier determines both the retailer’s optimal production in the retail market and his incentives to divert the input and not honouring his debt. As a result, both the demand for input and the line of credit $L$ are a function of $\omega$, and financial constraints arise endogenously in equilibrium. Second, I introduce product market competition in the retail market. Parameter $N$ captures competitive pressure, which crucially affects the profitability of investment and in turn, the input price. Assuming that incumbents are identical simplifies the analysis while preserving the key role of competition.

I assume that the supplier is a monopolist to the retailer, which in turn represents the only access to the retail market. This is consistent with Petersen and Rajan (1997), McMillan and Woodruff (1999) and Cunat (2007), who find that more trade credit is extended where suppliers have low competition. The scarcity of alternative sources of input increases the value of the commercial relationship for retailers and lowers the incentives to strategically default on their debt. This reduces agency costs and enables the extension of trade credit by suppliers. Formally, the supplier’s market power allows me to study how agency problems affect input prices and thus the competitiveness of the retailer in the product market.

### 3 Equilibrium Analysis

I consider subgame perfect Nash equilibria of this model. I start with a preliminary result that facilitates subsequent analysis, and then solve recursively:

**Lemma 2** In equilibrium, the retailer pays all his debt and does not allocate any input to alternative uses, i.e., $q_e = I$.

The result is intuitive. The retailer’s profit function (1) shows that honouring the debt is an all-or-nothing decision. This property is shared with Burkart and Ellingsen (2004),
and follows from the assumption that market revenues are contractible. More specifically, when market revenues are smaller than debt value \( \omega I \), the profitability of investment is zero. Alternatively, the same input might be diverted, generating a net profit of \( \beta > 0 \) per unit. Thus, either the retailer produces enough output to enjoy market revenues after repayment, or diverts it all. In equilibrium, the supplier only extends credit when repayment is incentive compatible. Therefore, whenever credit is extended, the debt is fully honoured.

Lemma 2 establishes that, besides honouring the debt in full, the retailer does not borrow any input to allocate it elsewhere. In particular, one may consider a case where the retailer finds it profitable to use a fraction of input borrowed for production —and pay the debt, and allocate the remaining part to an alternative use, generating a unit revenue of \( \beta \). Notice that this is never the case because in equilibrium the input price must be such that \( \omega \geq c_a \), whereas we have \( c_a > \beta \) by assumption. In words, the supplier always sets an input price weakly higher than its cost, which in turn is higher than the revenues of allocating the input to alternative uses. Thus, given a contract such that repayment is incentive compatible, it cannot be profitable for the retailer to buy input and allocate it elsewhere.

### 3.1 Production

At \( t = 2 \) retailer and incumbents simultaneously take production decisions. Their unit costs — \( \omega \) and \( c \) respectively, are known to all market participants. Moreover, the retailer’s production is limited to \( I \leq L \), where neither the credit line offered by the supplier nor the quantity of input borrowed have been observed by the incumbents.

Incumbent \( i \) maximizes net profits \( \pi_i = (M - \sum_{-i} q_{-i} - q_i - q_e) q_i - cq_i \). Here, \( \sum_{-i} q_{-i} \) represents the production of all other incumbents. By symmetry, each of them produces the same quantity, which satisfies the best reply

\[
q_{-i}(q_i) = \frac{M - q_i - c}{N + 1}, \forall i.
\]

(2)

Thus, the sum of all incumbents’ production can be represented as \( Nq_i \).

From Lemma 2 it follows that retailer’s profits are those obtained by producing with a unit cost \( \omega \). More specifically, input price and line of credit must be such that all input borrowed is used for production, i.e., \( q_e = I \). As a result, retailer’s profit function (1) collapses to \( \pi_e = (M - Nq_i - q_e) q_e - \omega q_e \). Moreover, production levels are upper bounded by the line of credit \( L \). I denote the retailer’s unconstrained best reply by \( q_e^u(q_i) \). Then, the best reply reads

\[
q_e(q_i) = \begin{cases} 
q_e^u(q_i) = \frac{M - Nq_i - \omega}{2} & \text{when } q_e^u(q_i) \leq L \\
L & \text{when } q_e^u(q_i) > L 
\end{cases}
\]

(3)
Response function (3) shows that the retailer’s financial constraint effectively acts as a capacity constraint. Given an input price $\omega$, the retailer’s optimal production is $q_e^u(q_i)$, and it is financially constrained when $q_e^u(q_i) > L$. The concavity of market profits implies that whenever the retailer is financially constrained, it exhausts all credit, i.e., $q_e(q_i) = L$.

Incumbents do not observe the line of credit offered by the supplier at $t = 1$, but they have a conjecture that is correct in equilibrium. As will become clear, there exists a one-to-one mapping from the publicly observed input price set by the supplier at $t = 0$ to the line of credit offered at $t = 1$. I derive the equilibrium production policies of market participants for a given $L$, and then show that the incumbents’ conjecture is correct. For simplicity, my notation does not differentiate between the conjecture of $L$ and the actual line of credit.

