Are the facts of UK inflation persistence to be explained by nominal rigidity or changes in monetary regime?[*]

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Abstract

It has been widely argued that inflation persistence since WWII has been widespread and durable and that it can only be accounted for by models with a high degree of nominal rigidity. We examine UK post-war data and find that the varying persistence it reveals is largely due to changing monetary regimes and that models with moderate or even no nominal rigidity are best equipped to explain it.

Keywords: inflation persistence, New Keynesian, New Classical, nominal rigidity, monetary regime shifts

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1 Introduction

Inflation persistence has been widely noted in the post-war period. Together with other facts of macroeconomic behaviour, notably output persistence, it has motivated the search for dynamic general equilibrium models that could account for such persistence. At the heart of models of this sort in current widespread use is nominal rigidity, or price and inflation stickiness, often modelled by contracts of the sort suggested by Calvo (1983) with a backward-looking element due to indexation (or in some versions rule of thumb behaviour by price setters unable to set their prices optimally). DSGE models with such a Phillips Curve are exemplified by Christiano et al (2005) and Smets and Wouters (2003); they have been dubbed 'New Keynesian' or 'New NeoKeynesian Synthesis' models. According to this line of theorising inflation persistence can be thought of as largely 'engineered into' the structure of the economy by the specification of the Phillips Curve itself. It should therefore be expected to be fairly constant with little effect from any changes in monetary regime. By contrast there is an alternative line of theorising going back to Lucas (1976) that would argue very differently. On this view, inflation persistence is reduced or final form behaviour reflecting the joint behaviour of forcing (error and other exogenous) processes that have natural persistence, a DSGE model with perhaps limited or even no nominal rigidity, and a monetary regime that may vary with political choices and perceptions. This final form behaviour will vary with regime and will not necessarily generate high persistence in all regimes.

This difference of approach has spawned a large body of empirical work examining the joint facts of inflation persistence and regime shift. Results have varied widely partly because of the difficulty of pinning down the nature and frequency of regime shifts; in general the more frequent the shifts, the more variable and the lower the persistence found. For the US Pivetta and Reis (2007) show that one can find a case in the univariate inflation data alone for there being constancy of inflation persistence from the 1960s to the present day; thus one cannot reject the null of constancy. Equally they agree that one cannot reject the null of a moderate decline as found by Cogley and Sargent (2002).

From the DSGE side there has also been much work on testing the capacity of various models to mimic among other things the facts of inflation persistence, in the form of the impulse response function of inflation to shocks — for example again Christiano et al (2005), Smets and Wouters (2003). At this stage the consensus favours the DSGE models with a fair degree of built-in rigidity; but there is still much work to be done. In particular, most attention has inevitably been paid to the main samples of data used rather than asking whether the models are robust across subsamples. An interesting question that has been largely uninvestigated is whether these models can pick up changes in the impulse response functions that could have been triggered by shifts in monetary regime.

It would clearly be most helpful for the investigation of these issues if one could achieve a reasonable agreement on what monetary regimes were in existence and when. Then one could separate the data into the relevant subsamples and estimate time-invariant time-series processes for each, thus establishing the facts of inflation persistence in each episode. Then also one could modify various contending versions of an appropriate DSGE model and test which of these versions could best account for the facts of each episode. This should enable one to answer the questions; which model version can best explain inflation persistence and are there model versions that cannot explain it at all? While there will still be many other facts that one would like such models to explain and therefore many further fences for these models to fall at, least we could have made some progress, in a Popperian way (Popper, 1934), in removing some model versions from contention in so far as they fail over inflation persistence.

It turns out that UK data is an answer to this implicit prayer. Whereas it has proved hard to reach agreement on what monetary regimes were in place in the US and indeed whether there was ever any change at all (except briefly at the start of the 1980s with the experiment in the control of bank reserves), for the UK there have been several well-documented changes in monetary regime. Furthermore it is possible, as we will show, to back up the massive documentary evidence econometrically.

Thus in this paper we focus on the phenomenon of inflation persistence in the UK over the post-war period. We begin with the facts of regime change, the sine qua non of our methods here. We review the shifts between fixed and floating exchange rates and within the latter between different sorts of monetary and other methods of inflation control. We test our documented split of regimes using a method recently suggested by Qu and Perron (2007) and we find reasonable support for our proposed splits. We are then able to proceed to the next stage which is to estimate the facts of inflation persistence in each episode; we proceed as simply as possible, estimating a parsimonious univariate ARMA for each. As one would expect in such subsamples the inflation process is clearly stationary (a main reason for nonstationarity is after all regime shift); furthermore we know from the DSGE models we set up that the final form of the inflation process will be an ARMA of finite order. We then use the parameters of this ARMA and

its implied impulse response function to assess the degree of persistence.

We then turn to the question of how much nominal rigidity is needed to account for the persistence revealed in each episode. We take a standard DSGE model of the open economy with exogenous capital and inject into it different degrees of nominal rigidity; we follow the widely-used procedure of taking a 'stripped down' model, where the Euler equations are converted into a forward-looking IS curve, and the remainder of the model consists of the equations for the monetary or other inflation-control regime in place together with the Phillips Curve (and its varying degree of nominal rigidity). We test our different model versions by asking whether each in turn could have generated the patterns of persistence we find in the actual data. To do this we generate the sampling variability within the model under each regime by the method of bootstrapping the model's estimated residuals; this permits us to find the statistical distribution of the $ARMA$ parameters in the inflation regression under the null hypothesis of each model and thus to reject or accept each model. We can also compare the impulse response functions we find in the data with the 95% bounds generated by each model; this test essentially replicates the other one in a more transparent way.

To anticipate our conclusions, first we do not find that inflation persistence is a stylised constant; it appears largely to disappear at various points in the post-war UK, notably most recently; this favours the view that this is indeed connected to several changes in monetary regime, with different regimes exhibiting very different degrees of persistence. Second, we find that while high stickiness can account best for some regimes and low stickiness best for others, the best overall model across all regimes is one with minimum stickiness.

In section 2 therefore we review earlier work on inflation persistence and its measurement. In section 3 we estimate $ARMA$ models for UK data in the various post-war regimes we identify. In section 4 we set out our various models for each monetary regime, calibrate and fit them to the data, to find the implied model errors later to be used in bootstrapping. In section 5 we carry out the bootstrap tests of the models. Section 6 concludes.

2   Inflation persistence and its measurement

A large econometric literature has found that post-war US inflation exhibits very high persistence, approaching that of a random walk process. Given similar evidence for other OECD countries, many macroeconomists have concluded that high inflation persistence is a 'stylised fact' and have proposed varied microeconomic interpretations. Roberts (1998), Ball (2000), Ireland (2000), Mankiw and Reis (2002), Sims (2001) and Woodford (2001) assume that private agents face information-processing constraints. Buiter and Jewitt (1981), Fuhrer and Moore (1995), Fuhrer (2000), Calvo, Celasun and Kumhof (2001), Christiano, Eichenbaum and Evans (2005) assume that high inflation persistence results from the structure of nominal contracts. Others like Rotemberg and Woodford (1997), Dittmar, Gavin and Kydland (2001) and Ireland (2003) generate the persistence through the data generating processes of the structural shocks hitting the economy. However, an alternative view is that the degree of inflation persistence is not an inherent structural characteristic of industrial economies, but in fact a function of the monetary policy regime (see also West, 1988).

Over the past decade we have observed substantial shifts in the monetary policy of a number of countries, particularly the widespread adoption of explicit inflation targets. There is a growing body of research supporting the view that the monetary regime in place has an impact on the persistence properties of inflation or in other words inflation persistence is not an inherent characteristic of industrial economies. Brainard and Perry (2000), Taylor (2000) and Kim, Nelson and Piger (2001) have found evidence that US inflation persistence during the Volcker-Greenspan era was substantially lower than during the previous two decades; Ravenna (2000) documents a large post-1990 drop in Canadian inflation persistence; Batini (2002) finds that UK and US inflation had no persistence during the metallic-standard era (prior to 1914), highest persistence during the 1970s and markedly lower persistence during the last decade.

As Nelson (2001) points out, monetary policy in the UK has undergone several regime changes over the last 50 years: from a fixed exchange rate with foreign exchange controls until 1972; to free-floating incomes policy with no domestic nominal anchor until 1978, followed by a system of monetary targeting until the mid-1980s; then back to exchange rate management, the period of 'shadowing' the Deutsche Mark, which culminated in the membership of the Exchange Rate Mechanism (ERM) from 1990-1992; finally since 1992, inflation targeting has been the official regime governing UK monetary policy, with interest rate decisions made by the UK government in collusion with the Bank of England up to May 1997 and after it by the Bank alone, under a new law mandating its procedures and target.

For the period as a whole, there have been large swings both in inflation and economic growth. Inflation was continuously in double digits during most of the 1970s, and returned there in the early 1980s and 1990s. Nelson (2001) documents that economic growth, which was already lower in the UK in comparison to its major trading partners in the 1960s, underwent a further slowdown after 1973, with partial recovery beginning only in the 1980s. There were recessions in 1972, 1974-75, 1979-81 and 1990-92. However, the disinflation of the early 1990s has been followed by a period of low and stable inflation and reasonably stable real GDP growth.

2.1 Measures of Persistence

A number of authors assume that inflation follows a stationary autoregressive process of order p $AR(p)$):

$$\pi_t = \mu + \sum_{j=1}^{p} \alpha_j \pi_{t-j} + \varepsilon_t$$

where ε_t is a serially uncorrelated but possibly heteroscedastic random error term. In order to facilitate the discussion that follows we first note that the above model may be reparameterised as:

$$\Delta \pi_t = \mu + \sum_{j=1}^{p-1} \delta_j \Delta \pi_{t-j} + (\rho - 1)\pi_{t-1} + \varepsilon_t$$

where

$$\rho = \sum_{j=1}^{p} \alpha_j$$

and

$$\delta_j = - \sum_{i=1+j}^{p} \alpha_i$$

In the context of this model persistence can be defined as the speed with which inflation converges to equilibrium after a shock in the disturbance term. Several measures of this have been proposed, including the sum of autoregressive coefficients, the spectrum at zero frequency,[1] the largest autoregressive root and the half life.[2]

Andrews and Chen (1994) argue that the cumulative impulse response function (CIRF) is a good way of summarising the information contained in the impulse response function,[3] and hence a good scalar measure of persistence. In a simple $AR(p)$ process, the CIRF is given by

$$CIRF = \frac{1}{1-\rho}$$

where ρ is the sum of the autoregressive coefficients. As there exists a monotonic relationship between CIRF and ρ it follows that one can use the sum of AR coefficients, $\rho = \sum \alpha_j$, as a scalar measure of persistence.[4]

An alternative measure of persistence widely used in the literature is given by the largest AR root γ, that is, the largest root of the characteristic equation

$$\lambda^K - \sum \alpha_j \lambda^{K-j} = 0$$

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[1] The spectrum at zero frequency is a well-known measure of the low frequency autocovariance of the series. For the $AR(p)$ process it is given by $h(0) = \frac{\sigma_\varepsilon^2}{(1-\rho)^2}$ where σ_ε^2 stands for the variance of ε_t. However, where one wants to test for changes in persistence over time, the use of spectrum at zero frequency becomes problematic as changes in persistence will be brought about not only by changes in ρ but also by changes in σ_ε^2.

