Targeting target shareholders*

Dan Bernhardt
University of Illinois
University of Warwick

Tingjun Liu
Cheung Kong Graduate School of Business

Robert Marquez
University of California at Davis

March 24, 2014

Abstract

We integrate heterogeneity and uncertainty in investor valuations into a model of takeovers. Investors have dispersed valuations, holding shares in firms they value more highly, and a successful offer must win approval from the median target shareholder. We derive the consequences for an acquiring firm's takeover offer—its size and cash/equity structure—and implications for takeover premia and firm returns. Cash offers are best for the acquirer when the acquirer's own valuation is high relative to target shareholders. Equity offers are best given the reverse. The acquirer's share price always rises following cash acquisitions, but can fall following equity offers. The combined target-acquirer return is always higher after cash acquisitions than equity acquisitions (which can be negative). We characterize how synergies and uncertainty about target shareholder valuations affect the optimal offer and probability a takeover succeeds.

*We thank Jie Gan and seminar participants at Arizona State University, Central University of Finance and Economics, and Cheung Kong Graduate School of Business for helpful comments. Address for correspondence: Dan Bernhardt (corresponding author), University of Illinois, Champaign IL 61801 danber@illinois.edu, tel: 217-244-5708, fax: 217-244-6678; Tingjun Liu, Cheung Kong Graduate School of Business, tjliu@ckgsb.edu.cn; Robert Marquez, University of California at Davis, rsmarquez@ucdavis.edu.

Targeting target shareholders

Abstract

We integrate heterogeneity and uncertainty in investor valuations into a model of takeovers. Investors have dispersed valuations, holding shares in firms they value more highly, and a successful offer must win approval from the median target shareholder. We derive the consequences for an acquiring firm's takeover offer—its size and cash/equity structure—and implications for takeover premia and firm returns. Cash offers are optimal when the acquirer's private valuation is high relative to target shareholders; while equity offers are optimal when the reverse holds. The acquirer's share price always rises following cash acquisitions, but can fall following equity offers. The combined target-acquirer return is always higher after cash acquisitions than equity acquisitions (which can be negative). We characterize how synergies and uncertainty about target shareholder valuations affect the optimal offer and probability a takeover succeeds.

Keywords: *Heterogeneous valuations; Mergers & Acquisitions; Optimal takeover offers; management guidance*

JEL codes: *G34, D83*

1 Introduction

> "I would rather see PolyMedica Corporation (PLMD) continue to operate as a stand-alone company than be taken over by something BIG in the near future. A takeover premium of let's say 20% would certainly be nice, but it's game over for us as stockholders in PLMD.... I have more faith in management producing higher returns than that!" 15-Feb-06 03:41 am. Yahoo Message Board.
>
> - PLMD closed at $43.43 on February 15, 2006. On August 28, 2007 Medco Health Solutions announced it would buy PLMD in an all-cash deal worth $1.5 billion. The purchase price valued PLMD at $53 per share, a 22% premium for that (presumably) disappointed shareholder.

Heterogeneity—in beliefs, derived utility, tastes, etc.—is a pervasive characteristic of many economic settings. The anecdote above highlights a form of heterogeneity that seems important in financial markets, namely that different investors attach very different valuations to stocks, and that some investors value their shares in a firm far above the market price. This paper integrates such investor heterogeneity into a theory of takeovers. We build a fully-equilibrium model that accounts for heterogeneous *investors* on both sides of the takeover. We investigate how the management of an acquiring firm should design its takeover bid—its size and cash/equity structure—in light of its own private valuations, and we derive the consequences for takeover premia paid, target and acquiring firm returns, and likelihood of successful takeovers. We show how our model can reconcile a broader set of empirical regularities than existing theories, and derive several new testable implications.

In our model, a potential acquirer develops a synergy with a target firm and would thus gain from acquiring it. An acquisition offer consists of either an amount of cash in exchange for a target shareholder's ownership interest, or an equity stake in the joint (merged) firm. To succeed, a takeover offer must win approval from a majority of shareholders. If the majority agrees to sell their shares, the target is absorbed by the acquirer, becoming a single entity.

We capture the existing lack of consensus about a firm's value by assuming that different shareholders hold different private valuations of their firms (Miller, 1977, Chen, Hong and Stein, 2002, and Bagwell, 1991 make similar assumptions).[1] In practice, institutional investors often substantially disagree over what a firm's future earnings and hence future share prices will be. One manifestation of this is the radically different one-year target share prices set for the same stock by analysts representing different institutional investors.[2] We similarly integrate private considerations for the management of the acquiring firm.

When valuations are heterogeneous, not only do a firm's shareholders disagree on their firm's value, but shareholders also have higher valuations than non-holders, reflecting that investors establish positions in stocks they deem "undervalued". The target's share price is determined by the private valuation of its marginal shareholder, who values the firm the least. A successful takeover offer, however, must win approval from the median shareholder who attaches a higher private valuation to the target. It follows that successful takeover offers must be at a premium over the extant share price. This effectively endows target shareholders with increased bargaining power, allowing the marginal shareholder to extract significant rents even when the acquirer makes a take-it-or-leave-it offer. Consistent with this prediction, takeover premia are often high even when there is no evidence of other interested bidders who might give rise to a bidding war (see Andrade et al., 2001, or Betton et al., 2008, for a survey, or Fishman, 1988), so that one would expect an acquiring firm to be able to extract substantial surplus in the absence of valuation heterogeneity at the target firm.

Beyond the simple prediction of takeover premia driven by investor heterogeneity, we analyze the acquiring firm's optimal choice of whether to use cash or equity in its takeover

[1]The literature on disagreement and differences of opinion between investors–Harris and Raviv, 1993, Morris, 1996–adopts a related approach.

[2]Inspection of share price targets reveals that for larger firms (e.g., with market caps exceeding $50 Billion), which are potential acquirers, the median difference in price targets is roughly 35-40% of the firm's current share price. The range of price targets (in percentage terms) is much greater for smaller firms with market caps between $100 million and $6 Billion that are potential targets: for most of these firms, the disagreement over price targets is greater than the outstanding share price. In addition, price targets are higher relative to share price for the vast majority of smaller firms, indicating that in percentage terms, private valuations of potential targets are both higher and more dispersed. Appropriately scaled year-ahead earnings forecasts reveal similar levels of disagreement between institutional investors. Institutional analysts have strong incentives to deliver accurate forecasts of earnings and share price targets—those who get them wrong are likely to be fired, while those that do well receive large bonuses (either from their employer or a competing institutional investor who hires them away. In our setting these large differences in assessments translate into large differences in private valuations. Papers that document upward-sloping supply curves for shares (i.e., heterogeenity in investor valuations) in takeover contexts, include Bagwell (1992), Bradley, Desai and Kim (1987) and Moeller, Schlingemann, and Stulz (2007).

bid. Unlike cash offers, equity offers require the acquiring firm's manager to cede some of his private valuation for his firm, but allow target shareholders to retain greater stakes in the target, and thus more of their private valuations. That is, equity offers mandate a transfer of private values from the acquiring firm's management and shareholders to the target firm's shareholders. The optimal means of payment therefore hinges on the private valuation of the acquirer's manager relative to that of the median target shareholder—cash is optimal when the acquirer's manager has a relatively high private valuation, and equity is optimal when the median target shareholder's valuation is relatively higher. Our prediction on the means of payment thus emphasizes the *contrast* in private valuations of management at the acquirer and the *median* target shareholder. This distinguishes it from theories that focus on a manager's desire to use equity when he believes the market overvalues his firm (Chatterjee, John and Yan 2012), which, in effect, is when the marginal shareholder valuation is high.

We establish that the return to the combined firm in a cash acquisition is always at least as high as that in an otherwise identical equity acquisition. We then show that an acquirer's stock price can fall following an (optimal) equity offer, but not after a cash offer. This reflects that the interests of the acquiring firm's management and its shareholders are aligned with cash offers, but not necessarily with equity offers. In particular, management and shareholders value cash similarly, so any cash offer that appeals to an acquiring firm's management also appeals to its shareholders. In contrast, with heterogeneous private valuations, they value equity differently, and when the acquiring firm's management's private valuation is lower than its shareholders, it may make an equity offer that its shareholders do not like. Moreover, equity offers are attractive precisely when the acquiring firm's management is lower than the median target shareholder's, suggesting that this is a likely circumstance.

These results are consistent with Andrade et al. (2001), who find that market reactions to cash acquisitions are positive, but those to equity acquisitions are mostly negative. Indeed, we find that after an optimal equity offer, the combined firm's share price can be less than the sum of their pre-acquisition standalone share prices. This possible drop in market assessment reflects that pre-merger, investers hold the firms they value most. However, when firms merge, investers must hold both firms, diluting their claims to their preferred pre-merger firms. As a result, the combined acquirer-target return can be negative when the synergies driving an acquisition are not large enough to compensate shareholders for this dilution.

These findings provide an alternative explanation for the observed negative returns for acquirers. For instance, Moeller, Schlingemann, and Stulz (2005) find that around aquisition announcements, acquiring-firm shareholders lose 12 cents per dollar spent on acquisitions. Our explanation is driven only by valuation heterogeneity and does not rely on stock market mispricing, as in Shleifer and Vishny (2003), asymmetric information, or irrationality. While not dismissing such possibilities, we offer a theoretical alternative with fully rational, optimizing behavior driven by synergies associated with wealth creation. The empirical literature concludes from observed negative acquirer and combined acquirer-target returns that there is "wealth destruction"—but our model suggests that what is perceived as "wealth destruction" may merely be a manifestation of what happens to the valuations of marginal shareholders. Relatedly, we provide an explanation for the so-called "diversification discount" found by Berger and Ofek (1995), Lamont and Polk (2001), and Graham et al. (2002)—mergers between less-related firms are associated with lower returns. Importantly, our analysis shows that this discount is not necessarily due to low synergies, but may just reflect large differences in valuations between target and acquiring firm shareholders.

We then investigate the implications of the fact that a target's share price only reveals the private valuation of its *marginal* shareholder, leaving potentially significant uncertainty about the *median* valuation. As a result, an acquirer does not know exactly how much to bid in order to assure itself of success. We show that if synergies are high enough, then increased uncertainty about the median target shareholder's valuation causes an acquirer to raise its offer in order to reduce the likelihood that its offer is rejected and have the synergies go unrealized. If, instead, synergies are lower, increased uncertainty causes the acquirer to lower its offer, since the cost of a failed offer is less and greater uncertainty increases the chance that even a low offer might be accepted. Thus, whether uncertainty about target shareholder valuations raises or reduces the optimal offer hinges crucially on the size of the synergies.

Most theories of takeovers do not provide a reason for why takeovers may fail. A corollary of our findings related to uncertainty about the median shareholder's valuation is that, in our model, offers fail with positive probability even when synergies are large enough that both the median target shareholder and acquirer could benefit from an acquisition. We also offer an explanation for why takeover bids may be rejected even though target shareholders understand that rejection will cause their share price to fall—the target's share price reflects the value of its marginal shareholder, the shareholder who most strongly favors the takeover, while a takeover's success hinges on the assessment of the target's median shareholder.

We predict that target's share price always rises following a takeover offer that is attractive to the marginal target shareholder who determines price, but which must also be attractive to the median shareholder to succeed. Its share price will rise further if a takeover succeeds, but fall if it fails. By contrast, an acquirer's share price will move in the same direction after a successful takeover, as it moved after the offer announcement. If, instead, a takeover fails, we predict that share prices will return to their original levels. These predictions allow us to distinguish empirically between our theory and theories based on informational asymmetries that are commonly used to explain declines in an acquirer's share price. Malmendier, Moretti and Peters (2012) observe that when an acquiring firm has private information about its value, equity offers suggest that its stock is overvalued. Hence, its share price could fall after an equity offer due to the bad news that it reveal. However, their subsequent predicted share price dynamics differ from ours: their model predicts that an acquirer's share price should rise with approval as long as synergies are positive or target shareholder approval reflects a positive assessment by target shareholders, and fall when takeovers fail. Importantly, Savor and Lu (2009) provide support for our theory: they find that in the three-day window around an announcement that a takeover failed for exogenous reasons, the acquirer's share price *rises* by 3%, just offsetting the 3% decline when the takeover was first announced.

We next present the model, and analyze optimal equity and cash offers. We then study which type of offer the acquiring firm finds optimal, and derive the consequences for market reactions. Following this, we analyze how the extent of uncertainty about the median shareholder's valuation affects offers, probability of success, and share price movements following announcement and shareholder vote. Proofs are in an appendix.

2 Base Model

Firms and Investors. The economy features a potential acquirer firm A and a potential takeover target T. We normalize each firm to have one share outstanding. Our base model focuses on two groups of risk-neutral investors who differ in their private valuations for the two firms. One group of investors are of types $\epsilon_A \geq 0$ who place values $V_A + \epsilon_A$ on firm A and V_T on firm T; the other group of investors are of types $\epsilon_T \geq 0$ who place values $V_T + \epsilon_T$ on firm T and V_A on firm A. Thus, a type ϵ_j shareholder has a per-share valuation $\pi_j(\epsilon_j) = V_j + \epsilon_j$, for firm $j = A, T$.