**Lemma 3** Suppose that incumbents’ conjecture of the credit line $L$ is correct in equilibrium. Then, retailer’s and incumbents’ production in equilibrium satisfy

\[
q_e^* = \begin{cases} 
q_e^u = \frac{M-(N+1)\omega+Nc}{N+2} & \text{when } q_e^u \leq L \\
L & \text{when } q_e^u > L
\end{cases},
\]

\[
q_i^* = \begin{cases} 
q_i^u = \frac{M-2c+\omega}{N+2} & \text{when } q_e^u \leq L \\
q_i^c = \frac{M-L-c}{N+2} & \text{when } q_e^u > L
\end{cases}
\]

and the corresponding profits are

\[
\pi_e^* = \begin{cases} 
\pi_e^u = \left[\frac{M-(N+1)\omega+Nc}{N+2}\right]^2 & \text{when } q_e^u \leq L \\
\pi_e^c = L \left[\frac{M-(N+1)\omega-L+Nc}{N+1}\right]^2 & \text{when } q_e^u > L
\end{cases},
\]

\[
\pi_i^* = \begin{cases} 
\pi_i^u = \left[\frac{M-2c+\omega}{N+2}\right]^2 & \text{when } q_e^u \leq L \\
\pi_i^c = \left[\frac{M-c-L}{N+1}\right]^2 & \text{when } q_e^u > L
\end{cases}
\]

Optimal production $\{q_e^*, q_i^*\}$ is obtained by solving for best replies (2) and (3); the profits functions follow from plugging the corresponding solutions into $\pi_e$ and $\pi_i$. I derive the expressions with more detail in the Appendix.

For a given input price $\omega$, production in the unconstrained equilibrium $\{q_e^u, q_i^u\}$ is that of a standard simultaneous Cournot-Nash game. Here, $q_e^u$ corresponds to the optimal production in Definition 1. For exposition, I no longer specify that it is a function of $\omega$. Notably, market outcomes in the constrained equilibrium $\{L, q_i^c\}$ are those of a game where the financially constrained retailer becomes a leader that must choose production $L$. Thus, the constraint can potentially provide a first-mover advantage. However, $L$ is itself a function of $\omega$, which is strategically set by the supplier.
Profit functions (6) show that, given a price $\omega$ and a line of credit $L$, financial constraints can only harm the retailer. Formally, simple algebra reveals that $\pi^c > \pi^u \iff L > q^u$. In words, retailer’s profits in the constrained equilibrium can only exceed those in the unconstrained one when the retailer is not constrained, i.e., never. This is not surprising because, everything else equal, financial constraints can only limit the retailer’s ability to maximize profits. Nonetheless, in equilibrium both input price and credit line are set by the supplier and depend, in turn, on whether the retailer is financially constrained. I study these in the next sections.

3.2 Line of credit

At $t = 1$ the supplier offers a credit line $L$ to the retailer, and the retailer borrows $q_e \leq L$. The following Lemma characterizes the equilibrium line of credit as a function of input price and market fundamentals.

**Lemma 4** Given input price $\omega$, in equilibrium the credit line $L$ makes the retailer indifferent between producing and diverting all input without honouring the debt, i.e., it satisfies

\[
L^* = \begin{cases} 
L^u = \frac{1}{\beta} \left[ \frac{M-(N+1)\omega+Nc}{N+2} \right]^2 & \text{when } \pi^u \geq \beta q^u \\
L^c = M - (N + 1)(\omega + \beta) + Nc & \text{when } \pi^u < \beta q^u \end{cases} \quad (8)
\]

Moreover, incumbents’ conjecture of $L$ is correct.

In the Appendix I derive (8); here I argue that this is the line of credit in equilibrium. Note first that retailer’s profits from diverting are maximized by exhausting all credit, i.e., diverting yields $\beta L$. Thus, the retailer is indifferent when the profitability of producing $q_e \leq L$ equals $\beta L$. From the concavity of market profits, it follows that additional credit would lead the retailer to divert and thus cannot be profitable for the supplier. In contrast, less credit keeps repayment incentive compatible, but it cannot increase supplier’s profits. More specifically, if the financial constraint is not binding ($q^u < L$), a marginally smaller line of credit has no effect in equilibrium. If, instead, it is optimal for the retailer to exhaust all credit ($q^u \geq L$), a smaller $L$ must reduce supplier’s profits whenever his sales are profitable, i.e., $\omega > c_u$. Hence, the supplier cannot do better than offering a credit line that makes the retailer indifferent between producing and diverting. This is, therefore, the conjecture of incumbents.