[2] The half-life is defined as the number of periods for which the effect of a unit shock to inflation remains above 0.5. It is a very popular measure of persistence particularly in the literature that evaluates the persistence of deviations from the purchasing power parity equilibrium. See for example Murray and Papell (2002) and Rossi (2001). For criticsm of this measure of persistence see Pivetta and Reis (2007).

[3] Impulse Response Functions (IRF) are an intuitive way to interpret measures of inflation persistence. IRF gives the response of inflation at various future dates to a shock that occurs today. CIRF as the name suggests is the concept of cumulative impact of a shock and is well documented in Hamilton (1994).

[4] Andrews and Chen (1994) note that that CIRF and thus ρ may not be sufficient to fully capture all the shapes in the impulse response functions. For e.g. CIRF and ρ will not be able to distinguish between two series in which one exhibits a large initial increase and then a subsequent quick decrease in the IRF while the other exhibits a relatively small initial increase followed by a subsequent slow decrease in the IRF.

It is easy to show that in the distant future, the impulse response of inflation to a shock becomes increasingly dominated by the largest root, so the size of γ is a key determinant of how long the effects of the shock will persist. When $\gamma = 1$, the process is infinitely persistent since, given a shock, we expect inflation never to revert to its initial value. When $\gamma = 0$, inflation is white noise and there is no persistence. In between, $0 < \gamma < 1$, the higher is γ, then the longer — to first approximation — it will take for inflation to come back to the original level, after a shock.

However, Phillips (1991), Andrews (1993a), Andrews and Chen (1994) and Pivetta and Reis (2007), criticise this measure of persistence. The main point of criticism is that the shape of the IRF depends on all the roots of the equation, not just the largest one. Hence, this statistic is a very poor summary of the impulse response function. According to Andrews and Chen (1994) and Marques (2004) ρ is more informative than the largest AR root as a measure of overall persistence. Despite this drawback, the largest AR root is still widely used as a measure of persistence. Levin, Natalucci and Piger (2004) argue that the largest AR root has intuitive appeal as a measure of inflation persistence, as it determines the size of the impulse response, $\frac{\delta \pi_{t+j}}{\delta \varepsilon_t}$, as j grows large. The other reason being that an asymptotic theory has been developed and appropriate software is available so that it is quite easy to compute asymptotically valid confidence intervals for the corresponding estimates.

Levin, Natalucci and Piger (2004) also show how the volatility of inflation can be decomposed into two sources: one due to the variance of the shocks to the autoregression and the other due to the propagation of shocks through autoregressive dynamics. The measure they use is the ratio of the total variance of inflation series to the variance of shocks to the autoregression:

$$\frac{Var(\pi_t)}{Var(\varepsilon_t)}$$

When the ratio is only slightly above unity, then that is consistent with a white noise process for the inflation series. If the ratio is nearer or above 2.0 then it means that the volatility of inflation contains a substantial propagation component.

Our approach here employs the finite-order ARMA representation of a time-series, from which the above infinite order AR representation can be derived. We show below that our two structural models imply an ARMA form for the inflation solution. To test each, we will set it up as the null hypothesis; under this hypothesis the final form is an ARMA, hence it is the appropriate representation. In an ARMA, the AR parameters indicate the pure length of time that whatever effect is left, after the MA coefficients have worked themselves out, takes to die out. The MA coefficients may effectively 'cancel' the effect of persistence coming from the AR coefficients so that this effect left over is small — thus indicating low persistence.

In the analysis below we measure the degree of persistence of the inflation process directly from the impulse response function (IRF) estimated from the best-fitting ARMA process. To obtain a single measure of persistence from the IRF we find the first order autoregressive parameter ρ that generates an IRF closest by least squares to the actual IRF. As will be seen below there is virtually no ambiguity in this measure of persistence in the inflation IRFs we find here.

3   Estimating time-series models on UK data

We begin by looking at the empirical evidence on UK inflation, carefully separating the data into periods of different regimes. We note that 'persistence' is not entirely a clear concept. A stationary time-series will typically consist of AR and MA; we confine ourselves to linear processes since the role of non-linearity seems to be basically secondary in this context. Persistence could naturally refer to the AR roots, ignoring the MA, which by construction must end sharply. However, the MA component can be inverted and turned into an infinite-order AR; this property is of course exploited in forming the widely-used VAR representation. We may note that a variable can have high AR roots and yet the MA terms may eliminate the effect of a shock after a period or two thus giving the high roots nothing to 'work on,' so resulting in little or no persistence. Such appears to be the case with inflation in at least some of the periods we deal with.

According to the simple models within which we organise our thinking, the inflation process invariably has one or more AR roots, contributed by the exogenous error processes, as well as MA terms, which result from the same processes. These seem to be critical in determining the extent of persistence; the monetary regime effectively operates in this MA part, dampening or not the direct effect of shocks on inflation. Thus basically the AR roots in our model reflect the more-or-less constant persistence of

exogenous error processes, while the MA terms reflect the activity of monetary reactions in 'closing down' such persistence in inflation, or not, as the regime dictates. In estimation we allow the data to determine the best-fitting process.

We estimate the degree of persistence in UK data regime by regime, in line with our brief earlier discussion of regimes which we elaborate in section 4 below. Inflation is calculated as quarter-on-quarter inflation annualised. It turns out that in each regime inflation is a stationary process. Further, as inflation is quarter-on-quarter one has to take into account seasonality, which we have done by using seasonal dummies. We begin by testing for the regime breakpoints of our theory above and examining the differences in mean inflation across the regimes. We then estimate the best-fitting ARMA processes for each regime and draw out its impulse response characteristics.

Plainly, the question of regime breaks is of the utmost importance for our subsequent analysis. Our regime identification is supported by a wealth of narrative evidence, detailed below. Thus the break-up of Bretton Woods and the UK's shift into floating is a matter of historical record; as were both the introduction of monetary targeting in 1979 and of inflation targeting in 1992. The period of 'exchange rate targeting' from 1986 until the 1992 exit from the ERM is also well documented. Nevertheless, it could be questioned whether there was statistical evidence from the macro time-series supporting the existence of these regime breaks. For this purpose we look at the evidence from the three endogenous macro variables identified in our models: output, inflation and the short-term interest rate. We estimate a VAR in the stationarised values of each viz, $\Delta\log(\text{output})$, inflation and $\Delta(\text{interest rate})$. Using the method of Qu and Perron (2007), we split the sample into three overlapping 20-year sub-samples that each contain two breaks according to our narrative analysis; this split was for computational reasons as running the whole sample in one proved to be too computationally burdensome for the programme to solve. The sub-samples were 1965-85; 1975-95 and 1985-2003. We looked for breaks in both parameters and covariance matrices. The results are reported in Table 1 which shows when each regime ends and the 95% confidence interval.

	Assumed end of regime	Estimated	95% Confidence Interval	
			Lower	Upper
Bretton Woods (1965-85)	1970Q4	1972Q2	1972Q1	1975Q3
Incomes Policy (1965-85)	1978Q4	1981Q3	1978Q1	1982Q1
(1975-95)	1978Q4	1979Q3	1978Q2	1980Q1
Money Targeting (1975-95)	1985Q4	1990Q2	1988Q3	1990Q3
(1985-2003)	1985Q4	1990Q2	1990Q1	1990Q3
Exchange Rate (1985-2003) Targeting	1992Q3	1994Q1	1993Q1	1994Q3

Table 1: Qu-Perron Structural Break Test

These tests generally confirm the existence of the assumed breaks and place them reasonably close to the assumed break date. They place the end of regimes rather later than we have assumed, in all cases. However for Incomes Policy the date lies within the 95% confidence interval and for Exchange Rate Targeting it is within a couple of quarters of it. The main one where the evidence disagrees materially is on the break between Monetary Targeting and Exchange Rate Targeting where it puts it in 1988-1990 against the end of 1985 as assumed here. Thus it confirms the existence of a break from Monetary to Exchange Rate targeting but puts it two years later. Since Inflation targeting starts soon after, this would imply that the Exchange rate targeting regime was rather brief, effectively confined to the period of formal membership of the Exchange Rate Mechanism. On this particular point we decided to allow the narrative evidence to stretch the Exchange Rate targeting sample to include the previous couple of years where there is known to have been 'shadowing' of the ERM, with an expressed target for the sterling-deutschemark rate.

Table 2 shows the mean and standard deviation of inflation for the different regimes (annualised quarterly rates of change, in fractions per annum).

The high water mark of inflation both in mean and variance was the Incomes Policy period of the 1970s. This followed the relatively tranquil period of Bretton Woods; and it was in turn followed by the period of Monetary Targeting when inflation was brought down dramatically. During the Exchange Rate Targeting regime it fell further; this was a period containing a severe recession also. At its end there again followed a period of relative tranquillity, under the new Inflation Targeting regime.

The best-fitting ARMA equation for each regime was chosen under the criterion of parsimony. Start-

Regime	Mean	Standard Deviation
FUS/Bretton Woods	0.036518	0.033962
IP	0.134800	0.081785
MT	0.095506	0.073607
FGR/ERM	0.057422	0.042047
IT (RPI)	0.024982	0.025131
Full Sample	0.062063	0.064810

Table 2: Summary statistics of inflation (annualised quarterly rates, fractions per annum)

ing with ARMA(1,0) we first raised the order of MA by one and then that of the AR by one, and so on upwards, each time doing an F-test to test (at 99%) whether the more parsimonious model was a valid restriction. The order was raised only if we reject the null hypothesis of a valid restriction. These tests can be seen in Table 3, the resulting parameters in Table 4: and the IRFs in Figure 1. In all cases the ARMA was of maximum order two, while in three cases we selected AR(1).