We denote the cumulative wealth of investors with private valuations of at least ϵ_j by $\tilde{G}_j(\epsilon_j)$. Thus, $\tilde{G}_j(\epsilon_j)$ decreases in ϵ_j. We assume that $\tilde{G}_j(0) > V_j$, which implies that the marginal shareholder's private valuation is strictly positive. Investors have no other wealth and no borrowing is allowed. Thus, an investor can invest any amount in each firm, up to his wealth limit. The limited access to capital means that the highest valuation investor does not hold the entire firm, giving rise to a downward sloping demand curve.[3] Market clearing pins down the private valuation $\underline{\epsilon}_j$ of the marginal shareholder of firm j:

$$V_j + \underline{\epsilon}_j = \tilde{G}_j\left(\underline{\epsilon}_j\right). \tag{1}$$

The trading prices of the firms reflect the valuations of their marginal holders:

$$P_j = V_j + \underline{\epsilon}_j, \quad j = A, T. \tag{2}$$

Equation (1) reflects the optimizing behavior of investors: a type-j investor invests all his wealth in firm j if his private valuation exceeds $\underline{\epsilon}_j$, and invests in neither firm if his private valuation is below $\underline{\epsilon}_j$. As a result, a firm's shareholders have higher valuations than non-holders.

[3]This formulation is standard when modeling heterogeneous shareholders (see Miller, 1977, or Bagwell, 1991). This reflects that what is crucial for our qualitative findings is that the induced demand curves slope down, and not the reasons why they do. One can alternatively provide primitives for downward sloping demand via risk averse agents whose private valuations enter mean returns. We forego this approach because qualitative outcomes are unchanged, and takeover offers then affect stock-holding choices on the intensive margin (how much to hold), rather than just on the extensive margin (with wealth constraints, the choice becomes whether to hold), complicating analysis and presentation.

The trading price of firm j reveals the wealth of shareholders with valuations exceeding $\underline{\epsilon}_j$, but nothing more. In particular, there is likely uncertainty over the valuation of the median target shareholder, i.e., about the value of ϵ_T^* such that $\tilde{G}_T(\epsilon_T^*) = \frac{V_T+\epsilon_T}{2}$, whose approval is required for a takeover to succeed. We let $H_j(\cdot|P_j)$ be the conditional distribution over $\tilde{G}_j$ given firm j's trading price. We assume that H_j is common knowledge. Most of the properties of H_j are unimportant for our analysis. However, the distribution of the median target shareholder's valuation, ϵ_T^*, features prominently in the analysis. It simplifies presentation to denote this conditional distribution over ϵ_T^* by $F_T(\cdot)$, with associated support $[\epsilon_T^l, \epsilon_T^h]$, where $\underline{\epsilon}_T < \epsilon_T^l < \epsilon_T^h$. This uncertainty means that although the acquiring firm can infer the valuation of the marginal shareholder from the market-clearing stock price, it is unlikely to know the median target shareholder's exact valuation.[4] For example, the unknown preferences of block shareholders could shift this valuation around substantially.

Acquirer Management's Valuations and Information. Like its shareholders, the acquirer management has a positive private valuation of firm A, attaching value $V_A + \epsilon_A^M$, where $\epsilon_A^M > 0$, but only values the target at V_T. We interpret $V_A + \epsilon_A^M$ as the manager's assessment of his firm's long-term value. We assume that the manager maximizes the long-term profits of shareholders based on his assessment of the firm value, or equivalently, the manager has an equity stake in the firm and maximizes his own profit.

Timing. The sequence of events is as follows. At $t = 0$, a synergy $S > 0$ develops between firms A and T. Synergy S is public information and the valuation of the joint firm is additive for all investors. Thus, a type ϵ_A investor values the joint firm at $(V_A + \epsilon_A) + V_T + S$. At $t = 1$, the acquiring firm's management makes an offer. At $t = 2$, target shareholders decide whether to accept or reject the offer. The offer is accepted if and only if at least 50% of target shareholders vote in favor. We assume that following a favorable vote, there

[4] For example, setting $V_T = 0$ for simplicity, if $P_T = 1$, it could be common knowledge that the wealth holdings of target shareholders come from a family indexed by α, $\tilde{G}_T^\alpha(\epsilon_T) = 1 - \alpha(\epsilon_T - 1) - (1-\alpha)(\epsilon_T - 1)^2$, for $\epsilon \in [1, 2]$ and $\alpha \in (0, 2)$. Thus, when $\alpha = 1$, the distribution of wealth is uniformly distributed on $[1, 2]$ and greater α means that most wealth is held by lower valuation shareholders. Then we have market clearing, as for all α, $\tilde{G}_T^\alpha(1) = 1 = P_T$. The relevant uncertainty concerns that over the median target shareholder's valuation $\epsilon_T^* \in [5/4, 7/4]$, which for a given α equals $\epsilon_T^* = 1 + \frac{-\alpha+\sqrt{\alpha^2+2(1-\alpha)^2}}{2(1-\alpha)}$, for $\alpha \neq 1$, and $\epsilon_T^* = 3/2$ for $\alpha = 1$. Then the uncertainty that the acquiring firm faces over the median target shareholder's valuation is that associated with the uncertainty over the distribution of α.

is a freeze-out of non-tendered shares, and the target is absorbed by the acquirer. This assumption mirrors general practice—freeze-outs occur in over 90% of US and UK takeovers (Gomes, 2001) in order to eliminate free riding.

Discussion. Our assumption that all acquiring-firm shareholders value the target at V_T and all target shareholders value the acquirer at V_A simplifies presentation, but is unimportant for our findings. It is designed to capture the fact that even within particular industries (e.g., biotechnology), few investors have positive private values for any given pair of firms. Here, the relevant pair of firms is the target and acquirer. Section 5 relaxes this structure so that some investors have private valuations of both firms, and shows how our results are robust.

Our model structure is designed to capture two key dimensions of valuation heterogeneities. First, $\underline{\epsilon}_j$ represents difference between how the marginal shareholder of firm j values firm j and how the marginal shareholder in another firm values firm j: it measures the extent to which shareholders of the two firms differ in their valuations of their respective firms, and it underlies the diversification discount that we will find. Section 5 shows that the diversification discount is reduced when more shareholders have private valuations in both firms.

Second, the difference in the valuations of the median and marginal target shareholder $(\epsilon_T^* - \underline{\epsilon}_T)$ is the key measure of dispersion in valuations among target shareholders. This dispersion drives the offer premia that we will find. One might measure this dispersion empirically by the dispersion in analyst forecasts of one-year-ahead earnings, or by the dispersion of one-year-ahead share price "targets" set by analysts. However, the most direct measure is the median of price targets set by analysts conditional on those targets exceeding the firm's extant share price. The institutional investors that employ these analysts are plausible shareholders in the firm, and their views may also be reflected in retail traders who may rely on these price targets in their trading decisions.

Importantly, the variation in share price targets and earnings forecasts is quite large relative to the current share prices of potential takeover targets. For example, for moderate-sized biotech firms, the *range* of price targets set by institutional investors often exceeds the current *level* of the stock price (i.e., disagrements routinely exceed 100%). This implies large differences in private values and means that our model can reconcile the *magnitudes* of offer premia found in the data.

Differences in private valuations are often high in percentage terms for young growth stocks—potential targets—because small differences in views (e.g., of the probability a drug works) imply large differences in discounted future cash flows. Arrival of information about success or future customer base may take years—so there is no reason for these differences to converge, or to be "arbitraged away". Differences in private valuations tend to be smaller in percentage, albeit not absolute, terms, for larger firms with established sources of revenues.

We assume away private valuations for synergies. None of our results are qualitatively affected as long as private valuations of synergies are small relative to those for a firm's assets in place.[5] This is the relevant scenario—synergies are typically tiny relative to a firm's assets, so disagreements about their values should be similarly tiny. Indeed, synergies are often well-understood. For example, the value-added of a pharmaceteutical firm's salesforce for a biotech firm when informing doctors and coordinating on delivery is likely well-understood. So, too, the value of access to internal capital is easy to assess.

For the ease of the analysis, we assume that the takeover opportunity is unexpected. What matters for our analysis is that it is not fully anticipated. The market response to takeover announcements makes clear that this is the relevant scenario—share prices would not move if the takeover were fully anticipated. If the market does anticipate a positive probability of a takeover, then the pre-merger share prices account for such a probability, reducing the absolute magnitudes of the predicted return effects that we find, but otherwise altering their qualitative properties.

3 Analysis

We first examine equity and cash offers assuming that the payment method (equity or cash) is exogenously determined. We then endogenize the method of payment.

[5]The discussion following Proposition 3 describes how the choice of optimal payment method is affected when disagreements about synergies are more substantial.

Exogenous Equity Offers. In an equity offer, an acquirer offers I shares of the joint firm in exchange for all of the target shares. We denote the valuation of the target shareholder who is indifferent between accepting and rejecting the offer of I by $\epsilon_E(I)$. This shareholder's payoff is $\pi_T = V_T + \epsilon_E$ if the takeover fails. To determine his payoff π_E if the takeover succeeds, note that immediately after a successful equity offer, trade will occur if $\underline{\epsilon}_A \neq \underline{\epsilon}_T$, until the marginal shareholders of the joint firm have the same private valuation. Denote the private valuation of the marginal shareholder in the joint firm by $\tilde{\underline{\epsilon}}_J$, where the tilde highlights that its realization, which is between $\underline{\epsilon}_A$ and $\underline{\epsilon}_T$, will hinge on the realized distributions of shareholder wealth distributions, $\tilde{G}_A(\cdot)$ and $\tilde{G}_T(\cdot)$. We can decompose the different scenarios as follows:

(*i*) If $\epsilon_E \geq \tilde{\underline{\epsilon}}_J$, then the median target shareholder will hold the joint firm, so his post-takeover per-share payoff is $\pi_E = \frac{V_T+V_A+S+\epsilon_E}{1+I} I$.

(*ii*) If $\epsilon_E < \tilde{\underline{\epsilon}}_J$, then the median target shareholder will not hold the joint firm. Instead, he will sell his shares at the market price, which is determined by the marginal holder of the joint firm, so his (random) post-takeover per-share payoff is $\pi_E = \frac{V_T+V_A+S+\tilde{\underline{\epsilon}}_J}{1+I} I$.

Summing over the two possibilities, we have

$$\pi_E = \frac{V_T + V_A + S + \max\left(\tilde{\underline{\epsilon}}_J, \epsilon_E\right)}{1+I} I.$$

Then the indifference condition $\pi_T = \pi_E$ implies that ϵ_E solves

$$\frac{V_T + V_A + S + \max\left(\tilde{\underline{\epsilon}}_J, \epsilon_E\right)}{1+I} I = V_T + \epsilon_E. \tag{3}$$

The value of $\tilde{\underline{\epsilon}}_J$ is pinned down by the market clearing condition:

$$\begin{aligned} &\frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{1+I} I \frac{\tilde{G}_T\left(\underline{\epsilon}_T\right) - \tilde{G}_T\left(\tilde{\underline{\epsilon}}_J\right)}{\tilde{G}_T\left(\underline{\epsilon}_T\right) - \tilde{G}_T\left(\bar{\epsilon}_T\right)} = G_A\left(\tilde{\underline{\epsilon}}_J\right) - \tilde{G}_A\left(\underline{\epsilon}_A\right) \quad \text{if } \underline{\epsilon}_A > \underline{\epsilon}_T \quad (i) \\ &\frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{1+I} \frac{\tilde{G}_A\left(\underline{\epsilon}_A\right) - \tilde{G}_A\left(\tilde{\underline{\epsilon}}_J\right)}{\tilde{G}_A\left(\underline{\epsilon}_A\right) - \tilde{G}_A\left(\bar{\epsilon}_A\right)} = \tilde{G}_T\left(\tilde{\underline{\epsilon}}_J\right) - \tilde{G}_T\left(\underline{\epsilon}_T\right) \quad \text{if } \underline{\epsilon}_T > \underline{\epsilon}_A \ (ii) \end{aligned} \tag{4}$$

The system of equations, (3) and (4), jointly determine the values of I and $\tilde{\underline{\epsilon}}_J$.

To simplify presentation, in the analysis that follows we assume:

A1. The *indifferent* target shareholder has a higher private valuation than the *marginal* acquiring firm shareholder: $\epsilon_E \geq \underline{\epsilon}_A$.

Approval of a takeover hinges on the median target shareholder's valuation. When $\epsilon_T^* > \underline{\epsilon}_A$, then even when all acquiring firm shareholders continue to hold the joint firm, so do at least half of the target shareholders (weighted by wealth), including the median target shareholder. We believe that this is typically the relevant scenario, i.e., that **A1** captures most real world settings. In this case, equation (3) simplifies to

$$I = \frac{V_T + \epsilon_E}{V_A + S}. \tag{5}$$

The acquiring firm's manager chooses I^* to maximize his expected payoffs, balancing the tradeoff that a higher offer, although more costly, is more likely to succeed. If a takeover succeeds, the joint firm's market value reflects the value attached by its marginal shareholder:

$$\widetilde{MV}_J \;=\; V_T + V_A + S + \tilde{\underline{\epsilon}}_J \;=\; P_T + P_A + S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T. \tag{6}$$

Because $\tilde{\underline{\epsilon}}_J$ is always less than $\max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$, the (random) market value of the joint firm, $\widetilde{MV}_J$, is always less than the sum of the two firms' pre-merger valuations, $P_T + P_A$, whenever the synergy is small relative to the marginal shareholder's valuation, i.e., whenever $S < \min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$. Denote the combined return over the takeover window from holding equal positions in the acquirer and the target by $\tilde{R}_E$:

$$\tilde{R}_E = \frac{\widetilde{MV}_J}{P_T + P_A} - 1 \;=\; \frac{P_T + P_A + S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T}{P_T + P_A} - 1 \;=\; \frac{S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T}{P_T + P_A}. \tag{7}$$

Recalling that $\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$ measures the extent to which shareholders of the two firms differ in their valuations of their respective firms, we have the following result:

Result 1 *The combined acquirer–target return $\tilde{R}_E$ following an equity acquisition is negative if the synergy S is less than* $\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$*. If, instead, the synergy S exceeds* $\max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$*, the combined acquirer–target return is positive.*

Thus, the combined return is negative if the synergy is less than the heterogeneity in valuations between shareholders of the two firms. This result reflects that a merger forces investors to hold firms they may otherwise not hold, diluting their claims to their favorite firms.