The credit policy (8) follows from solving $\pi_e = \beta L$ for $L$ both when the financial constraint is binding and when it isn’t. Figure 1 illustrates the two cases. If $\pi^u \geq \beta q^u$, the retailer makes higher profits by producing the optimal quantity of output $q^u$ than by diverting the
input required for such level of production and not honouring the debt. Then, the retailer is not financially constrained and it is indifferent between producing and diverting when the line of credit is $L^u$, which satisfies $\pi^u_e = \beta L$. Alternatively, when $\pi^u_e < \beta q^u_e$, if the retailer could borrow the optimal quantity of input for production, it would rather divert. Then, the supplier must limit the line of credit to make repayment incentive compatible. The retailer is indifferent when the credit line is $L^c$, which satisfies $\pi^c_e = \beta L$. Note in Figure 1 that $L^c = L^u$ when $\pi^u_e = \beta q^u_e$, so the line of credit is continuous on $q^u_e$. Thus, the retailer is financially constrained if and only if $\pi^c_e < \beta q^c_e$. As will become clear, this depends on the input price $\omega$, which is strategically set by the supplier.

### 3.3 Input price

At $t = 0$ the supplier sets input price $\omega$ to maximize profits $\pi_u = (\omega - c_u)q_e$, subject to the credit policy $L^*$. Crucially, the price not only maximizes supplier’s profits given a demand for input (production) in either equilibrium, i.e. for $q^u_e$ or $L^c$, but it also determines the equilibrium itself. For instance, the price maximizing supplier’s profits for a demand $q^u_e$ must be such that the retailer is not financially constrained, i.e. $q^u_e \leq L^u$. The following proposition characterizes the supplier’s pricing policy in equilibrium, which depends on market fundamentals.
Proposition 5 Define the price policies \( \omega^u = \frac{M+(N+1)c_u+Nc}{2(N+1)} \) and \( \omega^c = \frac{M+(N+1)(c_u-\beta)+Nc}{2(N+1)} \) satisfying \( \omega^u > \omega^c \); and the policy \( \overline{\omega} = \frac{M-(N+2)\beta+Nc}{N+1} \). Market fundamentals determine equilibrium outcomes such that

1. For \( \overline{\omega} > \omega^u \) the supplier sets input price \( \omega^u \); the retailer is not financially constrained and does not exhaust all credit;

2. For \( \overline{\omega} \in [\omega^c, \omega^u] \) the supplier sets an input price \( \overline{\omega} = \frac{M-(N+2)\beta+Nc}{N+1} \); the retailer is not financially constrained but exhausts all credit;

3. For \( \overline{\omega} < \omega^c \) the supplier sets input price \( \omega^c = \frac{M+(N+1)(c_u-\beta)+Nc}{2(N+1)} \); the retailer is financially constrained and therefore exhausts all credit.

I derive the price cutoffs of Proposition 5 in the Appendix; here I provide the main intuition. When the retailer is not constrained (\( q_e = q^u_e \)), the supplier maximizes profits by setting a price \( \omega^u \). Instead, if financial constraints are binding (\( q_e = L^c \)), the supplier’s profits are maximized for \( \omega^c \). The relation \( \omega^u > \omega^c \) shows that the retailer always pays a lower price for input when it is financially constrained. Notably, these pricing policies maximize supplier’s profits conditional on the retailer’s financial constraints. However, whether the retailer is constrained depends on the input price. The previous analysis showed that the retailer is constrained when \( \pi^u_e < \beta q^u_e \). Here, price \( \omega \) determines the profitability of producing \( \pi^u_e \), and therefore whether there is credit rationing in equilibrium. A graphical argument follows from Figure 1. For a given input diversion value \( \beta \), a small price \( \omega \) makes producing relatively profitable, and the retailer is not constrained. Increasing \( \omega \) lowers market profits and financial constraints eventually become binding.

The pricing policy \( \overline{\omega} \) satisfies \( \pi^u_e = \beta q^u_e \) and is such that the retailer is financially constrained if and only if \( \omega > \overline{\omega} \). When market fundamentals satisfy \( \omega^u < \overline{\omega} \), there are no constraints in equilibrium. Intuitively, the supplier’s price policy for a non-constrained retailer \( \omega^u \) makes repayment incentive compatible for the optimal production \( q^u_e \). Thus, unconstrained price and production are an equilibrium. Moreover, the price policy for a constrained retailer \( \omega^c \) does not provide incentives to divert, so financial constraints do not arise in equilibrium. Formally, it holds both \( q^u_e(\omega^u) < L^u(\omega^u) \) and \( q^c_e(\omega^c) < L^c(\omega^c) \). In this equilibrium, market outcomes are characterized by \( \omega^u \). Production levels are given by \( q^u_e \) and \( q^u_i \) in (4) and (5); and the line of credit is \( L^u \) in (8).