F-Test for Restriction					
	FUS/Bretton Woods	IP	MT[5]	FGR/ERM	IT (RPI)
ARMA(1,0)→ARMA(1,1)	6.862936*	0.681275	19.298391*	0.003338	0.693967
ARMA(1,0)→ARMA(2,0)	1.282354	0.749222	7.106008*	0.013357	0.332882
ARMA(2,0)→ARMA(2,1)			0.103106		

Table 3: F-Tests to Find Best-Fitting ARMA

Different Monetary Regimes					
	FUS/Bretton Woods	IP	MT	FGR/ERM	IT (RPI)
AR(1)	-0.592304	0.727366	0.927892	0.623726	0.202142
	(0.138349)	(0.133271)	(0.038179)	(0.164163)	(0.155071)
MA(1)	0.952206		-0.997381		
	(0.066378)		(0.056533)		
R^2	0.352409	0.565766	0.596308	0.630215	0.697537
S.E. of regression	0.027094	0.053893	0.046768	0.025569	0.013821
AIC	-4.281325	-2.861028	-3.099847	-4.329306	-5.618613
SIC	-4.068176	-2.632006	-2.814374	-4.089336	-5.415864

N.B. Figures in parentheses are the standard errors.

Table 4: Best Fitting ARMAs for UK Monetary Policy Regimes

Below the IRFs in Figure 1 we show — where the ARMA order is higher than AR(1) — the first order autoregression, ρ, that fits best: the ρ whose IRF minimises the sum of least-squares deviations from the actual IRF. Summarising these results, we find very low persistence under Bretton Woods and again under Money Targeting and Inflation Targeting, but the two other regimes exhibit high persistence. We now turn to the specification and calibration of the New Keynesian and New Classical models within each regime we have identified.

4   The Structural Model — with New Keynesian or New Classical Phillips curve

The objective of this section is to set out simple models with varying price stickiness, derived from micro-foundations in for example the manner of Ireland (1994), Clarida, Gali and Gertler (1999) and McCallum and Nelson (1999, 2000). Appendix C shows the steps in detail, and derives a basic template for each model/regime; the models in the text are based on these. For simplicity we will distinguish between sticky-price 'New Keynesian' models, based on Calvo contracts and flexible-price 'New Classical' models with a simple one-quarter information lag. The root model is identical between New Keynesian and New Classical, apart from the Phillips Curve and the information assumptions (there is an information lag in the New Classical model only). Within the New Keynesian model we will distinguish in turn between

FUS (IRF) IP (IRF) MT (IRF)

—ARMA(1,1) —AR(1) [0.249]   —ARMA(1,0) —AR(1) [0.735]   —ARMA(1,1) —AR(1) [-0.062]

FGR (IRF) IT_RPI (IRF)

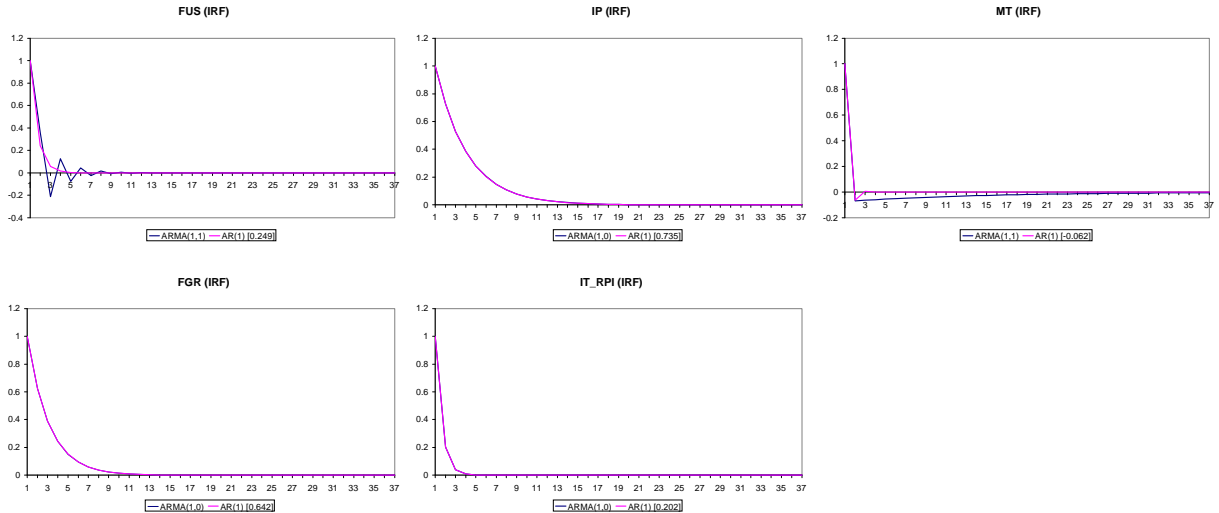—ARMA(1,0) —AR(1) [0.642]   —ARMA(1,0) —AR(1) [0.202]

Figure 1: Impulse Response Function (Best-Fitting ARMA, and closest AR(1) representation)

three degrees of stickiness: high (with a strong backward-looking element), medium (where backward and forward-looking elements are of similar size) and low (with the forward-looking element is dominant).

In all the models the first equation is the IS curve of the expectational variety that includes $E_t y_{t+1}$ as in Kerr and King (1996), McCallum and Nelson (1997) and Rotemberg and Woodford (1997).[6] This modification imparts a dynamic, forward-looking aspect to saving behaviour and leads to a model of aggregate demand that is tractable and also usable with a wide variety of aggregate supply specifications. This optimising IS function can be regarded as a transformation of the structural consumption Euler equation, with the market-clearing condition for output substituted into it; the error term captures stochastic movements in government spending, exports etc. In the case of fixed exchange rate regimes[7] we have an additional expenditure switching effect in the IS curve.

The second equation in the models is the New Keynesian or New Classical Phillips curve. The former is derived from Calvo contract price-setting with the addition of backward-looking indexation. The latter can be regarded as the equation of a clearing labour market equating the marginal product of labour with the Euler equation for labour supply, with a one-period information lag among households creating the inflation surprise term.

The last set of equations relate to monetary policy. Under fixed exchange rates the Euler equation for household choice of foreign versus home bonds creates the equation of uncovered interest parity; with the nominal exchange rate fixed, inflation at home changes the real exchange rate and this feeds into net exports and the real interest rate and so the IS curve. Under floating exchange rates we identify three variants of policy: one with no nominal anchor and an incomes policy which we model directly, one with monetary targeting and a demand for money, and one with direct setting of interest rates through a Henderson-McKibbin-Taylor rule ('inflation targeting').

Nelson (2000) provides estimates for the UK of the Taylor rule for several different monetary regimes in the period 1972-97; prior to the Bank of England receiving operational independence.[8] His results suggest that prior to 1992, it is difficult to characterise UK monetary policy using a standard Taylor rule. During these regimes policy makers were constantly changing the rules, what they were targeting and the targets themselves. In this chapter we impose the restrictions that we think existed in those periods.[9] Thus our models can be thought of as simple approximations of actual policy behaviour during

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[6] Fane (1985) and Koenig (1989, 1993a, 1993b) represent previous efforts in the same direction. However, they only show that some comparative-static properties of their models are like those of an IS-LM setup. Specifically, they do not develop dynamic equations analogous to IS and LM functions, as in McCallum and Nelson (1997). Auerbach and Kotlikoff (1995) derive IS and LM equations from an overlapping generations framework under the very restrictive assumption of rigid prices.

[7] The period of Bretton Woods and shadowing the Deutsch Mark.

[8] In a famous paper, Taylor (1993) showed that US monetary policy after 1986 is well characterised by a rule for the Federal Funds rate whereby the interest rate responds to output gap and inflation deviation from target. There has subsequently been an explosion of theoretical and empirical work in this area. See for example Clarida et al. (2000)

[9] For our choice of regime dates see Minford (1993), Nelson (2000) and Budd (2002).

each regime.

We set all the models out first with a New Classical Phillips Curve. These are our New Classical models. In a succeeding subsection we set out the New Keynesian equivalents.

4.1 Fixed Exchange Rate Regime (US) or Bretton Woods (1956:1 TO 1970:4)

Our first regime is the Bretton Woods fixed exchange rate system. This is not easy to model because of its progressive deterioration in the 1960s when 'one-off' exchange rate changes became commonplace means of adjustment. Another important factor causing change was the progressive dismantling of direct controls — including a relaxation of controls on international capital flows — which, while certainly adding to the potential macro-economic benefits from international economic activity, undoubtedly made fixed exchange rates inherently more difficult to sustain. Furthermore, countries within the system came to attach different priorities to inflation and unemployment as the immediate objective of policy. There was also disagreement about how the burden of domestic policy adjustment should be shared between surplus and deficit countries, including the US, the country of the anchor currency. The system eventually collapsed under the weight of the outflows from the US dollar, which, under the parity system, had to be taken into other countries' official reserves, on such a scale that the dollar's official convertibility into gold had eventually to be formally suspended in 1971.