The share price of the joint firm is:

$$\tilde{P}_J = \frac{\widetilde{MV}_J}{1+I} = \frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{1+I}. \tag{8}$$

Interpreting $\tilde{P}_J I^*$ as the cash equivalent of the equity offer, we have:

Proposition 1 *Suppose $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. Then, any equity offer that is accepted by a majority of target shareholders has a cash equivalent that is at a premium over the target's market value: $\tilde{P}_J I^* > P_T$.*

The intuition for this premium is that a takeover dilutes a target shareholder's claim to the target—he now only has a claim of $\frac{I}{1+I}$ to his private valuation ϵ_T—for which he must be compensated. This dilution affects every target shareholder, but the resulting loss is more severe for the median target shareholder than the marginal shareholder. When the median target shareholder is indifferent between accepting and rejecting, the marginal target shareholder, who determines the prices of the target and joint firm, must be strictly better off.

Together, the continuity of payoffs and the strict inequality imply that the result extends as long as $\underline{\epsilon}_T$ is not too much larger than $\underline{\epsilon}_A$.[6] Empirically, the acquiring firm is typically larger than the target, in which case the sufficient conditions for the result hold as long as private valuations do not *decrease* too rapidly in the common value component of the firm. We believe the opposite scenario is far more common. Accordingly, we now assume

A2: Monotonicity. The private valuation of the marginal shareholder in the acquiring firm, $\underline{\epsilon}_A$, and the size of synergies S are nondecreasing in V_A.

A2 delivers an unambiguous interpretation of firms with larger market capitalization: they have both larger private valuation and common valuation components. Under **A2**, when V_A increases, the joint firm becomes more expensive and the fraction of the joint firm offered to the target falls. Thus, target shareholders' claims to the target are further diluted. In turn, the cash equivalent of the offer must rise to compensate for the greater dilution:

Result 2 *Under **A2**, in a successful equity offer, the cash equivalent $\tilde{P}_J I^*$ increases in V_A.*

[6]This holds for our subsequent results that also assume $\underline{\epsilon}_A \geq \epsilon_T$.

The proof of Proposition 1 establishes Result 2 in the simplified setting where the acquirer knows the median target shareholder's valuation (i.e., where $\epsilon_T^h - \epsilon_T^l$ is small). This novel prediction can reconcile Moeller et al.'s (2007) *empirical* finding that shareholders in smaller acquiring firms earn systematically more in acquisitions.

Noting that $\tilde{P}_J I^*$ is the target's stock price after a takeover, Proposition 1 implies:

Result 3 *Suppose $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. Then in a successful equity offer, the target's return $R_T = \frac{I\tilde{P}_J - P_T}{P_T}$ is always positive.*

In contrast, the *acquirer's* return after a successful equity takeover, $R_A = \frac{P_J - P_A}{P_A}$, can be negative. To see this, substitute I^* from (5) into (8) to obtain the joint firm's share price,

$$\tilde{P}_J = \frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{V_T + V_A + S + \epsilon_E^*} (V_A + S) \,. \tag{9}$$

Note from equation (9) that $\tilde{P}_J < V_A + S$ because $\epsilon_E^* > \underline{\epsilon}_J$ means that the ratio of the relative valuations of the marginal and median target shareholder, $\frac{V_T+V_A+S+\tilde{\epsilon_J}}{V_T+V_A+S+\epsilon_E^*}$, is less than one. Therefore, from equation (2), if $S < \underline{\epsilon}_A$, then $\tilde{R}_A = \frac{\tilde{P}_J - P_A}{P_A} < 0$, i.e., the acquirer's return is negative whenever the synergy is small. Indeed, even when $S > \underline{\epsilon}_A$, the acquirer's return can still be negative when there is enough dispersion in target shareholder valuations that $\frac{\epsilon_E^* - \tilde{\epsilon}_J}{V_T+V_A+S+\epsilon_E^*}(V_A + S)$ is large. Thus, our model can reconcile the negative returns for acquirers that Moeller, Schlingemann, and Stulz (2005) find. Further, the combined return is negative when the synergy is small (Result 1), while the target's return is always positive (Result 3). These results will hold when we endogenize the acquiring firm's optimal offer.

Exogenous Cash Offers. With a cash offer, the acquirer offers cash C to target shareholders in exchange for all of their shares. We denote by the valuation of the target shareholder who is indifferent between accepting and rejecting $\epsilon_C(C)$. Immediately after a successful cash acquisition, the joint firm is held only by the acquiring firm's shareholders, while all type ϵ_T investors hold cash. Then, since type ϵ_A and ϵ_T investors value the joint firm at $V_A + V_T + S - C + \epsilon_A$ and $V_A + V_T + S - C + \epsilon_T$ respectively, any target shareholders with private values $\epsilon_T > \underline{\epsilon}_A$ will purchase claims to the joint firm from marginal acquiring firm shareholders. This transaction will raise the value of the marginal holder of the joint firm. Therefore, the share price of the joint firm will satisfy

$$\tilde{P}_J \; > \; V_A + V_T + S - C + \underline{\epsilon}_A. \tag{10}$$

Rearranging (10) yields $\tilde{P}_J + C > V_A + V_T + S + \underline{\epsilon}_A$. Hence, provided that $\underline{\epsilon}_A \geq \underline{\epsilon}_T$,

$$\begin{aligned} \tilde{R}_C &= \frac{\tilde{P}_J + C}{P_A + P_T} - 1 > \frac{V_A + V_T + S + \underline{\epsilon}_A}{P_A + P_T} - 1 = \frac{S - \underline{\epsilon}_T}{P_A + P_T} \\ &\geq \frac{S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T}{P_A + P_T} = \tilde{R}_E, \end{aligned}$$

where we use the fact that $\tilde{\underline{\epsilon}}_J$ lies between $\underline{\epsilon}_A$ and $\underline{\epsilon}_T$. Summarizing, we have:

Result 4 *Suppose that $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. Then, ceteris paribus, the combined acquirer and target return in a cash acquisition exceeds that in an equity acquisition.*

As we noted earlier, the acquiring firm is typically larger than the target. If $\underline{\epsilon}_j$ increases in V_j, then this would suggest that $\underline{\epsilon}_A \geq \underline{\epsilon}_T$ in most situations. When this is so, the result shows the combined acquirer and target return in a cash acquisition exceeds that in an equity acquisition. This result reflects the fact that the private valuation of the marginal holder of the joint firm in a cash offer exceeds $\underline{\epsilon}_A$, whereas that marginal valuation in an equity offers is between $\underline{\epsilon}_A$ and $\underline{\epsilon}_T$. These results can explain the empirical finding that, in most takeovers, the combined returns in cash offers exceed those in equity offers (Andrade et al., 2001).

To determine the optimal offer, C^*, note that just after a successful cash offer, former shareholders of the target for whom $V_A + V_T + S + \epsilon_T - C > \tilde{P}_J$ wish to buy shares in the joint firm. Analogously, original shareholders of the acquiring firm for whom $V_A + V_T + S + \epsilon_A - C < \tilde{P}_J$ want to sell. Market clearing determines $\tilde{P}_J$. There are two possible situations:

(i) If $\frac{V_T + V_A + S - C^* + \epsilon_C^*}{\tilde{P}_J} > 1$, the median target shareholder's private valuation exceeds that of the marginal shareholder of the joint firm. Thus, the median target shareholder derives an added benefit by holding $\frac{C^*}{P_J}$ shares of the joint firm for each share held in the target, receiving a per-share payoff of $(V_T + V_A + S - C^* + \epsilon_C^*) \frac{C^*}{P_J} > C^*$ from the takeover.

(ii) If $\frac{V_T + V_A + S - C^* + \epsilon_C^*}{\tilde{P}_J} \leq 1$, then the median target shareholder will not hold the joint firm, so his post-takeover per-share payoff is C^*.

As the offer C^* leaves the target shareholder with value ϵ_C^* indifferent between accepting and rejecting, the indifference conditions corresponding to these two scenarios yield:

$$\begin{aligned} V_T + \epsilon_C^* &= \frac{V_T + V_A + S - C^* + \epsilon_C^*}{\tilde{P}_J} C^* \quad \text{if } \frac{V_T + V_A + S - C^* + \epsilon_C^*}{P_J} > 1 \quad (i) \\ V_T + \epsilon_C^* &= C^* \qquad\qquad\qquad\qquad \text{otherwise.} \qquad\qquad (ii) \end{aligned} \tag{11}$$

Equation (11) (i) reveals that if the marginal joint firm shareholder has a lower private valuation than the median target shareholder, then $C^* < V_T + \epsilon_C^*$, i.e., the optimal cash offer is less than the median target shareholder's valuation. The median target shareholder uses the cash received for his shares to purchase shares in the joint firm at its market price, which is determined by the marginal holder of the joint firm. As the marginal joint firm shareholder has a lower private valuation than the median target shareholder, this purchase provides the median target shareholder with an added private benefit, making him willing to tender at a lower price (as do all shareholders with higher valuations). In Lemma 1 we relax assumption **A1** in order to identify sufficient conditions for the median target shareholder to hold and not to hold the joint firm, respectively:

Lemma 1 *Define* $\tilde{F}_A(\epsilon) \equiv 1 - \frac{\tilde{G}_A(\epsilon)}{V_A + \underline{\epsilon}_A}$ *for* $\epsilon \in [\underline{\epsilon}_A, \bar{\epsilon}_A]$. *With an optimal cash offer, if* $(V_A + S)\tilde{F}_A\left(\min\left\{\epsilon_T^l, \bar{\epsilon}_A\right\}\right) > V_T + \epsilon_T^h$, *the original median target shareholder holds the joint firm, and* $C^* < V_T + \epsilon_C^*$. *If, instead,* $\underline{\epsilon}_A > \epsilon_T^h$, *the original median target shareholder does not hold the joint firm, and* $C^* = V_T + \epsilon_C^*$.

Lemma 1 is intuitive. $\tilde{F}_A(\epsilon)$ is the number of shares of the acquiring firm held by ϵ_A-type investors with private valuation below ϵ. The first part of the lemma essentially says that if the value of the synergies plus the market value of the portion of the acquiring firm held by shareholders with lower private valuations than the median target shareholder's is large relative to the target's market value, then in an optimal cash offer the target shareholders, with the cash they received, cannot drive the joint firm's market value up past the median target shareholder's valuation. That is, $\tilde{P}_J$ becomes high relative to the cash that target shareholders receive, so target shareholders do not purchase enough of the joint firm to drive its price up past the value to the original median target shareholder. The second part of the lemma follows from (10): in the less plausible scenario where the private valuation of the marginal holders of the acquiring firm always exceeds the median target shareholder's value, the median target shareholder will not hold the joint firm.

When the median target shareholder holds the joint firm, the cash offer that makes him indifferent between accepting and rejecting is less than his valuation of the target, $V_T + \epsilon_C^*$. Proposition 2 shows that even when this is so, the offer still exceeds the target firm's pre-acquisition price, $P_T = V_T + \underline{\epsilon}_T$, as long as the acquirer's market value is high enough. This is because then the joint firm is expensive, so the median target shareholder only purchases a small claim and the added private benefit received is small. Thus, to make him indifferent, a premium relative to the pre-acquisition price must be offered. Indeed, as the acquirer's market value grows arbitrarily larger than the target's, the offer approaches $V_T + \epsilon_C^*$:

Proposition 2 *Suppose* $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. *Then in a successful cash offer, the offer represents a premium if the acquirer is larger than the target. More precisely, if* $V_A > P_T - S$ *then*

$$P_T < C^* \leq V_T + \epsilon_C^*. \tag{12}$$

Further, as the acquirer's market value grows arbitrarily larger than the target's value, the offer approaches the pre-acquisition value of the median target shareholder, $V_T + \epsilon_C^*$*:*

$$\lim_{\frac{V_T+\epsilon_T^h}{V_A+S}\left(\epsilon_T^h-\underline{\epsilon}_T\right)\to 0} C^* - (V_T + \epsilon_C^*) = 0. \tag{13}$$

Corollary 1 *Suppose* $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. *Then the target's return is positive in a successful cash acquisition if* $V_A > P_T - S$.

Boone and Mulherin (2007) report that the mean ratio of the target-to-bidder equity value is 0.45, and the median ratio is only 0.27. Miao and Hackbarth (2007) document that the acquirer is especially likely to be larger than the target in cash acquisitions. Thus, Proposition 2 and Corollary 1 apply to most cash acquisitions.

When $\underline{\epsilon}_A \geq \underline{\epsilon}_T$, the private valuation of the marginal holder of the joint firm exceeds $\underline{\epsilon}_T$. This means that the added benefit the median target shareholder derives from holding the joint firm is not too high. As a result, the cash offer must be at a premium.

3.1 Optimal Payment Method

We now let the acquirer choose the type of offer—cash, equity, no offer—to make.[7] We first examine an acquiring firm manager's willingness to make an equity offer. Prior to a takeover, his per-share payoff is $\pi_{AM} = V_A + \epsilon_M^A$; if an offer I is accepted, his post-merger per-share payoff is $\frac{V_T+V_A+S+\epsilon_A^M}{1+I}$. Thus, the manager's expected per-share payoff is

$$\pi_{AM}^E(I) = \text{pro}(\epsilon_E(I) \geq \epsilon_T^*)\frac{V_T + V_A + S + \epsilon_A^M}{1+I} + \text{pro}(\epsilon_E(I) \geq \epsilon_T^*)\left(V_A + \epsilon_M^A\right)], \tag{14}$$

where $\text{pro}(\epsilon_E(I) \geq \epsilon_T^*)$ denotes the probability that offer I is approved. More specifically, $\epsilon_E(I)$ is the indifferent shareholder given offer I, as determined by the system of equations (3) and (4), and $\text{pro}(\epsilon_E(I) \geq \epsilon_T^*)$ is the probability that $\epsilon_E(I)$ exceeds the median target shareholder's valuation, which is necessary for the offer's approval. The optimal I^* maximizes π_{AM}^E, trading off optimally between the probability of winning against the size of the payoff when a takeover succeeds. We now relax the structure in Assumption **A1** slightly. To guarantee that the median target shareholder holds the joint firm following an optimal equity offer it suffices that:

A3: The *median* target shareholder's private valuation always exceeds the private valuation of the *marginal* acquiring firm shareholder: $\epsilon_T^l \geq \underline{\epsilon}_A$.