A similar intuition applies for \( \omega^c > \overline{\omega} \), which implies that the retailer is financially constrained in equilibrium. In particular, \( \omega^c \) is such that makes the retailer constrained, and thus it is an equilibrium. Furthermore, with a price \( \omega^u \) repayment of debt corresponding to \( q^u_e \) is not incentive compatible, hence it is not an equilibrium price. More formally, we have
\( L^c(\omega^c) < q^u(\omega^c) \) and \( L^u(\omega^u) < q^u(\omega^u) \). Here, equilibrium outcomes are characterized by input price \( \omega^c \) and correspond to retailer’s production \( L^c \) in (8) and incumbents’ production \( q^i \) in (5).

When fundamentals are such that \( \overline{\omega} \in [\omega^c, \omega^u] \), neither the unconstrained outcome nor the constrained one are an equilibrium for the associated price policies \( \omega^u \) and \( \omega^c \). This is, with \( \omega^u \) debt repayment is not incentive compatible when producing \( q^u_e \); with \( \omega^c \) honouring the debt is incentive compatible for a production \( q^e_c \). Then, the supplier sets price \( \overline{\omega} \), which satisfies \( q^u_e = L^u = L^c \). Thus, the retailer is not constrained, but it exhausts all credit. The result follows from the distinct responsiveness of optimal production and credit line to the input price. In particular, lowering the price would lead to a situation where \( q^u_e < L^u < L^c \), so the retailer would not exhaust all credit. However, it would then be optimal for the supplier to set a higher price \( \omega^u \) rather than a lower one. Similarly, a price higher than \( \overline{\omega} \) yields \( L^c < L^u < q^u_e \), thereby making the retailer financially constrained. But then the optimal price is lower, i.e. \( \omega^c \), not higher.

### 3.4 Equilibrium outcomes

The following Corollary provides an intuitive overview of the main equilibrium outcomes that is convenient for subsequent discussion.

**Corollary 6** When the retailer does not exhaust all credit, i.e. for \( \omega^u < \overline{\omega} \), there is double marginalization over the vertical chain, and market outcomes are those of a Cournot-Nash.

With financial constraints, i.e. when \( \omega^c > \overline{\omega} \), the retailer’s production is that of a Stackelberg leader with a unit cost \( c_u + \beta \), and incumbents act as laggard firms. Furthermore, supplier’s profits are those of the Stackelberg leader, and retailer’s profits equal its agency rents, i.e. \( \pi^c_e = \beta L^c \).

The first statement is straightforward and highlights that contractual frictions need not affect market outcomes when agency costs are small. The second statement is proved formally in the Appendix; here I develop the main intuition. When the retailer is financially constrained (\( \omega^c > \overline{\omega} \)), it exhausts all credit. As a result, input price determines retailer’s production ex ante and provides leadership with respect to incumbents, which therefore become laggards. Moreover, because the retailer exhausts all credit, there is no double marginalization over the vertical chain. Hence, the supplier effectively delegates production to the retailer, conceding just enough rents to make repayment incentive compatible, i.e. \( \beta \) for each unit of input extended in credit. It then becomes optimal to set a price \( \omega \) such that the line of credit equals to the production of a Stackelberg leader with a unit cost \( c_u + \beta \). Here, \( c_u \) is
the actual production cost whereas \( \beta \) are retailer’s agency rents per unit of input extended in credit.

4 Product Market Competition

I characterize equilibrium outcomes as a function of the number of incumbents and compare them with those of a game where there are no contractual frictions. Comparative statics shed light on the interaction between financial constraints and product market competition. The next Corollary follows directly from Proposition 5:

**Corollary 7** Consider the cutoffs \( N_1 = \frac{M-c_u-4\beta}{2\beta-c+c_u} \), \( N_2 = \frac{M-c_u-3\beta}{\beta-c+c_u} \) and \( \overline{N} = \frac{M-c_u-\beta}{\beta-c+c_u} \) that satisfy \( N_1 < N_2 < \overline{N} \). Equilibrium is such that

1. If competitive pressure is low, i.e., \( N < N_1 \), the retailer is not financially constrained and does not exhaust all credit;

2. If competitive pressure is moderately low, i.e., \( N \in [N_1, N_2] \), the retailer is not financially constrained but exhausts all credit;

3. If competitive pressure is moderately high, i.e., \( N \in [N_2, \overline{N}] \), the retailer is financially constrained and therefore exhausts all credit;

4. If competitive pressure is sufficiently high, i.e., \( N \geq \overline{N} \), no credit is extended and the retailer does not enter the market.