Here we have made drastic simplifications, ignoring parity changes and assuming a high degree of capital mobility throughout. Equations (1) and $(3) - (7)$ when put together are the IS or demand side of the model; the errors entering here from a variety of exogenous shocks, apart from R_{FUSt}, are aggregated into u_{FUSt}.

$$\widetilde{y}_t = \gamma(E_t\widetilde{y}_{t+1}) - \phi(R_t - E_tP_{t+1} + P_t) + \lambda(E_tNX_{t+1}) + u_{FUS_t} \tag{1}$$

$$\widetilde{y}_t = \delta(P_t - E_{t-1}P_t) + v_{FUS_t} \tag{2}$$

$$NX_t = a_{FUS_0}Q_{FUS_t} + a_{FUS_1}y_{FUS_t}^F \tag{3}$$

$$Q_{FUS_t} = S_{FUS_t} + P_{FUS_t}^f - P_t \tag{4}$$

$$R_t = R_{FUS_t}^F + (E_tS_{FUS_{t+1}} - S_{FUS_t}) \tag{5}$$

$$S_{FUS_t} = \overline{S}_{FUS} \tag{6}$$

$$R_{FUS_t}^F = \rho_{FUS}R_{FUS_{t-1}}^F + \eta_{FUS_t} \tag{7}$$

$$u_{FUS_t} = \rho_{FUS_0}u_{FUS_{t-1}} + \varepsilon_{FUS_t} \tag{8}$$

$$v_{FUS_t} = \rho_{FUS_1}v_{FUS_{t-1}} + x_{FUS_t} \tag{9}$$

$$y_{FUS_t}^F = \rho_{FUS_4}y_{FUS_{t-1}}^F + \theta_{FUS_t} \tag{10}$$

$$P_{FUS_t}^f = \rho_{FUS_5}P_{FUS_{t-1}}^f + \kappa_{FUS_t} \tag{11}$$

In the equations above, $\widetilde{y}_t$ is the output gap defined as $\log GDP - \log GDPtrend$, R_t the nominal interest rate is the Bank of England base rate, P_t is the price level, NX_t is net exports, Q_{FUS_t} is the real exchange rate, S_{FUS_t} is the nominal exchange rate defined as £/\$, $P_{FUS_t}^f$ is the US price level (CPI), $y_{FUS_t}^F$ is US GDP which is used as a proxy for world income and $R_{FUS_t}^F$ is the US federal funds rate (nominal). Equations (1) and (2) are based on aggregate demand and supply specifications that are designed to reflect rational optimising behaviour on the part of the economy's private actors. Equation (1) is a forward looking open economy IS curve. The error term u_{FUS_t} can be interpreted as the demand shock to the economy which we have modelled as an $AR(1)$ process. Equation (2) is a standard New Classical Phillips Curve where v_{FUS_t} is the productivity shock modelled as an $AR(1)$. Equation (3) simply puts forth the idea that the net exports of a country is a function of the real exchange rate and the world income. If the real exchange rate appreciates or the world income is higher, then there would be a greater demand for the domestic exports. Equation (4) is the definition of real exchange rate and equation (5) is the Uncovered Interest Rate Parity (UIP) condition. Equation (6) simply states that the nominal exchange rate is fixed, as we are in a fixed exchange rate regime. We have modelled the world interest rate, GDP and prices as an $AR(1)$ process. The error terms in equation (7) to (11) are all $i.i.d.$

The solution for inflation is an ARMA whose order of both AR and MA components in principle depends on the number of error processes identified. Appendix C shows the method of derivation for this and other regimes, for both models.

4.2 Incomes Policy Regime (1971:1 to 1978:4)

Sterling was floated in June 1972.[10] 1972 was also the year of the Heath government's 'U-turn' in macro-economic policy. The view of the government was that it could stimulate output and employment through expansionary monetary and fiscal policies, while at the same time keeping inflation under control through statutory wage and price controls.[11] The opinion of the day was that the break-out of inflation in the 1970s largely reflected autonomous wage and price movements, and that the appropriate policy response was to take actions that exerted downward pressure on specific products, rather than to concentrate on a monetary policy response. Examples of non-monetary attempts to control inflation included statutory incomes policy announced in November 1972 and the voluntary incomes policy pursued by the Labour government from 1974; the extension of food subsidies in March 1974 budget; and the cuts in indirect taxation in the July 1974 mini-Budget.

From late 1973 policy makers did start paying heed to the growing criticism of rapid money growth that they had permitted. However, there was an unwillingness to make the politically unpopular decision of raising nominal interest rates. The Bank of England was given instructions from the Government that the growth of broad money, the Sterling M3 aggregate, was to be reduced — however, the nominal interest rates must not be increased. The result was the *'Corset,'* the introduction of direct quantitative control on £M3, which imposed heavy marginal reserve requirements if increases in banks' deposits exceeded a limit. While this control did result in a reduction in the observed £M3 growth, it did so largely by encouraging the growth of deposit substitutes, distorting £M3 as a monetary indicator and weakening its relationship with future inflation.[12] For the rest of the 1970s monetary policy often looked restrictive as measured by £M3 growth, but loose as measured by interest rates or monetary base growth.

In July 1976 targets were announced for £M3 monetary aggregate.[13] From then on UK had a monetary policy that reacted to monetary growth and to the exchange rate. Depreciation of the exchange rate in 1976 was a major factor that triggered a tighter monetary policy during 1976-1979. However, we must not over emphasise the monetary tightness as the nominal interest rate was cut aggressively – by more than 900 basis points from late 1976 to early 1978 — ahead of the fall in inflation from mid-1977 to late 1978. Reflecting the easier monetary policy, money base (£M0) growth, which had been reduced to single digits in the late 1977, rose sharply and peaked at more than 18% in July 1978; inflation troughed at 7.6% in October 1978 and continued to rise until May 1980, when it was 21%. Furthermore, the nominal Treasury bill rate from July 1976 to April 1979 averaged 9.32%. In real terms it was well below zero, indicating the continued tendency of the policy makers until 1978 to hold nominal interest rates well below the actual and prospective inflation rate.[14]

Nelson (2000) finds that the estimated long-run response of the nominal interest rate to inflation was well below unity during the 1970s. Moreover, the real interest rate was permitted to be negative for most of the period. These results suggest that UK monetary policy failed to provide a nominal anchor in the 1970s. However, we note that there was a determinate inflation rate during this period, even though there was clearly no orthodox monetary anchor. What we have chosen to do from a modelling viewpoint is treat Incomes Policy as the determinant of inflation and to assume that interest rates 'fitted in' with what the model dictated was necessary to achieve that inflation rate and the accompanying output rate. Plainly this is a drastic over-simplification since interest rates were independently set at quite inappropriate levels; however, introducing such contradictory monetary policy poses too much of a modelling challenge for this exercise — it could well be that there was such monetary indeterminacy, and incomes policy so incredible, that we were here in a 'non-Ricardian' period where fiscal policy was left to determine inflation. However exploring such possibilities lies well beyond the scope of this chapter.

$$\widetilde{y}_t = \gamma(E_t\widetilde{y}_{t+1}) - \phi(R_t - E_t\pi_{t+1}) + u_{IP_t} \tag{12}$$

$$\widetilde{y}_t = \delta(\pi_t - E_{t-1}\pi_t) + v_{IP_t} \tag{13}$$

$$\pi_t = \pi_{t-1}(1-c) + \tau_{IP_t} \tag{14}$$

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[10] The float of the exchange rate was announced on the 23 June 1972. See Bank of England (1972).

[11] From 1973 to 1980, the government periodically used the Supplementary Special Deposits Scheme, called the *'Corset,'* as a quantitative control on the expansion of the banks' balance sheets and therefore of the £M3 monetary aggregate.

[12] As Nelson (2000) points out it is likely that this served prinicipally as a device for restricting artificially the measured growth of £M3 without changing the monetary base or interest rates, rather than as a genuinely restrictive monetary policy measure. See also Minford (1993).

[13] The value of this target was 11% from May 1976 to April 1978 and 10% from May 1978 to April 1970. These are the mid-points of the successive targets announced for the annual £M3 growth.

[14] Judd and Rudebusch (1998) report average real interest rate for the US for the period 1970-78 to be 2 basis points. Hence, the phenomenon of low or negative real interest rates in the 1970s was more pronounced in the UK.

$$u_{IP_t} = \rho_{IP_0} u_{IP_{t-1}} + \varepsilon_{IP_t} \tag{15}$$

$$v_{IP_t} = \rho_{IP_1} v_{IP_{t-1}} + x_{IP_t} \tag{16}$$

$$\tau_{IP_t} = \rho_{IP_3} \tau_{IP_{t-1}} + \zeta_{IP_t} \tag{17}$$

In the equations above, π_t is the inflation quarter-on-quarter annualised and c is the incomes policy restraint. As before equation (12) is a forward looking IS curve and equation (13) is a New Classical Phillips curve. Equation (14) states that inflation at time t is set by incomes policy at some fraction of the actual inflation in period $t-1$ but subject to an error, the 'break-down' of policy, which we have modelled as an $AR(1)$. During this period there was a serious credibility problem. So, if the government came along and announced that it would cut inflation by 80 percent that simply would not be believable. However, if the government announced that it would cut inflation by say 20 percent then that would definitely be more credible and policy makers would be in a position to gradually get inflation expectations and hence inflation under control. Furthermore, it should be remembered that during this period there were no explicit targets. However, from policymakers' behaviour we do know that there existed implicit targets, and c helps us operationalise that. We have modelled the IS and PP forecast error as $AR(1)$ processes. ε_{IP_t}, x_{IP_t} and ζ_{IP_t} are all $i.i.d.$

Substituting equation (17) in (14) the solution for π_t in this regime is:

$$\pi_t = \{\rho_{IP_3} + 1 - c\}\pi_{t-1} - \rho_{IP_3}(1-c)\pi_{t-2} + \zeta_{IP_t} \tag{18}$$

The implied solution for inflation is an $AR(2)$.

4.3   Money Targeting Regime (1979:1 to 1985:4)

In 1979 inflation was rising rapidly from an initial rate of over 10 percent. The policy of wage controls that had been used to hold down inflation in 1978 had crumbled in the 'winter of discontent' of that year when graves went undug and rubbish piled up in the streets. The budget was in crisis, the deficit already up to 5 percent of GDP and headed to get worse due to large public sector pay increases promised by the previous government. Milton Friedman (1980) advised a gradual reduction in the money supply growth rate and a cut in taxes in order to stimulate output. The first part was accepted, but the opinion was that tax rates needed to remain high to try and reduce the deficit which was important in conditioning financial confidence.

As mentioned earlier monetary aggregate targeting was introduced in the UK in 1976 in conjunction with the International Monetary Fund (IMF) support arrangement. Figure 6 in the appendix plots the growth of £M0 from 1970 till the end of 2003 and of £M3 from 1979 to 1985. The previous government was quite successful in shrinking the Public Sector Borrowing Requirement (PSBR) from 10 percent in 1975 to less than 4 percent in 1977. However, the policies lacked long-term durability. To achieve durability policy was cast in the form of a Medium Term Financial Strategy (MTFS), a monetary and fiscal policy programme announced by the Conservative Government in its annual budget in 1980. This strategy consisted first of a commitment to a five-year rolling target for gradually decelerating £M3. Second, controls were removed, including the 'Corset', exchange controls and incomes policy. Third, the monetary commitment was backed up by a parallel reduction in the PSBR/GDP ratio.