Because $\tilde{\epsilon}_J \leq \max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$, Assumption **A3** ensures that $\tilde{\epsilon}_J \leq \underline{\epsilon}_E(I^*)$. This is still stronger structure than we need for the median target shareholder to hold the joint firm following the optimal offer: typically the optimal offer risks failure, and targets some $\epsilon_E > \epsilon_T^l$. With this structure, the probability that an optimal equity offer is accepted is just $\text{pro}(\epsilon_E(I) \geq \epsilon_T^*) = F_T(\epsilon_E(I))$. Substituting for I using equation (5), and omitting the I index on ϵ_E, we write the acquiring manager's expected per-share payoff as:

$$\pi_{AM}^E(\epsilon_E) = F_T(\epsilon_E)(V_A + S)\left(\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon_E}\right) + (1 - F_T(\epsilon_E))\left(V_A + \epsilon_M^A\right). \tag{15}$$

Without loss of generality, we focus on $\epsilon_E^* \in \left[\epsilon_T^l, \epsilon_T^h\right]$, because an offer that exceeds ϵ_T^h always wins, and thus is dominated by offering ϵ_T^h; and offering less than ϵ_T^l always loses,

[7]In practice, an acquirer may not always be able to choose between cash or equity offers; for example, financial constraints may mandate equity offers. Our main results extend to those situations.

and is thus equivalent to offering ϵ_T^l. For the optimal offer to make the acquirer manager strictly better off (i.e., $\pi_{AM}^E(\epsilon_E^*) > \pi_{AM}$), it must have a strictly positive probability of being approved by a majority of target shareholders (i.e., $\epsilon_E^* > \epsilon_T^l$). The converse is also true:

Lemma 2 *The optimal equity offer has a positive probability of being approved by a majority of target shareholders if and only if it renders the acquirer manager strictly better off.*

In order for $\pi_{AM}^E(\epsilon_E^*) > \pi_{AM}$, synergies must be large enough to compensate the manager for the dilution in his claim to his private valuation of his firm:

Lemma 3 *The optimal equity offer has a strictly positive probability of being approved by a majority of target shareholders if*

$$S \geq \frac{V_T + S + \epsilon_T^h}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\epsilon_T^h. \tag{16}$$

If, instead,

$$S \leq \frac{V_T + S + \epsilon_T^l}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\epsilon_T^l, \tag{17}$$

then any offer that the acquiring firm's management would like target shareholders to approve has zero probability of being approved (i.e., $\epsilon_E^* = \epsilon_T^l$).

The first half of the lemma says that a sufficient condition for an optimal equity offer to be accepted with positive probability is that synergies are high enough that there is positive surplus from a takeover even when the median target shareholder has the high private valuation, ϵ_T^h. The second half says that a sufficient condition for optimal equity offer to always be rejected is that synergies are low enough that there is no surplus from a takeover, even when the median target shareholder has the low private valuation, ϵ_T^l.

Now suppose that it is optimal for the acquirer to make a cash offer C. Then its manager's expected per-share payoff would be:

$$\pi_{AM}^C(C) = \text{pro}(\epsilon_C(C) \geq \epsilon_T^*)\left(V_T + V_A + S + \epsilon_A^M - C\right) + (1 - \text{pro}(\epsilon_C(C) \geq \epsilon_T^*))\left(V_A + \epsilon_M^A\right)], \tag{18}$$

where $\epsilon_C(C)$ is the value of ϵ_C corresponding to C. The optimal C^* maximizes π_{AM}^C. As with equity, the acquiring firm's manager must gain from a successful cash offer:

Lemma 4 *The optimal cash offer C^* has a positive probability of being approved by a majority of target shareholders if and only if $\pi^C_{AM}(C^*) > \pi_{AM}$.*

From equation (18), the acquirer manager's per-share expected profit is

$$\pi^C_{AM}(C) - \pi_{AM} = \text{pro}(\epsilon_C(C) \geq \epsilon^*_T)\,(V_T + S - C)\,. \tag{19}$$

This, combined with $C \leq V_T + \epsilon_C(C)$, yields a lower bound on the manager's profit:

$$\pi^C_{AM}(C) - \pi_{AM} \geq \text{pro}(C - V_T \geq \epsilon^*_T)\,(V_T + S - C) \tag{20}$$

$$= F_T(C - V_T)\,(V_T + S - C)\,. \tag{21}$$

Equation (21) and the optimality of C^* yield a sufficient condition for the acquiring firm's manager to make a cash offer:

Lemma 5 *An optimal cash offer by the acquiring firm's management has a strictly positive probability of being approved by a majority of target shareholders if synergies exceed the lower bound on the private valuation of the median target shareholder, i.e., if $S \geq \epsilon^l_T$.*

We next examine when each type of offer is optimal. The choice between cash and equity boils down to whether $\pi^C_{AM}(\epsilon^*_C) > \pi^E_{AM}(\epsilon^*_E)$, in which case a cash offer is made, or $\pi^C_{AM}(\epsilon^*_C) < \pi^E_{AM}(\epsilon^*_E)$, in which case an equity offer is optimal. Cash and equity have competing merits. Equity offers require an acquiring firm's manager to cede some of his private valuation for his firm. This works in favor of using cash and the effect rises with the manager's valuation for his firm, ϵ^M_A. Conversely, equity offers allow target shareholders to retain stakes in the target and thus some of their private valuations. This works in favor of using equity and the effect rises with the median target shareholder's valuation, ϵ^*_T. There is one additional effect in play with cash offers: as long as the price of the joint firm is less than the median target shareholder's valuation, the median target shareholder derives an added private benefit from holding the joint firm, which allows the acquirer to reduce its offer, making a cash offer more attractive.

The resulting choice of means of payment depends on how the private valuation of the acquirer's management compares to the private valuation of the median target's shareholder, and how the private valuation of the marginal holder of the acquiring firm compares to the private valuation of the median target's shareholder. We show that if $\epsilon_A^M > \epsilon_T^h$, then an acquiring firm's management prefers a cash offer to equity offers, but equity offers become more attractive when ϵ_A^M is small:

Proposition 3 *If the acquirer manager's private valuation always exceeds the median target shareholder's (i.e., if $\epsilon_A^M \geq \epsilon_T^h$), then he prefers a cash offer to an equity offer, i.e., $\pi_{AM}^C(C^*) \geq \pi_{AM}^E(I^*)$. If, instead, (a) the median target shareholder's private valuation always exceeds the acquirer manager's, i.e., if $\epsilon_A^M \leq \epsilon_T^l$, and (b) following the optimal cash offer, the median target shareholder does not hold the joint firm (e.g., if $\epsilon_T^h \leq \underline{\epsilon}_A$), then the acquirer prefers to make an equity offer, i.e., $\pi_{AM}^E(I^*) \geq \pi_{AM}^C(C^*)$.*

To gain intuition, consider the simple case in which ϵ_T^* is known with certainty and the median target shareholder derives no private benefits from holding the joint firm (e.g., if $\epsilon_T^* \leq \underline{\epsilon}_A$). Note that regardless of whether equity or cash is used, the acquiring firm's optimal offer leaves the median target shareholder indifferent between accepting and rejecting the offer. Thus, the acquiring firm's management prefers cash to equity if and only if the sum of its payoff plus that of the median target shareholder is higher with cash. Equity and cash offers differ in their impacts on the loss of private valuations in a merger. With equity, the acquirer holds a fraction $\frac{1}{1+I}$ of the joint firm and the target holds the remaining fraction $\frac{I}{1+I}$. Hence, the total loss of private valuation with an equity offer is $\frac{I}{1+I}\epsilon_A^M + \frac{1}{1+I}\epsilon_T^*$. In contrast, the loss with a cash offer is ϵ_T^* (given our premise that the median target shareholder does not hold the joint firm). Thus, the loss with the equity offer is greater if and only if

$$\frac{I}{1+I}\epsilon_A^M + \frac{1}{1+I}\epsilon_T^* \geq \epsilon_T^* \Leftrightarrow \epsilon_A^M \geq \epsilon_T^*,$$

which is exactly the condition from the proposition.

Existing theories (e.g., Chatterjee, John and Yan 2012) predict that a manager wants to use equity when the market overvalues his firm's equity. The results in Proposition 3 are consistent with such theories in that we show that equity is preferred when an acquirer's private valuation is low relative to its marginal shareholder's private valuation $\underline{\epsilon}$ (i.e., its equity is overvalued).[8] However, our analysis also indicates that this comparison is incomplete. Proposition 3 shows that the choice between cash and equity should also reflect the private valuations of target shareholders: equity is preferred to cash when the acquiring firm's manager has a low private valuation *relative* to the *median* target shareholder. Thus, the target's market value, as determined by its marginal shareholder, does not directly enter this calculation.

Proposition 3 establishes that the acquirer is more likely to use cash if its manager's private valuation for his firm is higher. Empirically, one can intepret the acquiring firm's manager as its CEO. As long as the size of the manager's holding of his firm increases with his private valuation, we have the following novel testable prediction:

Corollary 2 *The greater is the acquirer manager's holding of his company, the more likely the acquirer is to offer cash.*

3.2 Stock Price Impacts of Optimal Offers

Having characterized the design of the optimal offer by the acquiring firm's manager, we now show that an optimally chosen equity offer may succeed, and yet cause the acquirer's stock price to fall.

Endogenous Equity Offers. We analyze endogenous (optimal) equity offers that are strictly preferred by the acquiring firm's management to all cash offers (and no offers). We first consider the returns to the target. Recalling that the target return in an optimal equity acquisition is always positive if $\underline{\epsilon}_A \geq \underline{\epsilon}_T$, we have:

[8] In deriving Proposition 3, we have assumed that all parties agree on synergies. If, instead, acquirer management and target shareholders disagree on synergies, then when the acquirer management perceives higher synergies than do target shareholders, it favors the use of cash (all else equal) because equity becomes more costly for the acquirer. Conversely, when the acquirer management perceives lower synergies than do target shareholders, it raises the attraction of equity.

Proposition 4 *Suppose $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. Then, a target firm's share price rises following a successful equilibrium equity acquisition.*

We now contrast this positive return for target shareholders with what acquiring firm shareholders may experience:

Lemma 6 *Suppose **A3** holds and that $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. Then following a successful optimally chosen equity offer, the acquiring firm's share price falls, i.e., $\tilde{P}_J < P_A$, if the synergies are small enough that*

$$S < \underline{\epsilon}_A + (\epsilon_T^l - \underline{\epsilon}_A)\frac{V_A + S}{V_T + V_A + S + \epsilon_T^l}. \tag{22}$$

The condition for the acquirer's stock price to fall following a successful equity offer is that the synergy be too small to compensate the marginal acquiring firm shareholder for the dilution to his private valuation. We now use this result to characterize the possible returns associated with endogenous equity offers. We establish the stronger result that not only may the acquirer's share price fall following an optimal equity offer, but it can fall by so much that the combined acquirer and target return is negative:

Proposition 5 *If $\epsilon_A^M < \underline{\epsilon}_A$, then the combined acquirer and target return can be negative, i.e., $R_E < 0$, following an optimal equity offer that the acquiring firm's management strictly prefers to any cash offer and to no offer.*

The direct corollary of Propositions 4 and 5 is

Corollary 3 *If $\epsilon_A^M < \underline{\epsilon}_A$, then the acquiring firm's share price can fall, i.e., $\tilde{P}_J < P_A$, following an optimal equity offer that the acquiring firm's management strictly prefers to any cash offer and to no offer.*

Proposition 5 indicates that a negative combined return need not imply that the merger destroys wealth, as the existing literature sometimes suggests. Rather, the combined return can be negative even when synergies are positive because pre-merger, shareholders hold the firms they value most, but post-merger, shareholders must hold both firms, diluting their claims to their preferred pre-merger firms. From equation (6), the resulting "value loss" is $\underline{\epsilon}_A + \underline{\epsilon}_T - \tilde{\epsilon}_J - S$. For instance, if $\underline{\epsilon}_A = \underline{\epsilon}_T \equiv \underline{\epsilon}$, then the value loss is simply $\underline{\epsilon} - S$.

The size of the lost value to an acquiring shareholder depends on his private valuation and the extent of the dilution of his claim to that private valuation. For this loss to occur following an optimal equity offer, it must be that the private valuation of the acquiring firm's management is less than that of its marginal shareholder's. Then, the marginal shareholder suffers a loss when its management's payoff is positive, but sufficiently small. Further, the attraction of equity offers relative to cash offers rises when ϵ_A^M is smaller—precisely because the acquiring firm's management does not mind diluting its private valuation by as much. Here, $\epsilon_A^M << \underline{\epsilon}_A$ captures shareholders who attach higher valuations to the firm's assets than management.[9]. More generally, more extensive investor heterogeneity, as captured by a larger value of $\underline{\epsilon}_A$, can cause the acquiring firm's share price to fall.

These results provide a novel explanation for the "diversification discount" observed in mergers of conglomerates in different industries: the "value loss" need not be because the synergies are smaller, but rather because shareholders in the two firms differ more substantively in their valuations, i.e., $\underline{\epsilon}$ is larger reflecting that the conglomerates are more dissimilar. That is, the diversification discount may reflect large differences in valuations between target and acquiring firm shareholders of each other's firm, and not low synergies. Section 5 investigates the diversification discount in more detail, showing how the magnitude of the discount is related to the "similarity" between the merging firms.