Cutoffs \( N_1 \) and \( N_2 \) solve \( \omega^u = \overline{\omega} \) and \( \omega^e = \overline{\omega} \) respectively for the number of incumbents. Moreover, cutoff \( \overline{N} \) satisfies \( \omega^e = c_u \). The results are illustrated in Figure 2, where I plot the main equilibrium outcomes as a function of \( N \) (solid). Moreover, I plot the outcomes of a setting where there are no contractual frictions (dashed). For this benchmark setting, I simply assume that input diversion yields zero revenues, so the retailer has no incentives to divert. Formally, if \( \beta = 0 \) it always holds that \( \pi_u^e \geq \beta q_u^e \), and the line of credit is unlimited, i.e., \( L^u \to \infty \).

The characterization of all functions plotted is given in the Appendix. With low competitive pressure \( (N < N_1) \) the retail market is relatively profitable and contractual frictions do not affect market outcomes. Corollary 6 establishes that the commercial relation between supplier and retailer is characterized by double marginalization, and production corresponds to a Cournot-Nash equilibrium. Figure 1 shows that input price \( \omega^u \) decreases with competitive pressure \( N \), but not enough to outweigh the decrease in production caused by higher

13
competition, so $q^u_e$ decreases too. More competition also raises agency costs, and therefore reduces the line of credit $L^u$. Crucially, $L^u$ decreases at a higher rate than the optimal production $q^u_e$ and as a result, $q^u_e = L^u$ when $N = N_1$. For higher competitive pressure, double marginalization is no longer an equilibrium because $q^u_e < L^u$.

With a moderately small number of competitors $N \in [N_1, N_2]$ the supplier sets a price $\overline{\omega}$ so that the optimal production equals the credit line: $q^u_e = L^u = L^c$. Even though the retailer is not constrained, it acquires input at a lower price $\overline{\omega} < \overline{\omega}^u$ and exhausts all credit. The benefits of this are twofold. First, a smaller marginal cost. Second, a first-mover advantage with respect to incumbents. Figure 2 shows that the input price $\overline{\omega}$ is such that the retailer’s optimal production $q^u_e$ is invariant to the number of competitors and in turn, so are his profits. However, this is not an equilibrium when competition is strong enough ($N \geq N_2$).
Then, the repayment of debt associated to this relatively large level of production \( q_u(\omega) \) is no longer incentive compatible, and the retailer is financially constrained.

Moderately high competition \( N \in [N_2, \overline{N}] \) yields financial constraints. The supplier sets an input price lower than in a setting with no frictions \( (\omega^c < \omega^u) \), but the retailer cannot borrow enough to produce optimally, i.e., \( L^c < q_u^u(\omega^c) \). Notably, Figure 1 shows that financial constraints can lead to higher production levels and higher profits for the retailer. From Corollary 6 it follows that this is because agency rents from producing the quantity of a Stackelberg leader are larger than the profits in a setting with double marginalization. Figure 2 shows that this holds when the number of incumbents is sufficiently close to \( N_2 \), but not with higher competitive pressure.

As the number of incumbents grows, the production of a Stackelberg leader approaches zero due to the relative inefficiency with respect to incumbents: \( c_u + \beta > c \). Notably, financial constraints provide a commitment device, but increase the supplier’s cost of accessing the retail market. When \( N \to \overline{N} \), market revenues converge to the effective marginal cost for the supplier, \( c_u + \beta \). In particular, \( \omega^c \to c_u \) to pay for the actual production cost, whereas the remaining \( \beta \) is captured by the retailer as an agency rent. With higher competitive pressure, i.e. for \( N \geq \overline{N} \), the supplier can no longer extend credit in a profitable way.

## 5 Optimal Contracts

In this section I assume that the supplier can set a non-linear input price. Notably, this is equivalent to considering the optimal contract for the supplier. The next proposition summarizes the main equilibrium outcomes:

**Proposition 8** In equilibrium, when the supplier can set non-linear prices, market outcomes are such that:

i) The supplier extends in credit a quantity of input \( L \) that equals the production of a Stackelberg leader with a unit cost \( c_u + \beta \);

ii) The retailer exhausts all credit, i.e. \( q_e = L \), and has profits that equal its agency rents, i.e., \( \pi_e = \beta L \);

iii) The supplier’s profits are those of a Stackelberg leader with a unit cost \( c_u + \beta \).

I argue that Proposition 8 follows directly from previous results. When the supplier can offer an optimal contract, retailer’s profits from producing must be equal to its agency rents, i.e., \( \pi_e = \beta L \). In particular, note that if retailer’s profits were lower, it would have
incentives to divert. Alternatively, if they were higher, the supplier could charge a bigger price while keeping repayment incentive compatible, and therefore it would not be maximizing profits. This argument implies that the quantity of input extended in credit must be equal to retailer’s production —equivalently in equilibrium the retailer must exhaust all credit. Otherwise, if $q_e < L$, the supplier would be granting the retailer with unnecessary rents to make repayment incentive compatible, and thus would not be maximizing profits. This demonstrates statement (ii) of the proposition.