Large misses of the £M3 target were permitted as early as mid-1980, with the MTFS being heavily revised in 1982. In October 1985 £M3 targeting was abandoned. It was however clear prior to the abandonment that key policy makers did not regard overshoots of the £M3 target as intolerable, as long as other measures of monetary conditions, such as interest rates or monetary base growth, were not indicating that monetary policy was loose. Formally, monetary targets continued to be a part of the MTFS right until 1996. However, by 1988, the targets had been so de-emphasised in monetary policy formation that Nigel Lawson, the Chancellor of the Exchequer could say "As far as monetary policy is concerned, the two things perhaps to look at are the interest rate and the exchange rate."[15]

Even though the logic behind the MTFS was well developed, it failed not only to command credibility, but also to be carried out in its own literal terms. Policy turned out to be more fiercely contractionary than gradualism had intended. As Minford (1993) puts it succinctly "The paradox was: tougher yet less credible policies, apparently the worst of both worlds."

$$\widetilde{y}_t = \gamma(E_t \widetilde{y}_{t+1}) - \phi r_t + u_{MT_t} \tag{19}$$

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[15] Testimony, 30 November 1988, in Treasury and Civil Service Committee.

$$\widetilde{y}_t = \delta(P_t - E_{t-1}P_t) + v_{MT_t} \tag{20}$$

$$M_t = P_t - \beta_{MT_4}R_t + \beta_{MT_5}(E_t\widetilde{y}_{t+1}) + \xi_{MT,0_t} \tag{21}$$

$$\Delta M_t = \overline{M} + \xi_{MT,1_t} \tag{22}$$

$$R_t = r_t + E_tP_{t+1} - P_t \tag{23}$$

$$u_{MT_t} = \rho_{MT_0}u_{MT_{t-1}} + \varepsilon_{MT_t} \tag{24}$$

$$v_{MT_t} = \rho_{MT_1}v_{MT_{t-1}} + x_{MT_t} \tag{25}$$

$$\xi_{MT,0_t} = \rho_{MT_6}\xi_{MT,0_{t-1}} + \epsilon_{MT_t} \tag{26}$$

In the equations above, r_t is the real interest rate and M_t is the money demand (or supply). Equations (19) and (20) are the IS and Phillips curve, respectively. Equation (21) the LM curve sets out a standard money demand schedule. The shock to money demand is persistent as seen in equation (26). Growth in money supply equals a exogenously specified target $\overline{M}$ [16] and a random shock (equation (22)). Equation (23) is the definition of nominal interest rate in the model. As before the IS and PP curve shocks have been modelled as $AR(1)$ processes. ε_{MT_t}, x_{MT_t} and ϵ_{MT_t} are all $i.i.d.$

4.4 Exchange Rate Targeting (1986:1 to 1992:3)

The next regime largely consists of informal linking of the Sterling to the Deutsche Mark (DM). This includes not only the 'shadowing' of the DM in 1986-88, but also the period from autumn 1990 during which UK was a formal member of the Exchange Rate Mechanism (ERM). The idea essentially was that, just as the other major European currencies were successfully aiming to hold inflation down by anchoring their currencies to the DM within the ERM, the UK too could lock in to Germany's enviable record of sustained low inflation even without actually joining the mechanism. The approach was never formally announced, but it became clear in practice that the Sterling/DM exchange rate, which had depreciated very sharply from DM 4 in July 1985 to DM 2.74 in early 1987, was not subsequently allowed to appreciate above DM 3 even though this meant a massive increase in UK foreign exchange reserves, and a reduction of interest rates from 11 percent to a trough of 7 percent during 1987 to prevent the appreciation. This had the effect of accommodating and aggravating the inflationary consequences of the earlier depreciation.

In the Spring of 1988, the exchange rate cap was lifted but by then the boom was already entrenched. Interest rates were pushed up to 15 percent by the Autumn of 1989 to bring the situation under control. A year later the UK also formally joined the ERM. The episode produced a painful recession in which inflation which had risen to over 7% fell back sharply. According to Nelson (2000) from 1987-1990, the Bundesbank's monetary policy, rather than a domestic variable, served as UK monetary policy's nominal anchor.

At the time of ERM entry UK policy needs appeared to coincide with those of its partners. In principle it seemed possible that with the enhanced policy credibility that ERM membership was expected to bring, UK could hope to complete the domestic economic stabilisation programme with lower interest rates than otherwise, and so at less cost in terms of loss of output. There was also a very strong non-monetary consideration, that the UK would have little influence on the outcome of the European Inter-Government Conference if it was not in the ERM.

However, things did not go as planned. German reunification meant that Germany needed to maintain a tight monetary policy at a time when the domestic situation in a number of ERM countries, including the UK, required monetary easing. Parity adjustment was against the ERM rules and seemed inconsistent with maintaining policy credibility. The UK was then confronted with a situation where tightening policy by raising rates made no economic sense in terms of domestic conditions. It then sought to maintain the parity through intervention in the hope that the pressures in Germany would abate. In reality those pressures did not ease soon enough and after heavy intervention, and a last bout of interest rate increases, the UK had no choice but to withdraw from the ERM in September 1992.

The model we use here is the same as the Bretton Woods model with the exception that Germany replaces the US throughout.

$$\widetilde{y}_t = \gamma(E_t\widetilde{y}_{t+1}) - \phi(R_t - E_tP_{t+1} + P_t) + \lambda(E_tNX_{t+1}) + u_{FGR_t} \tag{27}$$

$$\widetilde{y}_t = \delta(P_t - E_{t-1}P_t) + v_{FGR_t} \tag{28}$$

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[16] We assume $\overline{M} = 0$ for convenience.

$$NX_t = a_{FGR_0}Q_{FGR_t} + a_{FGR_1}y_{FGR_t}^F \tag{29}$$

$$Q_{FGR_t} = S_{FGR_t} + P_{FGR_t}^f - P_t \tag{30}$$

$$R_t = R_{FGR_t}^F + (E_t S_{FGR_{t+1}} - S_{FGR_t}) \tag{31}$$

$$S_{FGR_t} = \overline{S}_{FGR} \tag{32}$$

$$R_{FGR_t}^F = \rho_{FGR}R_{FGR_{t-1}}^F + \eta_{FGR_t} \tag{33}$$

$$u_{FGR_t} = \rho_{FGR_0}u_{FGR_{t-1}} + \varepsilon_{FGR_t} \tag{34}$$

$$v_{FGR_t} = \rho_{FGR_1}v_{FGR_{t-1}} + x_{FGR_t} \tag{35}$$

$$y_{FGR_t}^F = \rho_{FGR_4}y_{FGR_{t-1}}^F + \theta_{FGR_t} \tag{36}$$

$$P_{FGR_t}^f = \rho_{FGR_5}P_{FGR_{t-1}}^f + \kappa_{FGR_t} \tag{37}$$

Here S_{FGR_t} is the nominal exchange rate £$/DM$, $R_{FGR_t}^F$ is the German nominal interest rate (day-to-day money rate) and $P_{FGR_t}^f$ is the German price level (CPI).

4.5 Inflation Targeting Regime (1992:4 to 2003:3)

Immediately following the UK's exit from the Exchange Rate Mechanism (ERM) in September 1992, inflation expectations were between 5 percent and 7 percent at maturities 10 to 20 years ahead — well above the inflation target of 1-4 percent at the time. Five years into the regime, by April 1997, inflation expectations had ratcheted down to just over 4 percent. A credibility gap still remained but it had narrowed markedly. The announcement of operational independence for the Bank of England in May 1997[17] caused a further decline in inflation expectations by around 50 basis points across all maturities. By the end of 1998, inflation expectations were around the UK's 2.5 percent inflation target, at all maturities along the inflation term structure. They have remained at that level since then.

Using the inflation target as a reference point for expectations is important during the transition to low inflation as the target then serves as a means of guiding inflation expectations downwards over time. It is widely thought, though not a feature of our models here, that lags in policy mean that inflation-targeting needs to have a forward-looking dimension. According to Haldane (2000) a successful inflation-targeting regime must have *'ghostbusting'* as an underlying theme; by which he means that policy makers take seriously the need to be pre-emptive in setting monetary policy, offsetting incipient inflationary pressures.[18] Nevertheless within our model here a forward element makes no sense and in fact causes indeterminacy; so we have framed interest rate policy in terms of current inflation and output.

$$\widetilde{y}_t = \gamma(E_t\widetilde{y}_{t+1}) - \phi(R_t - E_t\pi_{t+1}) + u_{IT_t} \tag{38}$$

$$\widetilde{y}_t = \delta(\pi_t - E_{t-1}\pi_t) + v_{IT_t} \tag{39}$$

$$R_t = \beta_{IT_0} + \beta_{IT_1}R_{t-1} + \beta_{IT_2}(\pi_t - \pi^*) \tag{40}$$

$$u_{IT_t} = \rho_{IT_0}u_{IT_{t-1}} + \varepsilon_{IT_t} \tag{41}$$

$$v_{IT_t} = \rho_{IT_1}v_{IT_{t-1}} + x_{IT_t} \tag{42}$$

In the equations above all variables are as defined earlier; π^* is the inflation target of the Bank of England. R_t the nominal interest rate is the Bank of England base rate is plotted in Figure 6 in the appendix. As before equation (38) and (39) are the IS and Phillips curve, respectively. Equation (40) is a Taylor rule with interest rate smoothing. As before the IS and PP errors have been modelled as $AR(1)$ processes. ε_{IT_t}, x_{IT_t} are *i.i.d.*

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[17] Autonomy of the Bank is enshrined in the Bank of England Act of 1998. This act confers instrument-independence on the Bank, though the government still sets the goals of policy. In the jargon, there is goal-dependence but instrument independence.

[18] Haldane (2000) goes on to say *"Like ghosts, these pressures will be invisible to the general public at the time policy measures need to be taken. Claims of sightings will be met with widespread derision and disbelief. But the central bank's job is to spot the ghosts and to exorcise them early. A successful monetary policy framework is ultimately one in which the general public is not haunted by inflationary shocks."*

4.6 The New Keynesian versions and their calibration:

In order to convert our small models to New Keynesian form, we merely need to substitute the New Keynesian Phillips curve for the New Classical Phillips Curve used above. This we write in the standard way with backward indexation as:

$$\pi_t = \nu E_t \pi_{t+1} + (1 - \nu)\pi_{t-1} + \zeta \widetilde{y}_t + v_{it}$$

where v_{it} represents the error term for the ith regime.

This replaces the second equation in our New Classical model above, viz:

$$\widetilde{y}_t = \delta(P_t - E_{t-1}P_t) + v_{it}$$

It is immediately apparent that, with the exception of the Incomes Policy regime where the solution for inflation remains the same, this raises the dynamic complexity of the solution greatly. There are now several (two or more) forward roots as well as at least one backward root in each model's characteristic equation; all must be stable in order for the model to have a stable solution (see Minford and Peel, 2002, chapter 2). This requires numerical analysis: because of the complexity of the equations, it is not possible to establish this analytically in any of these cases. We have therefore calibrated each model so that it satisfied this stability condition when subjected to simulation analysis. In general we have found this has meant keeping the value γ, the forward-looking term in output in the IS curve, somewhat below 1; we have varied it from 1 according to the demands of stability.