The declining share price does *not* necessarily indicate that the acquirer blundered or that there is some form of winner's curse. In fact, all agents are rational and there are no informational asymmetries in our model. Indeed, one can distinguish between the predictions of our model and those of takeover theories based on asymmetries of information. Malmendier, Moretti and Peters (2012) observe that when an acquiring firm has private

[9]Agency considerations (e.g., a managers' empire building motives) could also lead to a decrease in the acquirer's return, just as when the manager's private valuation differs from target shareholders. However, one must be careful when interpreting a low value of ϵ_A^M as reflecting agnecy concerns: presumably, a manager's private benefit of control does not vary with the payment method, so agency considerations will not have the same differential implications for the choice between cash and equity that differences between a manager's private valuation and that of the median target shareholder do.

information about its value, equity offers would suggest that its stock is overvalued, so that its share price could fall following an equity offer due to the bad news revealed. However, subsequently, the acquirer's share price should rise with approval as long synergies are positive or if approval reflects positive private target shareholder information; and should fall when takeovers fail due to any negative information revealed by the rejection about the acquirer and the loss of synergies. In contrast, in our setting, if an acquiring firm's stock price falls following an equity offer and there is uncertainty over whether the offer would be accepted, then it should fall further following acceptance, but rise following rejection.

It is difficult to test these predictions directly due to the endogeneity and selection issues associated with accepted and rejected offers (for example, outside of our model, the takeover negotiation process may feature the *possibility* of a subsequent offer if an initial offer is rejected). Savor and Lu (2009) had the insight that one can get clean identification by focusing on takeovers that fail for exogenous reasons, an approach that Masulis et al. (2012) also employ. Then our model predicts that the acquirer's share price should rise back to its original level when the failure of a takeover is announced (as the transaction is unwound); but Malmendier et al. predict that the acquirer's share price should fall (due to the lost synergies). Consistent with our model, in the three day window around the announcement of a takeover's failure, Savor and Lu find abnormal acquirer returns of 3 percent, which just offset the negative abnormal acquirer's returns of 3 percent when a takeover with equity was first announced. These twin results provide strong confirmation of our theory.

Endogenous Cash Offers. We now analyze endogenous cash offers. These offers (a) maximize the payoff of the acquiring firm's management (i.e., ϵ_C^* is optimally chosen), (b) have positive probabilities of being approved (i.e., $\epsilon_C^* > \epsilon_T^l$), and (c) are preferred to equity offers. We highlight a sharp contrast between optimal equity and cash offers: unlike equity offers, any cash offer that is individually rational for an acquiring firm's manager is also preferred by its marginal shareholder. As a result, for endogenous cash offers, we have:

Proposition 6 *An acquiring firm's share price rises following a successful equilibrium cash acquisition.*

Intuitively, all parties value cash in the same way. Hence, a cash offer that appeals to the acquiring firm's management also appeals to its shareholders, so the joint share price increases. This result is consistent with Andrade et al.'s (2001) empirical finding that acquiring firms' share prices tends to drop following stock acquisitions, but not cash acquisitions.

We now compare combined acquirer and target returns in cash and equity acquisitions. Recall that when the payment method was exogenous and if $\underline{\epsilon}_A \geq \underline{\epsilon}_T$, the combined return in a cash acquisition exceeded that in an equity acquisition (Result 4), and the target's return is positive (i.e., the target's share price increases) as long as the target is not much larger than the acquirer (Corollary 1). These results extend when the acquiring firm's manager selects his preferred payment method:

Proposition 7 *Consider two equilibrium takeover offers with the same values of $V_A, V_T, S, \underline{\epsilon}_A, \underline{\epsilon}_T$ where $\underline{\epsilon}_A \geq \underline{\epsilon}_T$, but with different values of ϵ_A^M, so that one acquisition is with equity and the other is with cash. Then the combined return in the cash acquisition exceeds that in the equity acquisition.*

Proposition 8 *Suppose $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. If $V_A > P_T - S$, then a target firm's share price rises following a successful equilibrium cash acquisition.*

Combining Propositions 5-8 reveals that, consistent with empirical findings, cash acquisitions are associated with positive and higher returns than equity acquisitions, the target experiences positive returns, but equity acquisitions can be associated with negative combined acquirer-target returns, even when equity acquisitions are optimal. Thus, our theory imposes many more restrictions on the data than existing theories.

4 Comparative Statics

In this section, we impose additional structure in order to derive explicit comparative static characterizations. Specifically, we assume that $\underline{\epsilon}_A = \underline{\epsilon}_T \equiv \underline{\epsilon}$, and that the median target shareholder's private valuation, ϵ_T^*, is uniformly distributed on $[\hat{\epsilon} - \alpha, \hat{\epsilon} + \alpha]$. The expected private valuation, $\hat{\epsilon}$, of the median target shareholder measures the extent of heterogeneity in valuations between target and acquiring firm shareholders; while α measures the extent to which an acquiring firm is uncertain about the private valuation of the median target shareholder, and $\hat{\epsilon} - \alpha > \underline{\epsilon}$ reflects that the acquiring firm knows the marginal shareholder valuation, which bounds its uncertainty over ϵ_T^*. We focus on cash offers (which would endogenously arise if, for example, ϵ_A^M is large enough); equity offers have qualitatively similar features. To avoid the complications in cash offers when the median target shareholder derives private benefits from holding the joint company, we assume that $\frac{V_T+\hat{\epsilon}+\alpha}{V_A+S} << 1$, in which case we can approximate this additional private benefit as zero.

For any cash offer C, the tendering decision of a target shareholder with private valuation ϵ is simple: accept if and only if $C \geq V_T + \epsilon$. The probability that an offer C is accepted is

$$\Pr(C) = \begin{cases} 1 & \text{if } \frac{C-V_T-(\hat{\epsilon}-\alpha)}{2\alpha} > 1 \\ \frac{C-V_T-(\hat{\epsilon}-\alpha)}{2\alpha} & \text{if } 0 < \frac{C-V_T-(\hat{\epsilon}-\alpha)}{2\alpha} \leq 1 \\ 0 & \text{if } \frac{C-V_T-(\hat{\epsilon}-\alpha)}{2\alpha} \leq 0. \end{cases}$$

Without loss of generality, we can focus on offers where $C \in [V_T + \hat{\epsilon} - \alpha, V_T + \hat{\epsilon} + \alpha]$. As a function of C, the expected payoff of the acquiring firm's management is:

$$\begin{aligned} \Pi_A &= \Pr(C)(V_T + S - C) + \left(V_A + \epsilon_A^M\right) \\ &= \frac{C - V_T - \hat{\epsilon} + \alpha}{2\alpha}(V_T + S - C) + V_A + \epsilon_A^M. \end{aligned}$$

The first term is the expected increment in value associated with a successful takeover offer, while the second term is the status quo (no acquisition) value. Differentiating with respect to C yields the first-order condition describing the optimal offer:

$$\frac{d\Pi_A}{dC} = 0 = \frac{S + \hat{\epsilon} + 2V_T - \alpha - 2C}{2\alpha}. \tag{23}$$

Since the second-order conditions are satisfied, (23) defines a global maximum. In addition, if the optimal offer C exceeds $V_T + \hat{\epsilon} - \alpha$, the offer must be individually rational because the acquiring firm could always offer $C = V_T + \hat{\epsilon} - \alpha$ and have its offer be rejected. Allowing for a boundary solution, the general solution for the optimal offer C^* is

$$C^* = \begin{cases} \text{no offer} & \text{if} \quad S < \hat{\epsilon} - \alpha \\ V_T + \frac{S+\hat{\epsilon}-\alpha}{2} & \text{if} \quad \hat{\epsilon} - \alpha < S < \hat{\epsilon} + 3\alpha \\ V_T + \hat{\epsilon} + \alpha & \text{if} \quad S \geq \hat{\epsilon} + 3\alpha. \end{cases} \tag{24}$$

We can now characterize how the optimal offer varies with the primitive parameters describing the economy:

Proposition 9 *When the acquiring firm's beliefs about the median target shareholder's private valuation are uniformly distributed,*

- *C^* increases in the extent $\hat{\epsilon}$ of heterogeneity in private valuations of target firm shareholders.*
- *C^* increases in the synergy, S.*
- *If synergies are small, $S \leq \hat{\epsilon}$, then C^* decreases with the extent of uncertainty α.*
- *If synergies are large, $S > \hat{\epsilon}$, then C^* first increases in α and then decreases, reaching a maximum at $\alpha = \frac{S-\hat{\epsilon}}{3}$.*

The optimal offer C^* increases in the extent of heterogeneity $\hat{\epsilon}$. This result reflects the central intuition of our paper: a successful offer must win approval from at least 50% of shareholders, who have higher valuations than the marginal shareholder that determines the price. The result that C^* rises with the synergy S is also intuitive, reflecting that the opportunity cost of rejection rises in S. The reason why increased uncertainty can cause an acquirer to reduce its offer is that greater uncertainty raises the likelihood that low offers are accepted. Further, the cost of having an offer rejected is not too great when synergies are small, so the marginal cost of a lower offer is small, and thus lower offers are optimal when synergies are small. If, instead, synergies are large, there is a range where the offer initially rises in α because the acquirer does not want to risk a failed offer. However, as the extent of uncertainty grows, the only way to ensure success is to keep raising the offer, which eventually becomes too costly. Beyond this point, the marginal increase in the probability that a higher offer succeeds is too small to justify increasing the offer further, and the optimal offer C^* begins to fall with α.

We can now solve for how the synergies and extent of uncertainty faced by the acquiring firm affect the equilibrium likelihood of a successful takeover. Substituting for C^* yields

$$\Pr\left(C^*\right) = \begin{cases} 0 & \text{if} \quad S \leq \hat{\epsilon} - \alpha & (i) \\ \frac{S-\hat{\epsilon}+\alpha}{4\alpha} & \text{if} \quad \hat{\epsilon} - \alpha < S < \hat{\epsilon} + 3\alpha & (ii) \\ 1 & \text{if} \quad S \geq \hat{\epsilon} + 3\alpha. & (iii) \end{cases}$$

Proposition 10 *The equilibrium probability of success falls with the extent of heterogeneity $\hat{\epsilon}$ and rises with the synergy S. If the synergy is less than the extent of heterogeneity, i.e., if $S < \hat{\epsilon}$, then the success probability rises with the extent of uncertainty, α. If, instead, the synergy exceeds the extent of heterogeneity, i.e., if $S > \hat{\epsilon}$, then the success probability falls with α.*

Few theories of takeovers provide a reason for why takeover bids sometimes fail. Greater heterogeneity reduces the probability of successful offers because higher offers are needed for success. Greater synergies induce the acquiring firm to raise its offer, increasing the probability of a successful offer. To understand why the success probability rises with the extent of uncertainty α when synergies are small, observe that when synergies are low, the realized private valuation of the median target shareholder must be low for target shareholders to accept an offer, and a higher α increases this probability. However, when synergies are high, but not so high that the acquiring firm finds it optimal to make an offer that always succeeds, the probability of success falls with α because the acquiring firm lowers its offer, trading off a reduced probability of success against the possibility of a better deal if the realized private valuation of the median target shareholder turns out to be low.

Some of the comparative static results in Propositions 9 and 10 are testable. For example, one can proxy the extent of heterogeneity in private valuations by the dispersion in earnings forecasts of analysts or share price targets associated with investment banks and other institutional investors. The propositions would then suggest that an acquiring firm's returns should be lower, ceteris paribus, when the variance of earnings forecasts or share price targets is greater, and that such takeovers should be more likely to fail.

4.1 Share price dynamics over the takeover process

Any offer that has a positive probability of being accepted is always at a premium over the target's stand-alone price, which is determined by the target shareholder with marginal valuation $\underline{\epsilon}$. Hence, following such a takeover offer, the target's share price will rise to reflect that (i) $C^* > V_T + \underline{\epsilon}$, and (ii) with probability $\frac{S-\hat{\epsilon}+\alpha}{4\alpha}$, we have $C^* > V_T + \epsilon_T^*$, in which case the takeover succeeds. In this case the target's share price will rise further to reflect the beneficial resolution of takeover uncertainty from the perspective of its marginal shareholder. However, with probability $\frac{3\alpha-S+\hat{\epsilon}}{4\alpha}$ the offer is rejected, in which case the target's share price will fall to its pre-takeover value, $V_T + \underline{\epsilon}$.

Moreover, a cash offer that appeals to the acquiring firm's management also appeals to its shareholders (Proposition 6). Hence, following a takeover bid in cash, the acquiring firm's share price will rise to reflect the positive probability that the bid will succeed. The share price would rise further upon acceptance, reflecting the beneficial resolution of takeover uncertainty from the perspective of the acquirer; but fall to its level prior to the emergence of synergies whenever its offer is rejected. Hence, we have the following testable predictions:

Corollary 4 *Suppose that synergies are large but not too large, i.e.,* $\hat{\epsilon} - \alpha < S < \hat{\epsilon} + 3\alpha$, *so that the equilibrium cash takeover bid is accepted with positive probability strictly less than one. Then, the share prices of both the target and acquiring firms rise when synergies emerge and a takeover bid is made, and rise further whenever a takeover bid succeeds. Both firms' share prices fall whenever a takeover offer fails.*

If, instead, an acquirer makes an equity bid rather than cash, the *target*'s share price exhibits similar dynamics. However, the *acquirer*'s share price dynamics are unchanged *only* if synergies are high enough that a successful takeover results in positive acquirer returns; this requires both that $S > \underline{\epsilon}$ and for α to be small enough relative to other parameters. Otherwise, the acquiring firm's share price will fall after an equity takeover bid, *and fall further if the takeover succeeds.* We emphasize that this prediction is the opposite of that implied by takeover theories based on asymmetric information, which predict that the acquirer's share price will fall when an equity takeover bid is made, *but rise if the takeover succeeds.*

5 Diversification Discount

In this section, we investigate the foundations of the "diversification discount"—exploring why mergers between less-related firms are associated with lower returns. To capture the notion of "more-related and"less-related" firms we enhance our basic model so that some investors have positive private valuations of both firms. More-related firms are then those in which more investors have positive provate valuations of both firms.