For statements i) and iii) consider the supplier’s optimal production and profits. Since the retailer exhausts all credit, i.e. $q_e = L$, the credit line acts as a commitment device in the retail market. Thus, the retailer becomes a leading producer, and the supplier can both determine its production and extract all its surplus except for the agency rents. It then follows that the supplier maximizes profits by extending in credit the input that corresponds to the production of a Stackelberg leader with a unit cost $c_u + \beta$, where $c_u$ is the actual production cost and $\beta$ is the agency cost for each unit of input lent.

When the supplier can set a non-linear price, contractual frictions always benefit the retailer. In particular, retailer’s profits equal its agency rents, and these are null when there exist no incentives to divert. Formally, $\pi_e = 0$ when $\beta = 0$. Notice also that retailer’s profits depend on contractual frictions (input diversion value $\beta$), but not on the presence of financial constraints. This leads us to a relevant insight that is captured by the following Corollary:

**Corollary 9** With non-linear input prices the retailer need not be financially constrained despite contractual frictions and regardless of market fundamentals. The following two-part tariff satisfies this condition:

$$
\omega_{tpt} = \frac{1}{2} \left[ (N + 2)(\beta + c_u) - \frac{N(M + Nc)}{(N + 1)} \right], \tag{9}
$$

$$
F_{tpt} = \frac{1}{4} \left[ M - N(\beta - c + c_u) - c_u - \beta \right] \left[ M - N(\beta - c + c_u) - c_u - 3\beta \right], \tag{10}
$$

where $\omega_{tpt}$ represents the unit price and $F_{tpt}$ the fixed fee.

I derive $\{\omega_{tpt}, F_{tpt}\}$ in the Appendix. The unit price $\omega_{tpt}$ is such that in a setting with double marginalization, the retailer’s production equals that of a Stackelberg leader with a unit cost $c_u + \beta$. The fixed fee $F_{tpt}$ extracts the remaining fraction of retailer’s surplus except for the agency rents, i.e. for $\beta L$. Notably, with this two-part tariff the retailer is never financially constrained. In particular, provided that producing is profitable, optimal production is determined by the marginal cost $\omega_{tpt}$. Here, the line of credit is such that the retailer can borrow just enough input to produce optimally.
Corollary 9 reveals a key limitation of the definition of financial constraints used here (Definition 1), which is seemingly standard. This is that financial constraints may not capture the presence of contractual frictions and agency costs, even though these affect market outcomes. The result highlights that taking production costs as given when studying firms’ credit rationing may neglect a crucial element of the whole picture, namely the fact that these costs are endogenous. The first part of the paper showed that with linear input prices financial constraints arise endogenously in equilibrium when agency costs are sufficiently large. In this section I show that with non-linear prices the retailer might never be financially constrained, even though market outcomes are affected by financial frictions.

6 Concluding Remarks

In this paper I model an endogenous relation between financial constraints and product market competition. I characterize a trade credit transaction where a supplier lends input to a competitive retailer, and contractual frictions may lead to financial constraints. By setting the input price, the supplier determines both retailer’s optimal demand for input and the quantity of input that can be extended in credit while making repayment incentive compatible. Credit rationing arises in equilibrium when the retailer can not borrow enough input to produce optimally for a given input price.

When the supplier sets linear prices, the retailer is financially constrained if competitive pressure is strong enough. I show that a financially constrained retailer faces lower input prices, and it can make higher profits due to its own financial constraints. Formally, this occurs when agency rents are larger than the profits of double marginalization in a setting with no frictions. When non-linear input prices are considered, the retailer might never be financially constrained despite the presence of contractual frictions and regardless of market fundamentals. The result reveals that seemingly standard definitions of financial constraints may neglect a key part of the analysis, namely the endogeneity of production costs.
7 Appendix

7.1 Proof of Lemmas 3-4

Lemma 3. Incumbents observe $\omega$ and have the right conjecture of $L$, so they know whether $q_u(q_i) \leq L$ or $q_u(q_i) > L$. When $q_u(q_i) \leq L$, equilibrium production levels $\{q_u^a, q_i^a\}$ solve the system of best response functions (3) and (2), which are derived in the main text. Note that when $q_u(q_i) > L$, the retailer exhausts all credit because market profits are quasiconcave in $q_e$ with a maximum at $q_e^u$. Incumbents’ response then is $q_i(L) = \frac{M - L - c}{N + 2}$. Profits functions are obtained by plugging in the production levels derived above so that

$$\pi_e^* = (M - Nq_i^* - q_e^* - \omega) \cdot q_e^*$$

and

$$\pi_i^* = (M - Nq_i^* - q_e^* - c) \cdot q_i^*$$

for retailers.