Values for ν, the forward-looking root in the Phillips Curve, can also produce a stability problem if close to unity. As it happens the value of this parameter is hotly disputed in recent empirical work. Thus Rudd and Whelan (2005), found the backward element predominant in fitting the inflation data at the single equation level and so set ν close to zero. Gali et al (2005) on the other hand argue on the basis of their own instrumental variable estimation procedure that it should be close to unity. We decided therefore to look at a range of values for ν.

For δ we took the usual calibration in this literature of 0.2. For other parameters in both models, we took values from Orphanides (1998), Dittmar, Gavin and Kydland (1999), McCallum and Nelson (1999a, 1999b), McCallum (2001), Rudebusch and Svensson (1999), Ball (1999) and Batini and Haldane (1999). Details can be found in appendix B.

5 Comparing Models and Data using the Bootstrap

5.1 Estimating the error processes

In each of the models we estimate the AR coefficients of the IS, Phillips Curve and, where applicable, money demand shocks. As the solution itself is a function of the errors, we iterate; we get a first approximation of the errors by using the calibrated parameter values along with the data in the IS/PP curve equation and for the expectational variables the values given by the solution's lagged terms ignoring the errors.[19] Once we have the shock data we run $AR(1)$ on it, to get our first estimates of ρ_0 and ρ_1 in the various models.[20]

To work out the 'true' errors and ρs we have used a rolling forecast programme. The programme works as follows. Our first estimates of the ρs enable it to work out the expectational variables in the model. Using the expectational variables the model solves for the endogenous variables for the current period and all periods in the future. The new error then is simply the difference between the left hand side and right hand side of the original equation where actual data is plugged in for current and lagged endogenous variables and the expected terms are from the current rolling forecast. Then it estimates $AR(1)$ on these new errors to get the new ρs, which can then be used to work out the new expectational variables. The model then solves again to get the new endogenous variables and then gets yet again a new set of errors. This iterative procedure is repeated until the errors and ρs converge to their 'true' values — as if the expectations that are model derived were used in the first place.

In addition all exogenous variables (foreign interest rate, foreign GDP and foreign prices) have autoregressive processes estimated for them.

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[19] This is clearly not ideal. However, it enables us to get some starting values of the errors.

[20] Please note that we have omitted the subscripts specific to each regime. eg. ρ_{FUS_0} etc

6 Bootstrapping

We now replicate the stochastic environment for each model-regime combination to see whether within it our estimated $ARMA$ equations could have been generated. This we do via bootstrapping the models above with their error processes.

The idea is to create pseudo data samples — here 1000 — for inflation. Within each regime we draw the vectors of $i.i.d.$ shocks in our error processes with replacement;[21] we then input them into their error processes and these in turn into the model to solve for the implied path of inflation over the sample period. We then run $ARMA$ regressions on all the pseudo-samples to derive the implied 95% confidence intervals for all the coefficient values found. Finally we compare the $ARMA$ coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals: under the null hypothesis of the model-regime being considered, these values represent the sampling variation for the $ARMA$ coefficients. We also show a portmanteau statistic (the m[odel]-metric), the 95% confidence limit for the joint distribution of the ARMA parameters; this is obtained by computing the Mahalanobis distance of the joint parameters from their joint means. The bootstraps create a distribution of this distance. The m-metric is then the percentile of this distribution given by the distance obtained from the actual data; plainly if this exceeds 95% the model is rejected at the 95% confidence level. (Minford et al, 2007 discusses these methods in much greater detail). Table 5 summarises the results of this exercise.

6.1 Results for the New Classical models:

		Estimated	95% Confidence Interval		M-metric
			Lower	Upper	
Bretton Woods	AR(1)	−0.592304	−0.825237	0.930382	67.3
(FUS)	MA(1)	0.952206	−0.969678	1.296733	
Incomes Policy	AR(1)	0.727366	0.199370	0.737302	94.0
(IP)					
Money Targeting	AR(1)	0.927892	−0.801210	0.968120	91.8
(MT)	MA(1)	−0.997381	−0.997490	1.531400	
Exchange Rate Targeting	AR(1)	0.623726	−0.222109	0.258290*	100.0
(FGR)					
Inflation Targeting (RPI)	AR(1)	0.202142	−0.208337	0.294338	79.7
(IT)					

Table 5: Confidence Limits the New Classical Model for Theoretical ARMAs

It can be seen from Table 5 that the model is accepted as a whole (based on the m-metric) for all regimes except for exchange rate targeting where the model falls well short of the estimated persistence.

The charts that follow show the impulse response functions with their 95% confidence intervals. Inflation persistence is fairly low under Bretton Woods; rises as it moves to a floating regime with incomes policy and then falls back sharply under monetary targeting — this was the period of the Thatcher government's 'monetarist' policies designed to squeeze high double-digit inflation out of the economy. Finally persistence rose again under exchange rate targeting until inflation targeting pushed it back down to the Bretton Woods level. The model fails as we have seen to generate enough persistence under exchange rate targeting but otherwise captures the shifts from low to high persistence and then back again to low. As we have seen, this is not because the persistence of the exogenous shocks changes across regimes but rather because the regimes themselves alter the response of inflation to this persistence.

6.2 Results for the New Keynesian models:

We now turn to the New Keynesian versions of the model. In the following tables we show the equivalent bootstrap results. We group them into three: high stickiness (low-ν), medium, and low stickiness (high-ν).

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[21]By drawing vectors for the same time period we preserve their contemporaneous cross-correlations. The errors also have zero means so that the resulting ARMAs are estimated without a constant, since the true constant must be zero.
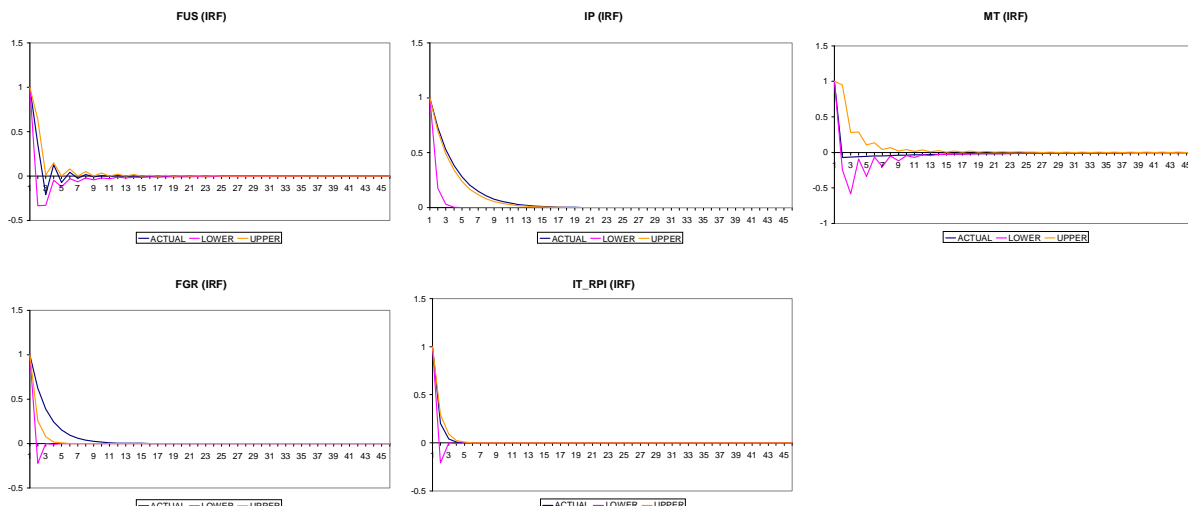
Figure 2: New Classical Impulse Response Functions with 95% Bounds

6.2.1 High stickiness (low-ν:)

The model is comprehensively rejected under all regimes (except Incomes Policy which is the same under both models). The reason seems to be the high level of persistence in the New Keynesian Phillips Curve itself, which is both forward-looking as in Calvo and also has a large backward-looking component. This Phillips Curve was constructed to generate persistence: persistence is as it were 'engineered into' the inflation process through it. However, the consequent difficulty is that policy regime changes have insufficient effect on the degree of persistence. Inflation targeting brings it down somewhat; but still nowhere near enough.

| | | Estimated | 95% Confidence Interval | | M-metric |
			Lower	Upper	
Bretton Woods	AR(1)	-0.592304	0.651404*	0.917036	100.0
(FUS)	MA(1)	0.952206	0.014508	0.814515*	
Money Targeting	AR(1)	0.927892	0.589790	0.972620	99.3
(MT)	MA(1)	-0.997381	0.001890*	0.997420	
Exchange Rate Targeting	AR(1)	0.623726	0.803951*	0.996775	100.0
(FGR)					
Inflation Targeting (RPI)	AR(1)	0.202142	0.333518*	0.685009	99.9
(IT)					

Table 6: Confidence Limits from the New Keynesian Model for Theoretical ARMAs

These results indicate that the high-stickiness New Keynesian version of the model is rejected for all four regimes (for Incomes Policy, ignored here, it is the same as the New Classical model and accepted). As we saw earlier the NC version was only rejected for Exchange Rate Targeting.

In Appendix C we examine from the model solutions what exactly is driving the differences between the two models. The New Keynesian model fails to fit the varying inflation persistence data essentially because persistence is 'hard-wired' into its Phillips Curve which contains the high autoregressive root, $1 - \nu$. Without a policy response to inflation, this backward root and the two forward roots (γ and ν) are the roots of the characteristic equation. This backward root can only be reduced by a powerful inflation response from interest rates that changes the configuration of all the roots in the characteristic equation. However, only under inflation targeting does this produce any material reduction and even this is insufficient to match the data. The result is that in all regimes, persistence is excessive compared with the data.