To ease exposition, we simplify the model so that the economy is symmetric with $V_A = V_T \equiv V$. We consider three groups of investors. Group-one investors place values $V + \epsilon_A$ on firm A and V on firm T; group-two investors place values $V + \epsilon_T$ on firm T and V on firm A; and group-three investors place the same value $V + \epsilon_{AT}$ on both firms T and A. The values of ϵ_A, ϵ_T, and ϵ_{AT} in the population of investors are each uniformly distributed on $[0, \bar{\epsilon}]$. Each investor has an equal amount of wealth, and their total wealth is $W > 2V$. We capture the closeness of the two firms with the fraction $\rho \in [0, 1]$ of investors who have positive private valuations of both firms. Thus, the total wealth of group-three investors is ρW, and the remaining wealth $(1 - \rho)W$ is divided evenly between group-one and group-two investors.

The symmetry that we assume is unimportant for the qualitative results, but it simplifies calculations and facilitates cleaner interpretations of how ρ affects the diversification discount. We show that the price of the merged firm is decreasing in ρ, and that when ρ is small enough—when the firms are more dissimilar from the perspective of most investors—then the price of the merged firm is less than the collective stand-alone values of the two firms. The extent of the diversification discount would only be greater were the private valuations of group-three investors less correlated (e.g., independent).

We first compute $\underline{\epsilon}_A$ and $\underline{\epsilon}_T$, which determine the standalone market values of the two firms. We first show that in our symmetric setting, in equilibrium, $\underline{\epsilon}_A = \underline{\epsilon}_T$. To see this, suppose without loss of generality that $\underline{\epsilon}_A > \underline{\epsilon}_T$, instead. Then, types $\epsilon_{AT} \geq \underline{\epsilon}_T$ and types $\epsilon_T \geq \underline{\epsilon}_T$ hold the cheaper firm T; only types $\epsilon_A \geq \underline{\epsilon}_A$ hold firm A. As a result, the total wealth of shareholders of firm T exceeds that of shareholders of firm A. But then the market-clearing conditions imply that the market value of T will be driven above that of A, contradicting the premise that $\underline{\epsilon}_A > \underline{\epsilon}_T$. Thus, $\underline{\epsilon}_A = \underline{\epsilon}_T \equiv \underline{\epsilon}$. The market-clearing conditions require that the wealth of $\epsilon_{AT} \geq \underline{\epsilon}$ investors is divided evenly between the two firms. Then, the market-clearing condition for each firm takes the form:

$$\frac{\bar{\epsilon} - \underline{\epsilon}}{\bar{\epsilon}} \left(\frac{1-\rho}{2} + \frac{\rho}{2} \right) W = \frac{\bar{\epsilon} - \underline{\epsilon}}{\bar{\epsilon}} \frac{W}{2} = V + \underline{\epsilon},$$

yielding

$$\underline{\epsilon} = \frac{W - 2V}{W + 2\bar{\epsilon}} \bar{\epsilon}. \tag{25}$$

If the two firms merge through an equity offer—if the acquirer offers I in exchange for all shares of T—then just after the merger, group 3 investors value the joint firm by more than group 1 and 2 investors with the same private valuation. Thus, trade will occur between group 3 investors with private valuations below $\underline{\epsilon}$, who hold neither firm and have cash on hand, and group 1 and 2 investors with private valuations slightly above $\underline{\epsilon}$. The joint firm's equilibrium market value of $2V + S + \underline{\epsilon}_J$ is pinned down by the market-clearing condition. That is, group 1 and 2 investors with private valuations between $\underline{\epsilon}$ and $\underline{\epsilon}_J$ sell their shares to group 3 investors with private valuations between $\frac{\underline{\epsilon}_J}{2}$ and $\underline{\epsilon}$. Thus, $\underline{\epsilon}_J \in [\underline{\epsilon}, 2\underline{\epsilon}]$.

For simplicity, we assume that $\underline{\epsilon}_J < \bar{\epsilon}$, which happens if there is sufficient dispersion in the private valuations of investors, where the required extent of dispersion is increasing in ρ. A sufficient condition for this to hold is that $W < 4V$ or $\bar{\epsilon} > \frac{W}{2}$. This assumption rules out the corner solution of $\underline{\epsilon}_J = \bar{\epsilon}$; when such a solution obtains, the qualitative features of our results do not change, but the algebra is more complicated because the market-clearing condition that we identify ceases to hold with equality.

We now solve for $\underline{\epsilon}_J$. Prior to the merger, group 3 investors divide their investments evenly between the two firms, allocating $\rho/2$ to each firm implying that the fraction of firm A initially held by group 1 investors is $(1-\rho)$. With the uniform distribution of private valuation for investors, the fraction of the acquiring firm initially held by group 1 investors with private valuations $\epsilon_A \in [\underline{\epsilon}, \underline{\epsilon}_J]$ is $\frac{\underline{\epsilon}_J - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}(1-\rho)$. So, too, the initial fraction of the target held by group 2 investors with private valuations $\epsilon_T \in [\underline{\epsilon}, \underline{\epsilon}_J]$ is $\frac{\underline{\epsilon}_J - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}}(1-\rho)$. Thus, just after the merger (before any trading takes place), the fraction of the joint firm held by group 1 and 2 investors with private valuations between $\underline{\epsilon}$ and $\underline{\epsilon}_J$ is

$$\frac{\underline{\epsilon}_J - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} (1-\rho) \left(\frac{1}{1+I} + \frac{I}{1+I} \right) = \frac{\underline{\epsilon}_J - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} (1-\rho) \,,$$

which they can sell for

$$\frac{\underline{\epsilon}_J - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} (1-\rho) \left(2V + S + \underline{\epsilon}_J \right).$$

The buyers are group 3 investors with private valuations $\epsilon_{AT} \in [\frac{\underline{\epsilon}_J}{2}, \underline{\epsilon}_J]$, who have wealth

$$\rho W \frac{\underline{\epsilon} - \frac{1}{2}\underline{\epsilon}_J}{\bar{\epsilon}}.$$

Equating demand and supply yields:

$$\frac{\underline{\epsilon}_J - \underline{\epsilon}}{\bar{\epsilon} - \underline{\epsilon}} (1-\rho) \left(2V + S + \underline{\epsilon}_J \right) = \rho W \frac{\underline{\epsilon} - \frac{1}{2}\underline{\epsilon}_J}{\bar{\epsilon}}. \tag{26}$$

Note that in this symmetric setting with uniform uncertainty, the size I of the equity offer does not enter the market-clearing condition (26). Define $\kappa \equiv \frac{\rho}{1-\rho} \frac{\bar{\epsilon}-\underline{\epsilon}}{2\underline{\epsilon}} W$. Substituting $\underline{\epsilon}$ by (25), yields

$$\kappa \equiv \frac{\rho}{1-\rho} \frac{W\left(\bar{\epsilon} + V\right)}{W - 2V}, \tag{27}$$

which is monotonically increasing in ρ, going from 0 to infinity as ρ goes from 0 to 1. Substituting in κ, the market-clearing condition simplifies to

$$\left(\underline{\epsilon}_J - \underline{\epsilon}\right)\left(\underline{\epsilon}_J + 2V + S\right) = \kappa \left(2\underline{\epsilon} - \underline{\epsilon}_J\right). \tag{28}$$

Equation (26) has a unique positive solution:

$$\underline{\epsilon}_J = \frac{1}{2} \left[\left(\left(2V + S + \kappa - \underline{\epsilon}\right)^2 + 4\underline{\epsilon}\left(2V + S + 2\kappa\right) \right)^{0.5} - \left(2V + S + \kappa - \underline{\epsilon}\right) \right], \tag{29}$$

where $\underline{\epsilon}$ is given by (25).

We denote by D the difference between the sum of the two firm's standalone market values and the joint firm's market value:

$$D = 2\underline{\epsilon} - \underline{\epsilon}_J - S. \tag{30}$$

Thus, when D is positive, it indicates that together the standalone market values of the two firms exceeds the joint firm's market value, i.e., that the combined return to the takeover is negative. We next explore how ρ affects D, and what it says about the "diversification discount." Substituting in (25) for $\underline{\epsilon}$ and (29) for $\underline{\epsilon}_J$, we can solve for:

Lemma 7 *The diversification discount is*

$$D = \frac{2W - 4V}{W + 2\bar{\epsilon}}\bar{\epsilon} - S -$$
$$\frac{1}{2}\left[\left(\left(2V + S + \kappa - \frac{W - 2V}{W + 2\bar{\epsilon}}\bar{\epsilon}\right)^2 + 4\frac{W - 2V}{W + 2\bar{\epsilon}}\bar{\epsilon}\left(2V + S + 2\kappa\right)\right)^{0.5} - \left(2V + S + \kappa - \frac{W - 2V}{W + 2\bar{\epsilon}}\bar{\epsilon}\right)\right],$$

where κ is given by (27).

We now derive key properties of the diversification discount:

Proposition 11 *The diversification discount D decreases monotonically in ρ. The maximal discount of $D = \frac{W-2V}{W+2\bar{\epsilon}}\bar{\epsilon} = \underline{\epsilon} - S$ occurs at $\rho = 0$, where firms are most dis-similar. For any $\rho < 1$, there exists an $\bar{S}(\rho) > 0$ such that for all $S < \bar{S}(\rho)$, the discount is positive, i.e., $D > 0$.*

One can interpret ρ as capturing the degree of similarity between the two industries for A and T. Thus, the proposition indicates that, ceteris paribus, the diversification discount is larger when the two firms are from less related industries (e.g., conglomerates), which is consistent with the empirical facts.

The intuition for the above result is closely related to that from the base model in which there are only two groups of investors, where each group has private valuation for only one firm. This base model delivers the intuition that the diversification discount reflects the differences in valuations between target and acquiring firm shareholders of each other's firm; and a merger dilutes a shareholder's holdings of his preferred firm. When we allow for investors with private valuations for both firms as we do here, this intuition extends in that the diversification discount reflects some measure of average differences in valuations between target and acquiring firm shareholders of each other's firm. Moreover, "the average differences in valuations" directly relate to the "closeness" of the two industries, which gives rise to our result that the magnitude of the discount falls with the closeness of the two industries.

6 Conclusion

We integrate heterogeneity and uncertainty in investor valuations into a model of takeovers. When investors have dispersed valuations, they hold shares in firms they value more highly. A successful takeover offer must be sufficiently generous to convince not only marginal target shareholders to sell their shares, but also a majority of those who value the firm more highly.

We derive the consequences for an acquiring firm's takeover offer—its size and cash/equity structure—and the implications for takeover premia and firm returns. We establish that cash offers are optimal when the acquirer's private valuation is high relative to target shareholders, as cash does not dilute the acquirer's claim to its private value; while equity offers become more attractive when the acquiring firm's manager's private valuation is small.

We then characterize how synergies and the extent of uncertainty about target shareholder valuations affect the optimal offer and probability a takeover succeeds. We establish that the optimal offer is increasing in synergies, but is only increasing in the extent of uncertainty about the median target shareholder's private valuation when synergies are large, and the uncertainty is not too high. We conclude by deriving the patterns of share price dynamics following successful and unsuccessful takeover bids.

Our simple model generates an extensive array of empirical predictions:

1. All accepted takeover offers are at significant premia over the target's market value.
2. The cash equivalent of successful equity offers are increasing in the acquirer's market capitalization.
3. Combined target-acquirer return are higher after (optimal) cash acquisitions than after (optimal) equity acquisitions.

4. Returns after an (optimal) equity acquisition are negative when synergeis are low.

5. While the acquirer's share price falls following a successful equity takeover whenever synergies are small, it rises following a cash acquisition.

6. Ceteris paribus, shareholders in smaller acquiring firms earn more in takeovers (Moeller et al., 2005)

7. Ceteris paribus, larger acquiring firms are more likely to make cash offers.

8. Greater dispersion in private valuations of a target raises target takeover premia (Chatterjee et al., 2012), but lower the acquirer's returns and reduce the probability that a takeover succeeds.

9. If an acquirer's share price falls following an equity offer, then it falls further when the target accepts an offer, but rise on announcement that a takeover fails (Savor and Lu, 2009). The target's share price should rise when the target accepts an offer, but fall when it fails.

10. The larger is the stake of the decision maker in the acquiring firm, the more likely it is to offer cash.

11. Combined acquirer-target returns are higher in mergers featuring "more similar" firms (Berger and Ofek 1995, Lamont and Polk 2001, Graham et al. 2002).