Lemma 4. The expression for $L^u$ follows from solving $\pi_e^u = \beta L$ for $L$. In particular, the equation reads

$$\left[\frac{M - (N + 1)\omega + Nc}{N + 2}\right]^2 = \beta L^u. \tag{11}$$

To obtain $L^c$ I solve $\pi_e^c = \beta L$ for $L$. This is,

$$L^c \left[\frac{M - (N + 1)\omega - L^c + Nc}{N + 1}\right] = \beta L^c. \tag{12}$$

Incumbents’ conjecture is correct because the supplier has no incentives to deviate and offer a different line of credit.

7.2 Proof of Proposition 5 and Corollary 6

Proposition 5. Lemma 2 shows that $I = q_e$, thus supplier’s profits read $\pi_u = (\omega - c_u)q_e$. Suppose that the retailer is not financially constrained, so he borrows $q_e^u$. Then the supplier sets input price to maximize

$$\pi_u = (\omega - c_u) \left[\frac{M - (N + 1)\omega + Nc}{N + 2}\right]. \tag{13}$$

The solution is given by $\omega^u$ in Proposition 5. Similarly, suppose that the retailer is constrained and thus he borrows $L^c$. Then the supplier sets a price to maximize

$$\pi_u = (\omega - c_u) [M - (N + 1)(\omega + \beta) + Nc]. \tag{14}$$

The solution is given by $\omega^c$ in Proposition 5.
For prices \( \omega^u \) and \( \omega^c \) to be an equilibrium they must be such that the retailer is not constrained and constrained respectively. The retailer is constrained when \( \pi^u_e = \beta q^u_e > 0 \) that follows from solving

\[
\left[ \frac{M - (N + 1)\omega + Nc}{N + 2} \right]^2 = \left[ \frac{M - (N + 1)\omega + Nc}{N + 2} \right] \beta. \tag{15}
\]

Thus, the retailer is financially constrained if and only if \( \omega > \omega^u \). Note that the same price function is obtained by solving \( q^u_e = L^c \), indicating that this price equals the production in the constrained and the unconstrained equilibria.

Note that \( \omega^u > \omega^c \). When \( \omega > \omega^u > \omega^c \), the retailer is not financially constrained neither for a price \( \omega^u \) nor for \( \omega^c \). Thus, in equilibrium there are no financial constraints and the supplier sets a price \( \omega^u \). When \( \omega^u > \omega^c > \omega \) the retailer is financially constrained both with prices \( \omega^u \) and \( \omega^c \). Thus, in equilibrium the retailer is constrained and the supplier sets a price \( \omega^c \).

When \( \omega \in [\omega^c, \omega^u] \) neither of the above is an equilibrium. Note that \( \omega \) satisfies \( q^c_e = L^u = L^c > 0 \), i.e.

\[
\frac{M - (N + 1)\omega + Nc}{N + 2} = \frac{1}{\beta} \left[ \frac{M - (N + 1)\omega + Nc}{N + 2} \right]^2 = M - (N + 1)(\omega + \beta) + Nc > 0. \tag{16}
\]

Furthermore, we have \( q^u_e < L^u < L^c \) for \( \omega < \omega^c \) and \( q^u_e > L^u > L^c \) for for \( \omega > \omega^c \). Thus, neither a price \( \omega < \omega^c \) or a price \( \omega > \omega^c \) can be an equilibrium. With a price \( \omega = \omega^c \) the supplier maximizes profits given the input demand and the line of credit and therefore it is an equilibrium.

**Corollary 6.** When the financial constraint is binding \( (\omega < \omega^c) \) the retailer’s production is \( L^c(\omega^c) \) or, equivalently,

\[
L^c = \frac{M - (N + 1)(c_u + \beta) + Nc}{2}. \tag{17}
\]

This also corresponds to the supplier’s input sales. Supplier’s profits are \( \pi^c_u = (\omega^c - c_u)L^c \) whereas retailer’s profits are given by \( \pi^c_e = (M - Nq^c_i - L^c - \omega^c)L^c \). Plugging both \( \omega^c \) and \( L^c \) into the profit functions yields

\[
\pi^c_u = \frac{[M - N(\beta - c + c_u) - c_u - \beta]^2}{4(N + 1)} \quad \text{and} \quad \pi^c_e = \beta L^c. \tag{18}
\]

Denote \( q_L \) the production of a Stackelberg leader with unit cost \( c_u + \beta \). From (2) it follows that incumbents’ best reply reads \( q_i(q_L) = \frac{M-q_L-c}{N+1} \). Thus, the leader maximizes
profits \( \pi_L = (M - Nq_i(q_L) - q_L)q_L - (c_u + \beta)q_L \). The first order condition yields

\[
q_L^* = \frac{M - (N + 1)(c_u + \beta) + Nc}{2}. \tag{19}
\]