By contrast, the new Classical model derives its inflation persistence properties from the autoregressive roots driving the errors. The monetary policy mechanism can either add further persistent errors
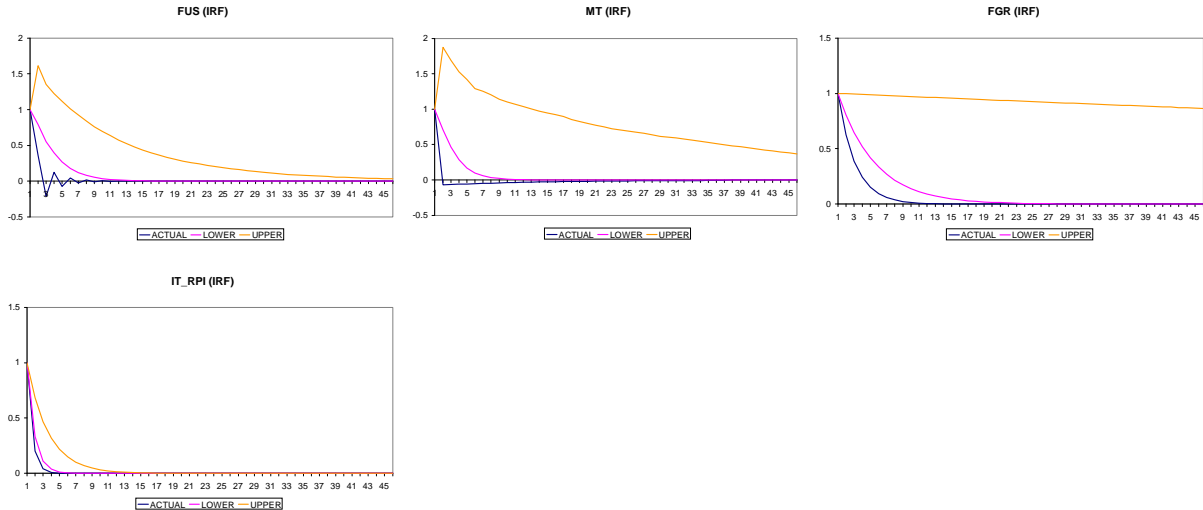
Figure 3: New Keynesian Impulse Response Functions with 95% Bounds (high stickiness)

or it can offset existing sources of persistence by reacting to an inflation shock with a future inflation reduction (as for example under Inflation targeting).

We can summarise the difference as that persistence in the New Keynesian model is set by the autoregressive roots essentially produced by the Phillips Curve's persistence, which can with difficulty be changed by the monetary regime whereas persistence in the New Classical model is set by the combination of largely fixed autoregressive roots coming from the exogenous processes and of a moving average process much affected by the monetary policy regime.

6.2.2 Medium stickiness (ν: is 0.5)

When the size of the backward-looking root is brought down to around 0.5, the model's implications are for substantially less persistence. This allows it to match the Exchange Rate Targeting regime well But although it gets closer to the persistence of the Monetary Targeting regime, it is still rejected and is massively rejected for both Bretton Woods and Inflation Targeting.

		Estimated	95% Confidence Interval		M-metric
			Lower	Upper	
Bretton Woods	AR(1)	−0.592304	0.255710*	0.991510	100.0
(FUS)	MA(1)	0.952206	−0.971190	0.746510*	
Money Targeting	AR(1)	0.927892	0.050886	0.906435	98.3
(MT)	MA(1)	−0.997381	−0.580258*	0.970441	
Exchange Rate Targeting	AR(1)	0.623726	0.523998	0.845061	63.6
(FGR)					
Inflation Targeting (RPI)	AR(1)	0.202142	0.596853*	0.819715	100.0
(IT)					

Table 7: Confidence Limits from the New Keynesian Model for Theoretical ARMAs

6.2.3 Low stickiness (ν:=0.9)

In this final version of the New Keynesian model the Phillips Curve is virtually entirely forward-looking, with the least stickiness of any of these Calvo contract models. The model now matches the Money Targeting and Inflation Targeting regimes but it is still too persistent for Bretton Woods and it is now not persistent enough for the Exchange Rate Targeting regime.
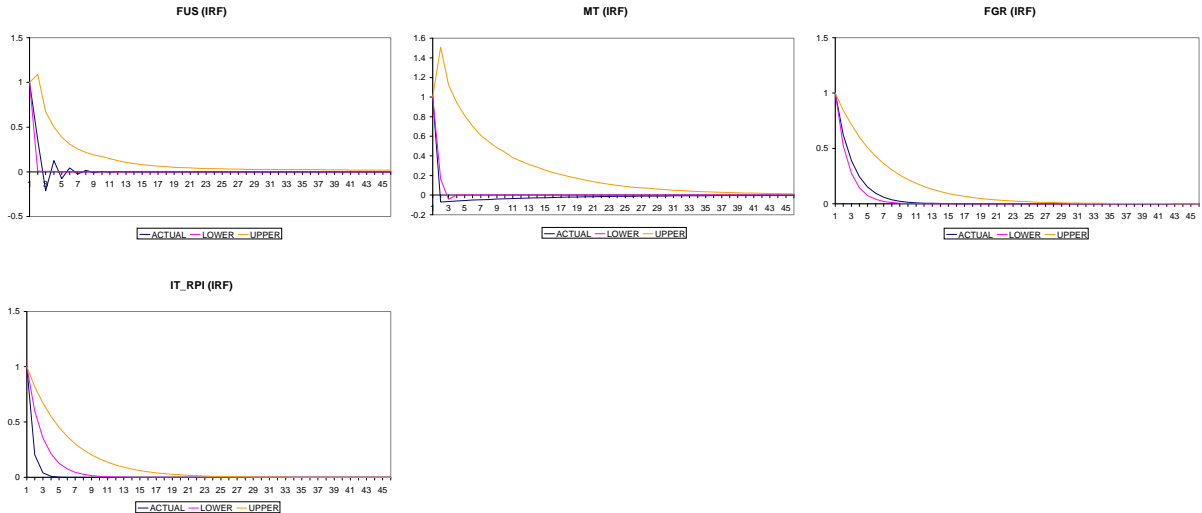
FUS (IRF)

MT (IRF)

FGR (IRF)

IT_RPI (IRF)

Figure 4: New Keynesian Impulse Response Functions with 95% Bounds (medium stickiness)

| | | Estimated | 95% Confidence Interval | | M-metric |
			Lower	Upper	
Bretton Woods	AR(1)	−0.592304	0.853110*	0.983040	100.0
(FUS)	MA(1)	0.952206	−0.997490	−0.421050*	
Money Targeting	AR(1)	0.927892	0.191180	0.932391	77.3
(MT)	MA(1)	−0.997381	−0.93970*	0.633995	
Exchange Rate Targeting	AR(1)	0.623726	−0.274871	0.407777*	100.0
(FGR)					
Inflation Targeting (RPI)	AR(1)	0.202142	−0.247884	0.566936	63.5
(IT)					

Table 8: Confidence Limits from the New Keynesian Model for Theoretical ARMAs

6.3 Comparing the models

The New Keynesian model in its most sticky form generates far too much persistence in all regimes. As the backward-looking root is brought down, it is able to encompass up to two of the regimes only. The persistence features in each case are largely fixed by the Phillips Curve, so that the regime itself has limited influence on the model's overall properties. With the New Classical model where the Phillips Curve itself has merely a one-period information lag, the persistence properties come from the natural autoregressiveness of the errors interacting with the regime. As the regime varies the basic autoregressiveness due to the errors is modified by the regime's responses; this enables the model to encompass most of the variation in persistence across regimes.

Thus if we ask which model version is the most likely, we can measure this by an overall likelihood. In each regime the likelihood of observing the data-generated ARMA parameters, under the null of each model, can be computed from the model's probability density function (we assume this is multi-variate normal by appeal to the central limit theorem since these parameters are sample means). The natural logs of these pdfs are shown in Table 9 together with the sum across all regimes for each model. This last figure represents the log of the joint likelihood.

The Table shows that for all the regimes other than for exchange rate targeting the model with least stickiness, the New Classical, is the most likely. This model is also the most likely overall. The next most likely is the medium stickiness New Keynesian model; even though it is rejected in three out of the four regimes, whereas the low stickiness version is only rejected in two, it avoids the disastrous performance of the latter in Bretton Woods.
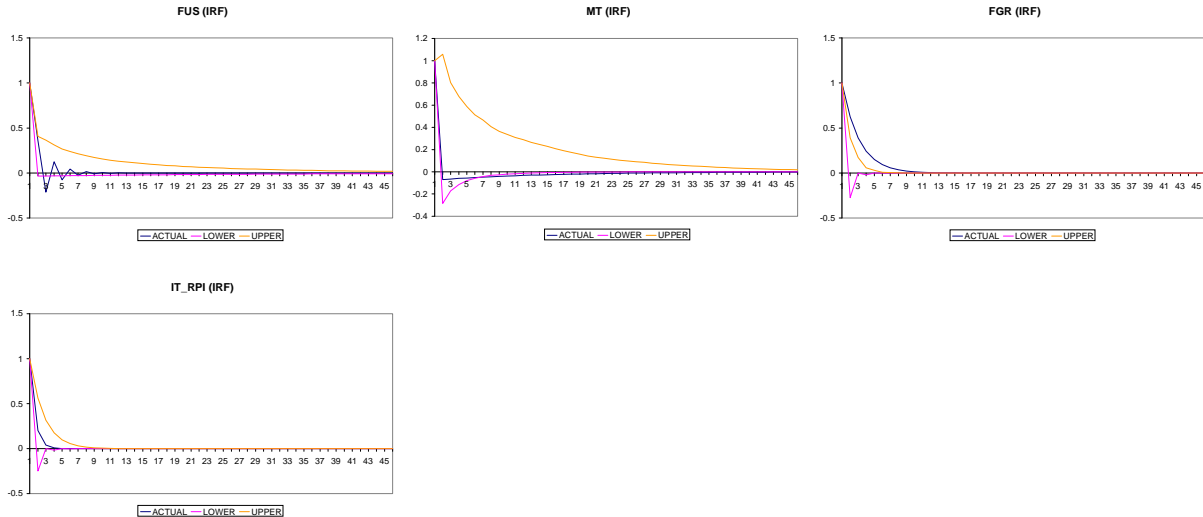
Figure 5: New Keynesian Impulse Response Functions with 95% Bounds (low stickiness)

| | Targeting Regimes | | | | |
	Bretton Woods	Monetary	Exchange Rate	Inflation	Total
New Keynesian					
High Stickiness	-82.71	-17.94	-6.50	-2.16	$-109.31\ (-26.6)^{+}$
Medium Stickiness	-32.16	-10.48	0.86	-9.91	$-51.69\ (-19.53)^{+}$
Low Stickiness	$-\infty$	-0.79	-19.10	0.33	$-\infty\ (-19.56)^{+}$
New Classical	-2.69	-6.44	-7.12	0.45	$-15.8\ (-13.11)^{+}$