7 Appendix

Proof of Proposition 1: The indifference condition (3) yields

$$\frac{I^*}{1+I^*} = \frac{V_T + \epsilon_E^*}{V_T + V_A + S + \max\left(\underline{\epsilon}_J, \epsilon_E^*\right)}, \tag{31}$$

which gives

$$\begin{aligned}\tilde{P}_J I - P_T &= \frac{V_T + V_A + S + \underline{\epsilon}_J}{1+I^*} I^* - V_T - \underline{\epsilon}_T \\ &= \frac{V_T + V_A + S + \underline{\epsilon}_J}{V_T + V_A + S + \max\left(\underline{\epsilon}_J, \epsilon_E^*\right)} \left(V_T + \epsilon_E^*\right) - V_T - \underline{\epsilon}_T.\end{aligned}$$

Next we provide a general proof for the proposition without imposing **A1**. Consider two cases. (1) $\underline{\epsilon}_J \geq \epsilon_E^*$. Then $\tilde{P}_J I - P_T = \epsilon_E^* - \underline{\epsilon}_T > 0$, establishing the proposition. (2) $\underline{\epsilon}_J < \epsilon_E^*$. Then, because $\underline{\epsilon}_J$ is between $\underline{\epsilon}_T$ and $\underline{\epsilon}_A$, the condition $\underline{\epsilon}_A \geq \underline{\epsilon}_T$ yields $\underline{\epsilon}_J \geq \underline{\epsilon}_T$. Thus, we have

$$\begin{aligned}\tilde{P}_J I - P_T &= \frac{V_T + V_A + S + \underline{\epsilon}_J}{V_T + V_A + S + \epsilon_E^*}\left(V_T + \epsilon_E^*\right) - V_T - \underline{\epsilon}_T \\ &\geq \frac{V_T + V_A + S + \underline{\epsilon}_T}{V_T + V_A + S + \epsilon_E^*}\left(V_T + \epsilon_E^*\right) - V_T - \underline{\epsilon}_T \\ &= V_T + \epsilon_E^* - \frac{\left(\epsilon_E^* - \underline{\epsilon}_T\right)\left(V_T + \epsilon_E^*\right)}{V_T + V_A + S + \epsilon_E^*} - V_T - \underline{\epsilon}_T \\ &= \frac{\left(\epsilon_E^* - \underline{\epsilon}_T\right)\left(V_A + S\right)}{V_T + V_A + S + \epsilon_E^*} \\ &> 0,\end{aligned} \tag{32}$$

establishing the proposition. Next, suppose the median target shareholder's valuation is known (ϵ_E^* is constant) and consider how the premium changes as V_A increases. Assume both $\underline{\epsilon}_A$ and S are nondecreasing in V_A. First assume V_A is small enough such that $\underline{\epsilon}_J < \epsilon_E^*$. Rewrite equation 32 as

$$\tilde{P}_J I - P_T = \left(1 - \frac{\epsilon_E^* - \underline{\epsilon}_J}{V_T + V_A + S + \epsilon_E^*}\right)\left(V_T + \epsilon_E^*\right) - V_T - \underline{\epsilon}_T.$$

Then, as V_A increases, $V_T + V_A + S + \epsilon_E^*$ increases while $\epsilon_E^* - \underline{\epsilon}_J$ does not increase (but is still positive). Thus $\left(1 - \frac{\epsilon_E^* - \underline{\epsilon}_J}{V_T + V_A + S + \epsilon_E^*}\right)$ increases and hence $\tilde{P}_J I - P_T$. When V_A increases to a critical value such that $\underline{\epsilon}_J = \epsilon_E^*$, $\tilde{P}_J I - P_T = \epsilon_E^* - \underline{\epsilon}_T$. If V_A increases further beyond that, $\tilde{P}_J I - P_T$ stays constant. $\square$

Proof of Lemma 1: To prove the first part of the lemma, suppose the conclusion is false, i.e., that $\tilde{P}_J \geq V_T + V_A + S - C^* + \epsilon_C^*$, so that $C^* = V_T + \epsilon_C^*$. After a successful cash offer, original shareholders of the acquiring firm for whom $V_A + V_T + S + \epsilon_A - C^* < P_J$ want to sell their shares. The value of their shares is $P_J \tilde{F}_A(\min\{(P_J - V_A - V_T - S + C^*), \bar{\epsilon}_A\})$. Substituting for P_J and C^*, the value of their shares is at least

$$\begin{aligned}\left(V_T + V_A + S - C^* + \epsilon_C^*\right)\tilde{F}_A\left(\min\{\epsilon_C^*, \bar{\epsilon}_A\}\right) &= (V_A + S)\tilde{F}_A\left(\min\{\epsilon_C^*, \bar{\epsilon}_A\}\right) \\ &\geq (V_A + S)\tilde{F}_A\left(\min\left\{\epsilon_T^l, \bar{\epsilon}_A\right\}\right).\end{aligned}$$

On the demand side, shareholders of the original target for whom $V_A + V_T + S + \epsilon_T - C^* > P_J$ wish to buy shares in the joint firm, and they have cash not exceeding $C^* = V_T + \epsilon_C^* \leq$

$V_T + \epsilon_T^h$ to invest. Thus, equating total demand with the value of the shares supplied yields $\left(V_T + \epsilon_T^h\right) \geq (V_A + S)\tilde{F}_A\left(\min\left\{\epsilon_T^l, \bar{\epsilon}_A\right\}\right)$, contradicting the lemma's premise, thus establishing the first part of the lemma. The proof for the second part of the lemma is given in the text. $\square$

Proof of Proposition 2: To prove the first statement, suppose instead that $C^* \leq P_T = V_T + \underline{\epsilon} < V_T + \epsilon_C^*$. Then the median target shareholder must hold the joint firm, i.e., equation (11) (i) must hold. Note that $(V_T + V_A + S - C + \epsilon_C^*)\, C$ increases in C for $C \in [0, P_T]$ under $V_A > P_T - S$. Therefore, from equation (11) (i),

$$\begin{aligned} V_T + \epsilon_C^* \;=\; \frac{V_T + V_A + S - C^* + \epsilon_C^*}{P_J} C^* &\leq \frac{V_T + V_A + S - P_T + \epsilon_C^*}{P_J} P_T \\ &= \frac{V_A + S + \epsilon_C^* - \underline{\epsilon}_T}{P_J}\left(V_T + \underline{\epsilon}_T\right) \\ &\leq \frac{V_A + S + \epsilon_C^* - \underline{\epsilon}}{V_T + V_A + S + \underline{\epsilon}_T - C^*}\left(V_T + \underline{\epsilon}_T\right), \end{aligned}$$

where the first equality follows from $P_T = V_T + \underline{\epsilon}_T$ and the second follows from $P_J \geq V_T + V_A + S + \min\left(\underline{\epsilon}_T, \underline{\epsilon}_A\right) - C^*$ and $\underline{\epsilon}_T \leq \underline{\epsilon}_A$. From this, we have

$$\begin{aligned} C^* &\geq V_T + V_A + S + \underline{\epsilon}_T - \frac{V_T + \underline{\epsilon}_T}{V_T + \epsilon_C^*}\left(V_A + S + \epsilon_C^* - \underline{\epsilon}_T\right) \\ &= P_T + \left(\epsilon_C^* - \underline{\epsilon}_T\right)\frac{V_A + S - P_T}{V_T + \epsilon_C^*} \;\geq\; P_T, \end{aligned}$$

a contradiction.

To prove the second statement, examine equation (11) (i):

$$\begin{aligned} C^* \;=\; \left(V_T + \epsilon_C^*\right)\frac{P_J}{V_T + V_A + S - C + \epsilon_C^*} &\geq \left(V_T + \epsilon_C^*\right)\frac{V_T + V_A + S - C + \underline{\epsilon}_T}{V_T + V_A + S - C + \epsilon_C^*} \\ &= \left(V_T + \epsilon_C^*\right) - \frac{\left(\epsilon_C^* - \underline{\epsilon}_T\right)\left(V_T + \epsilon_C^*\right)}{V_T + V_A + S - C + \epsilon_C^*} \\ &\geq \left(V_T + \epsilon_C^*\right) - \frac{V_T + \epsilon_C^*}{V_A + S}\left(\epsilon_C^* - \underline{\epsilon}_T\right). \end{aligned}$$

Rearranging, we have

$$C^* - \left(V_T + \epsilon_C^*\right) \geq -\frac{V_T + \epsilon_C^*}{V_A + S}\left(\epsilon_C^* - \underline{\epsilon}_T\right).$$

Taking limits on both sides yields

$$\lim_{\frac{V_T + \epsilon_T^h}{V_A + S}\left(\epsilon_T^h - \underline{\epsilon}_T\right) \to 0} C^* - \left(V_T + \epsilon_T^*\right) \geq 0.$$

However, because $C^* \leq (V_T + \epsilon_C^*)$, we also have $\lim_{\frac{V_T+\epsilon_T^h}{V_A+S}\left(\epsilon_T^h - \epsilon_T\right)\to 0} C^* - (V_T + \epsilon_C^*) \leq 0$. Thus, the relationship must hold as an equality. $\square$

Proof of Lemma 2: Note that for all $\epsilon_E \in \left[\epsilon_T^l, \epsilon_T^h\right]$, $\pi_{AM}^E(\epsilon_E) - \pi_{AM} = F_T(\epsilon_E)\,\Pi(\epsilon_E)$, where

$$\begin{aligned}\Pi(\epsilon_E) &= S + \frac{\epsilon_A^M - \epsilon_E}{V_T + V_A + S + \epsilon_E}(V_A + S) - \epsilon_A^M \\ &= S + \left(\frac{\epsilon_A^M}{V_T + V_A + S + \epsilon_E} - \frac{1}{\frac{V_T+V_A+S}{\epsilon_E} + 1}\right)(V_A + S) - \epsilon_A^M. \end{aligned} \tag{33}$$

Note that if $\epsilon_E^* = \epsilon_T^l$, then $F_T(\epsilon_E^*) = 0$ and $\pi_{AM}^E(\epsilon_E^*) = \pi_{AM}$, this proves the "if" part by contradiction. We next prove the "only if" part by contradiction. Suppose instead that $\pi_{AM}^E(\epsilon_E^*) = \pi_{AM}$, then $\Pi(\epsilon_E^*) = 0$. Equation (33) shows $\Pi(\epsilon_E)$ strictly falls in ϵ_E. Then $\Pi\left(\epsilon_E = \frac{\epsilon_T + \epsilon_E^*}{2}\right) > 0$. As $F_T\left(\epsilon_E = \frac{\epsilon_T+\epsilon_E^*}{2}\right) > 0$, we have $\pi_{AM}^E\left(\epsilon_E = \frac{\epsilon_T+\epsilon_E^*}{2}\right) > \pi_{AM}$. This contradicts the optimality of ϵ_E^*. $\square$

Proof of Lemma 3: Note that for all $\epsilon_E \in \left[\epsilon_T^l, \epsilon_T^h\right]$, we have

$$\begin{aligned}\pi_{AM}^E(\epsilon_E) - \pi_{AM} &\geq F_T(\epsilon_E)\left[S + \frac{\epsilon_A^M - \epsilon_E}{V_T + V_A + S + \epsilon_E}(V_A + S) - \epsilon_A^M\right] \\ &= F_T(\epsilon_E)\left[\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon_E}S + \frac{\left(\epsilon_A^M - \epsilon_E\right)V_A}{V_T + V_A + S + \epsilon_E} - \epsilon_A^M\right] \\ &= F_T(\epsilon_E)\left[\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon_E}S - \frac{V_T + S + \epsilon_E}{V_T + V_A + S + \epsilon_E}\epsilon_A^M - \frac{V_A}{V_T + V_A + S + \epsilon_E}\epsilon_E\right] \\ &= F_T(\epsilon_E)\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon_E}\left(S - \frac{V_T + S + \epsilon_E}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M - \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\epsilon_E\right).\end{aligned} \tag{34}$$

To prove the first part of the proposition, note that if $S \geq \frac{V_T+S+\epsilon_T^h}{V_T+V_A+S+\epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T+V_A+S+\epsilon_A^M}\epsilon_T^h$, then $\pi_{AM}^E\left(\epsilon_E = \frac{1}{2}\epsilon_T^l + \frac{1}{2}\epsilon_T^h\right) - \pi_{AM} > 0$. As $\pi_{AM}^E(\epsilon_E^*) \geq \pi_{AM}^E\left(\epsilon_E = \frac{1}{2}\epsilon_T^l + \frac{1}{2}\epsilon_T^h\right)$, it follows that $\pi_{AM}^E(\epsilon_E^*) > \pi_{AM}$. To prove the second part, note that if $\underline{\epsilon}_J \leq \epsilon_T^l$, then (34) holds with equality. If $S \leq \frac{V_T+S+\epsilon_T^l}{V_T+V_A+S+\epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T+V_A+S+\epsilon_A^M}\epsilon_T^l$, then $\pi_{AM}^E(\epsilon_E) - \pi_{AM} \leq 0$ for all $\epsilon_E \in \left[\epsilon_T^l, \epsilon_T^h\right]$. Thus, $\pi_{AM}^E(\epsilon_E^*) - \pi_{AM} \leq 0$. As $\pi_{AM}^E(\epsilon_E^*) - \pi_{AM} \geq 0$, we have $\pi_{AM}^E(\epsilon_E^*) - \pi_{AM} = 0$. Thus, by Lemma 2, the acquirer's optimal offer is never approved. $\square$

Proof of Lemma 4: Refer to equation (19) and note that $(V_T + S - C)$ strictly decreases in C while pro$(\epsilon_C(C) \geq \epsilon_T^*)$ is continuous in C. The proof follows from a similar argument as that of Lemma **2**. $\square$

Proof of Lemma 5: Refer to equation (21). If $S > \epsilon_T^l$, $\pi_{AM}^C \left(C = V_T + \frac{1}{2}\epsilon_T^l + \frac{1}{2}S\right) - \pi_{AM} > 0$. As $\pi_{AM}^C\left(C^*\right) \geq \pi_{AM}^C \left(C = V_T + \frac{1}{2}\epsilon_T^l + \frac{1}{2}S\right)$, it follows that $\pi_{AM}^C\left(C^*\right) > \pi_{AM}$. The result then follows from Lemma 4. □