Plugin this expression into the profit function I obtain

\[
\pi_L^* = \frac{[M - N(\beta - c + c_u) - c_u - \beta]^2}{4(N + 1)} = \pi_u^c \tag{20}
\]

### 7.3 Proof of Corollary 7 and characterization of Figure 2

**Corollary 7.** Consider the expressions for \( \omega^u, \omega^c \) and \( \overline{\omega} \) in Proposition 5. Solving \( \omega^u = \overline{\omega} \) for \( N \) yields the cutoff \( N_1 \), which satisfies \( \overline{\omega} > \omega^u \) if and only if \( N < N_1 \). Similarly, solving \( \omega^c = \overline{\omega} \) for \( N \) yields the cutoff \( N_2 \), which satisfies \( \omega^c > \overline{\omega} \) if and only if \( N > N_2 \).

The cutoff \( \overline{N} \) is obtained by solving for the input price that equals the marginal cost under financial constraints, i.e. solving \( \omega^c = c_u \) for \( N \). The cutoff also satisfies a null line of credit, so it can be derived by solving \( L^c = 0 \) for \( N \).

**Figure 2.** The full characterization of cutoffs \( N_1, N_2 \) and \( \overline{N} \), and input prices \( \omega_u, \overline{\omega} \) and \( \omega_c \), is given by Proposition 4.

When \( N < N_1 \), production levels and profits are obtained by plugging \( \omega_u \) into \( \{q_u^e, q_u^i, \pi_u^e, \pi_u^i\} \) in Lemma 2, whereas supplier’s profits satisfy \( \pi_u^u = (\omega_u - c_u)q_u^u \). The exercise yields

\[
q_u^e = \frac{M - (N + 1)c_u + Nc}{2(N + 2)} \quad \pi_u^e = (q_u^e)^2
\]

\[
q_u^i = \frac{M - 2c + \frac{M}{2} + (N + 1)c_u + Nc}{2(N + 2)} \quad \pi_u^i = (q_u^i)^2
\]

\[
\pi_u^u = \frac{[M - (N + 1)c_u + Nc]^2}{4(N + 1)(N + 2)}
\]

When \( N \in [N_1, N_2] \), production levels and profits are obtained by plugging \( \overline{\omega} \) into the same functions. I obtain

\[
q_u^e = \frac{M - \beta(N + 2) + Nc}{N + 1} \quad \pi_u^e = (q_u^e)^2
\]

\[
q_u^i = \frac{M - c - \beta}{N + 1} \quad \pi_u^i = (q_u^i)^2
\]

\[
\pi_u^u = \beta \left[ \frac{M - \beta(N + 2) + Nc}{N + 1} - c_u \right]
\]

When \( N \in (N_2, \overline{N}] \), production levels and profits are obtained by plugging \( \omega_c \) into \( L^c \) in Lemma 4 for the retailer’s production; both \( \omega_c \) and \( L^c(\omega_c) \) and into \( \{q_c^i, \pi_c^e, \pi_c^i\} \) and into
\[ \pi_u^c = (\omega_c - c_u) L^c \] for the remaining functions. It yields

\[ L^c = \frac{M - (N + 1)(c_u + \beta) + N\epsilon}{2} \quad \pi_e^c = \beta L^c \]
\[ q_e^c = \frac{M + N(\beta - c + c_u) - 2\epsilon + c_u + \beta}{2(N + 1)} \quad \pi_i^c = \left( \frac{M - c}{2(N + 1)} \right)^2 - \frac{(\beta - c + c_u)^2}{4} \]
\[ \pi_u^c = \frac{[M - N(\beta - c + c_u) - c_u - \beta]^2}{4(N + 1)} \]

### 7.4 Proof of Corollary 9

The supplier can set an optimal contract where the retailer is not financially constrained by offering input at a price such that optimal retailer’s production is \( q_L^* \) and extracting his surplus with a fixed fee so that \( \pi_e = \beta q_L^* \).

The equilibrium production levels in a setting with no financial constraints \( q_e^u \) and \( q_i^u \) are provided by (4) and (5) in the main text. Solving \( q_e^u = q_L^* \) for \( \omega \) yields \( \omega_{tpt} \) in the corollary. This unit price generates the input demand that equals the optimal production of a Stackelberg leader with unit cost \( \beta + c_u \).

For the supplier to extract all the retailer’s surplus while making repayment incentive compatible he must set a fixed fee \( F = (M - Nq_i^u - q_L^*)q_L^* - (\omega + \beta)q_L^* \). The fixed fee equals retailer’s market revenues net of the unit price \( \omega q_L^* \) and the agency cost \( \beta q_L^* \). Algebra manipulation yields \( F_{tpt} \). Solving \( F = 0 \) for \( N \) shows that the fixed fee is negative when \( N \in (N_2, N) \).
References