Proof of Proposition 3: For all $\epsilon_T^* \in \left[\epsilon_T^l, \epsilon_T^h\right]$, define $\pi_{AM}^C(\epsilon_T^*) \equiv \pi_{AM}^C(C = V_T + \epsilon_T^*)$. Then, for all ϵ_T^*, we have

$$\begin{aligned}
\pi_{AM}^C(C^*) - \pi_{AM}^E(I^*) &\geq F_T\left(\epsilon_T^*\right)\left[\epsilon_A^M - \epsilon_T^* - \frac{\epsilon_A^M - \epsilon_T^*}{V_T + V_A + S + \epsilon_T^*}\left(V_A + S\right)\right] \\
&= F_T\left(\epsilon_T^*\right)\left(\epsilon_A^M - \epsilon_T^*\right)\frac{V_T + \epsilon_T^*}{V_T + V_A + S + \epsilon_T^*},
\end{aligned} \tag{35}$$

where (15) and (21) are used. This expression is nonnegative for all $\epsilon_T^* \in \left[\epsilon_T^l, \epsilon_T^h\right]$ if $\epsilon_A^M \geq \epsilon_T^h$, and in particular, $\pi_{AM}^C\left(\epsilon_E^*\right) \geq \pi_{AM}^E\left(\epsilon_E^*\right)$. As $\pi_{AM}^C\left(\epsilon_C^*\right) \geq \pi_{AM}^C\left(\epsilon_E^*\right)$, we have $\pi_{AM}^C\left(\epsilon_C^*\right) \geq \pi_{AM}^E\left(\epsilon_E^*\right)$. This proves the first part. To prove the second part, note that if the median target shareholder does not hold the joint firm (e.g., if $\epsilon_T^h \leq \underline{\epsilon}_A$), then (35) holds with equality:

$$\pi_{AM}^C(\epsilon_T^*) - \pi_{AM}^E(\epsilon_T^*) = F_T\left(\epsilon_T^*\right)\left(\epsilon_A^M - \epsilon_T^*\right)\frac{V_T + \epsilon_T^*}{V_T + V_A + S + \epsilon_T^*}.$$

Thus, if $\epsilon_A^M \leq \epsilon_T^l$, $\pi_{AM}^C\left(\epsilon_T^*\right) \leq \pi_{AM}^E\left(\epsilon_T^*\right)$ for all $\epsilon_T^* \in \left[\epsilon_T^l, \epsilon_T^h\right]$, and hence, $\pi_{AM}^C\left(\epsilon_C^*\right) \leq \pi_{AM}^E\left(\epsilon_C^*\right)$. As $\pi_{AM}^E\left(\epsilon_E^*\right) \geq \pi_{AM}^E\left(\epsilon_C^*\right)$, we have $\pi_{AM}^E\left(\epsilon_E^*\right) \geq \pi_{AM}^C\left(\epsilon_C^*\right)$. □

Proof of Proposition 4: Follows directly from Result 3. □

Proof of Lemma 6: We have from equations (9) and (2) that

$$\begin{aligned}
P_J - P_A &= \frac{V_T + V_A + S + \underline{\epsilon}_J}{V_T + V_A + S + \epsilon_E^*}\left(V_A + S\right) - \left(V_A + \underline{\epsilon}_A\right) \\
&= \left(V_A + S\right) - \frac{\epsilon_E^* - \underline{\epsilon}_J}{V_T + V_A + S + \epsilon_E^*}\left(V_A + S\right) - \left(V_A + \underline{\epsilon}_A\right) \\
&= S - \underline{\epsilon}_A - \frac{\epsilon_E^* - \underline{\epsilon}_J}{V_T + V_A + S + \epsilon_E^*}\left(V_A + S\right) \\
&\leq S - \underline{\epsilon}_A - \frac{\epsilon_E^* - \max\left(\underline{\epsilon}_A, \underline{\epsilon}_T\right)}{V_T + V_A + S + \epsilon_E^*}\left(V_A + S\right).
\end{aligned} \tag{36}$$

Note the right-hand-side of equation (36) is decreasing in ϵ_E^* for $\epsilon_E^* \in \left[\epsilon_T^l, \epsilon_T^H\right]$, thus

$$P_J - P_A \leq S - \underline{\epsilon}_A - \frac{\epsilon_T^l - \max\left(\underline{\epsilon}_A, \underline{\epsilon}_T\right)}{V_T + V_A + S + \epsilon_T^l}\left(V_A + S\right),$$

which, combined with the condition $\underline{\epsilon}_A \geq \underline{\epsilon}_T$, establishes the lemma. $\square$

Proof of Proposition 5: Let ϵ_E^* be the median target shareholder value corresponding to the equity offer. If the success probability is strictly positive, Lemma 2 and equation (34) in the proof of Lemma 3 yield:

$$S > \frac{V_T + S + \epsilon_E^*}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\epsilon_E^*. \tag{37}$$

Next, consider a case in which $\underline{\epsilon}_A = \underline{\epsilon}_T \equiv \underline{\epsilon}$, and consider the limiting case in which ϵ_T^h is arbitrarily close to $\underline{\epsilon}$. Then ϵ_E^* approaches $\underline{\epsilon}$. Then the RHS of (37) equals

$$\begin{aligned}\frac{V_T + S + \underline{\epsilon}}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\underline{\epsilon} &= \frac{(V_T + S)\,\epsilon_A^M + \underline{\epsilon}\epsilon_A^M + \underline{\epsilon}V_A}{V_T + V_A + S + \epsilon_A^M} \\ &< \frac{(V_T + S)\,\underline{\epsilon} + \epsilon_A^M\underline{\epsilon} + V_A\underline{\epsilon}}{V_T + V_A + S + \epsilon_A^M} \;=\; \underline{\epsilon}.\end{aligned}$$

It then follows that there exists S such that the RHS of (37) $< S < \underline{\epsilon}$. In light of (37), an equity offer can be made that maximizes the payoff of the acquirer's management and has a strictly positive probability of success. Furthermore, in light of Proposition 3, the equity offer is preferred to a cash offer. In addition, from (7), we have $R_E < 0$. $\square$

Proof of Proposition 6: Since the marginal holder of the joint firm in a cash offer has a private valuation of at least $\underline{\epsilon}_A$, the price of the joint firm satisfies

$$P_J \geq V_A + V_T + S - C + \underline{\epsilon}_A = P_A + V_T + S - C.$$

Using (19) and Lemma 4, we have

$$V_T + S - C > 0.$$

Combining these inequalities yields $P_J > P_A$. $\square$

Proof of Proposition 7 and 8: Follows from the same arguments as in the proofs of Result 4 and Corollary 1. $\square$

Proof of Proposition 9: Follows directly from equation (24). Note that the condition in the third bullet of the proposition is $S < \hat{\epsilon}$, which differs from the condition $S < \hat{\epsilon} + 3\alpha$, as in the second line of equation (24) because in the proposition we consider what happens when α increases from zero. $\square$

Proof of Proposition 11. We use (28) to prove the proposition. To show D monotonically decreases in ρ, suppose $1 \geq \rho_1 > \rho_2 \geq 0$. Denote the corresponding values of κ by κ_1 and κ_2, and those of $\underline{\epsilon}_J$ by $\underline{\epsilon}_{J,1}$ and $\underline{\epsilon}_{J,2}$. Note also that $\underline{\epsilon}$ is independent of ρ. Then (28) gives $\left(\underline{\epsilon}_{J,2} - \underline{\epsilon}\right)\left(\underline{\epsilon}_{J,2} + 2V + S\right) = \kappa_2\left(2\underline{\epsilon} - \underline{\epsilon}_{J,2}\right)$. The three terms $\underline{\epsilon}_{J,2} - \underline{\epsilon}$, $\underline{\epsilon}_{J,2} + 2V + S$, and $2\underline{\epsilon} - \underline{\epsilon}_{J,2}$ are positive. Thus, $\kappa_1 > \kappa_2$ yields $\left(\underline{\epsilon}_{J,2} - \underline{\epsilon}\right)\left(\underline{\epsilon}_{J,2} + 2V + S\right) < \kappa_1\left(2\underline{\epsilon} - \underline{\epsilon}_{J,2}\right)$. Next, note $\underline{\epsilon}_{J,1}$ satisfies $\left(\underline{\epsilon}_{J,1} - \underline{\epsilon}\right)\left(\underline{\epsilon}_{J,1} + 2V + S\right) = \kappa_1\left(2\underline{\epsilon} - \underline{\epsilon}_{J,1}\right)$. We now show that $\underline{\epsilon}_{J,1} > \underline{\epsilon}_{J,2}$. Suppose instead that $\underline{\epsilon}_{J,1} \leq \underline{\epsilon}_{J,2}$. Then

$$\left(\underline{\epsilon}_{J,1} - \underline{\epsilon}\right)\left(\underline{\epsilon}_{J,1} + 2V + S\right) \leq \left(\underline{\epsilon}_{J,2} - \underline{\epsilon}\right)\left(\underline{\epsilon}_{J,2} + 2V + S\right) < \kappa_1\left(2\underline{\epsilon} - \underline{\epsilon}_{J,2}\right) \leq \kappa_1\left(2\underline{\epsilon} - \underline{\epsilon}_{J,1}\right),$$

which contradicts the condition $\left(\underline{\epsilon}_{J,1} - \underline{\epsilon}\right)\left(\underline{\epsilon}_{J,1} + 2V + S\right) = \kappa_1\left(2\underline{\epsilon} - \underline{\epsilon}_{J,1}\right)$. Therefore, $\underline{\epsilon}_J$ monotonically increases in ρ. In light of (30) and the fact that $\underline{\epsilon}$ is independent of ρ, D monotonically increases in ρ. Next, note that (28) yields $\underline{\epsilon}_J = 2\underline{\epsilon}$ if $\rho = 1$. Because $\underline{\epsilon}_J$ monotonically increases in ρ as we have shown above, $\underline{\epsilon}_J < 2\underline{\epsilon}$ for all $\rho < 1$, which, combined with (30), establishes the rest of the proof. □

References

[1] Andrade Gregor, Mark Mitchell and Erik Stafford, 2001, "New Evidence and Perspectives on Mergers," *Journal of Economic Perspectives*, 15, 103-120.

[2] Bagwell, Laurie Simon, 1991, "Share Repurchase and Takeover Deterrence," *Rand Journal of Economics,* 22, 72-88.

[3] Bagwell, Laurie Simon, 1992, "Dutch Auction Repurchases: An Analysis of Shareholder Heterogeneity," *Journal of Finance*, Vol. 47, No. 1, pp. 71-105.

[4] Berger, Phillip G. and Eli Ofek 1995, "Diversification's effect on firm value," *Journal of Financial Economics*, 37, 39-65.

[5] Betton, Sandra, Espen Eckbo, and Karin Thorburn, 2008, "Corporate Takeovers," in B. E. Eckbo (ed.), *Handbook of Corporate Finance: Empirical Corporate Finance, Volume 2,* North-Holland/Elsevier, Handbooks in Finance Series, Chapter 15, 291-430.

[6] Boone, Audra, and Harold Mulherin, 2007, "How Are Firms Sold?" *Journal of Finance*, 62, 847-75.

[7] Chatterjee, Sris, Kose John and An Yan, 2012, "Takeovers and Divergence of Opinion", *Review of Financial Studies*, 25, 227-277.

[8] Chen, Joseph, Harrison Hong and Jeremy C. Stein, 2002, "Breadth of Ownership and Stock Returns," Journal of Financial Economics, 66, 171-205.

[9] Fishman, Michael, 1988, "A Theory of Preemptive Takeover Bidding," *RAND Journal of Economics,* 19(1): 88-101.

[10] Gomes, Armando, 2001, "Takeovers, Freezeouts, and Risk Arbitrage," *Working Paper,* University of Pennsylvania.

[11] Graham, John R., Michael L. Lemmon and Jack G. Wolf, 2002, "Does Corporate Diversification Destroy Value," *Journal of Finance, 57, 695-720*

[12] Hackbarth, Dirk and Jianjun Miao, 2007, "The Timing and Returns of Mergers and Acquisitions in Oligopolistic Industries," Working Paper, Boston University.

[13] Harris, Milton and Artur Raviv, 1993, "Differences of Opinion Make a Horse Race," *Review of Financial Studies*, 6, 473-506.

[14] Lamont, Owen and Christopher Polk, 2001, "The Diversification Discount: Cash Flows versus Returns," *Journal of Finance*, 56, 1693-1721.

[15] Malmendier, Ulrike, Enrico Moretti and Florian Peters, 2012, "Winning by Losing: Evidence on the Long-Run Effects of Mergers", working paper.

[16] Masulis, Ronald, Peter Swan and Brett Tobiansky, 2012, "Do Wealth Creating Mergers and Acquisitions Really Hurt Acquirer Shareholders?", AFA 2012 Chicago Meetings Paper.

[17] Miller, Edward, 1977, "Risk, Uncertainty, and Divergence of Opinion," *Journal of Finance*, 32, 1151-1168.

[18] Moeller, Sara, Frederik Schlingemann, and Rene Stulz, 2005, "Wealth Destruction on a Massive Scale? A Study of Acquiring-Firm Returns in the Recent Merger Wave" *Journal of Finance*, 757-782.

[19] Moeller, Sara, Frederik Schlingemann, and Rene Stulz, 2007, "How do Diversity of Opinion and Information Asymmetry Affect Acquirer Returns?" *Review of Financial Studies*, Vol. 20, No. 6, pp. 2047-2078.

[20] Morris, Stephen, 1996, "Speculative Investor Behavior and Learning," *Quarterly Journal of Economics*, 111, 1111-1133.

[21] Savor, Pavel and Qi Lu, 2009, " Do Stock Mergers Create Value for Acquirers?" *Journal of Finance*, 64, 10611097.

[22] Shleifer, Andrei and Robert Vishny, 2003, "Stock Market Driven Acquisitions," *Journal of Financial Economics*, 70, 295-311.