$^{+}$Numbers in parentheses correspond to the total for the last three regimes

Table 9: Log-likelihood of Observing the Data-Generated ARMA Parameters Under Each Model and Regime

7 Conclusions

UK inflation persistence varies strikingly across the many monetary regimes pursued in the UK during the postwar period. It started low under Bretton Woods, then rose sharply during the next decade as the exchange rate floated without a monetary anchor, fell to virtually nil under the succeeding monetarist regime of the 1980s, before rising again to a high level when the pound was tied to the Deutschemark; finally on the introduction of inflation targeting from 1992 inflation persistence dropped back again to the level last seen under Bretton Woods. These facts cannot be accounted for easily by models of nominal rigidity of the sort modelled in Calvo contracts with an element of lagged indexation. These models effectively build persistence into the Phillips Curve and this degree of persistence is consequently not at all sensitive to variations in the monetary regime. Thus not surprisingly they find it hard to match the variation of persistence revealed in the facts. By contrast a model with minimal rigidity, such as the flexprice model with a one-quarter information lag, 'New Classical' in nature, has reasonable success in picking up these variations. These models rely for inflation persistence on the autoregressiveness in the error processes themselves; however different monetary regimes moderate this natural persistence more or less. We conclude in short that inflation persistence is not a constant resulting from the inherent nominal rigidity of the monetary transmission process, but is rather the product of monetary policy interacting with the natural autoregressiveness of exogenous processes and is best captured by models with little nominal rigidity. Of course whether these models can also successfully address other macroeconomic regularities has been questioned; but that is something we leave to later work. We hope merely to have established in a Popperian way a negative finding: namely that the facts of UK inflation persistence reject models of nominal rigidity but not models of minimal rigidity.

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8   Appendix A: Data

8.1   Data Set (Base year 2000)

1. UK Base Rate: Bank of England Base Rate % (EP). Series UKPRATE. in DataStream.

2. UK Gross Domestic Product: Gross Domestic Product (GDP) at Factor Cost £ millions. Seasonally Adjusted. Series YBHH Table 1.1 Monthly Digest of Statistics.

3. UK M0: Wide Monetary Base. Seasonally Adjusted. Series AVAE Table 6.2 Economic Trends.

4. UK M4: Money Stock. Seasonally Adjusted. Series AUYN Table 6.2 Economic Trends.

5. UK Net Exports: Current Account Balance £ millions, Office of National Statistics (ONS). Series UKHBOG.. in DataStream. Calculated as fraction of GDP at Factor Cost.

6. UK RPI Price Index: Retail Price Index (RPI), Office of National Statistics (ONS). Not Seasonally Adjusted. Series UKRP....F in DataStream.

7. UK RPIX Price Index : Retail Price Index All items excluding Mortgage Payments, Office of National Statistics (ONS). Not Seasonally Adjusted. Series CHMK Table 3.1 Economic Trends.

8. Sterling/US Dollar: International Financial Statistics for UK. Inverse of Market Rate US Dollars per Pound.

9. Sterling/Deutsche Mark: International Financial Statistics for Germany and UK. Used Market Rate Deutsche Mark per US Dollar and Market Rate US Dollars per Pound to calculate.

10. US Gross Domestic Product: Gross Domestic Product (AR), Bureau of Economic Analysis. Series USGDP...D in DataStream.

11. US Interest Rate: Federal Funds Rate %, Federal Reserve. Series USFEDFUN in DataStream.

12. US Price Index: Consumer Price Index (CPI) All Urban All Items, Bureau of Labour Statistics. Not Seasonally Adjusted. Series USCONPRCF in DataStream.

13. German Gross Domestic Product: Gross Domestic Product, Deutsche Bank. Seasonally Adjusted. Series BDGDP...D in DataStream.

14. German Interest Rate: Day-to-day Money Rate %, Eurostat. Series BDESSFRT in DataStream.

15. German Price Index: Consumer Price Index (CPI), Deutsche Bank. Seasonally Adjusted. Series BDCONPRCE in DataStream.
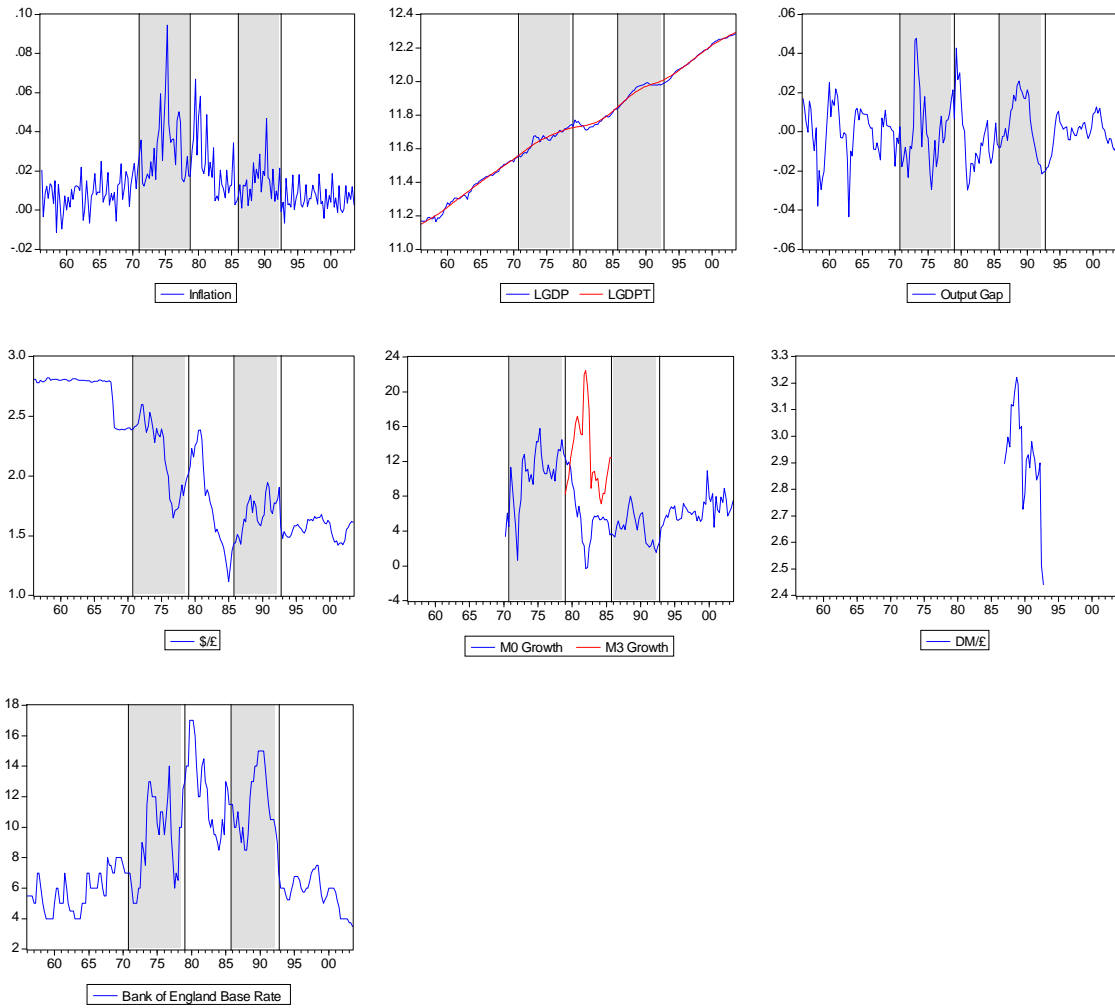
8.2 Graphs

Figure 6: Historical Data

8.3 APPENDIX A: Calibration and estimation:

8.4 Calibrated and Estimated Parameters of the Models

New Classical		New Keynesian	
Parameter	Calibrated Value	Parameter	Calibrated Value
ϕ	0.4	ϕ	0.4; 0.3 (MT)
δ	0.5	ζ	0.2
λ	0.2	λ	0.1 (FUS/FGR);0.17 (MT); 0.3 (IT)
γ	1	γ	0.3/0.7 (FUS/FGR);0.4/0.7 (MT); 0.5/1.0 (IT)*
$a_{FUS,0}$	2	$a_{FUS,0}$	2
$a_{FUS,1}$	0.65	$a_{FUS,1}$	0.65
$a_{FGR,1}$	2	$a_{FGR,1}$	2
$a_{FGR,1}$	0.65	$a_{FGR,1}$	0.65
c	0.2	β_{MT_4}	0.2
β_{MT_4}	0.15		
β_{MT_5}	1	β_{MT_5}	0.5
β_{IT_0}	0.025	β_{IT_0}	0.025
β_{IT_1}	0.85	β_{IT_1}	0.85
β_{IT_2}	1.5	β_{IT_2}	1.5
β_{IT_3}	0.5	ν	0.35(FUS);0.17(MT);0.1(FGR);0.3(IT)+

Table 10: Calibrated Parameters

*The first value shown is that used for the high stickiness version. the second for the medium and low stickiness versions.

+These values are those used for the high stickiness cases; for medium stickiness the value was 0.5, for low stickiness 0.9 in all regimes.

New Classical							
Regime	Estimated Parameters of errors and exogenous processes						
	$\rho(R^F)$	$\rho_0(u)$	$\rho_1(v)$	$\rho_3(IP)$	$\rho_4(y^F)$	$\rho_5(P^F)$	$\rho_6(\Delta M)$
Fixed Exchange Rate: US (FUS)	0.958062	0.521085	0.717961		0.833633	1.0	
Incomes Policy (IP)		0.575105	-0.05911	-0.211389			
Money Targeting (MT)		0.345169	0.270198				0.908337
Fixed Exchange Rate: Germany (FGR)	0.990000	0.650013	0.722195		0.478226	1.0	
Inflation Targeting (IT) RPI		0.886310	0.047119				

New Keynesian			
Regime	Estimated Parameters		
	$\rho(R^F)$	$\rho_0(u)$	$\rho_1(v)$
Fixed Exchange Rate: US (FUS)	0.958062	0.968885	-0.338905
Money Targeting (MT)	0.003876	0.671948	0.075834
Fixed Exchange Rate: Germany (FGR)	0.990000	0.911366	-0.730040
Inflation Targeting (IT) RPI		0.502884	-0.392109

Table 11: Estimated Parameters of error processes

9   Appendix C: A Basic Open Economy DSGE Model — the derivation from it of the New Keynesian and New Classical models

Consider an economy populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment. We assume that there are no market imperfections. We have for ease of argument deleted all shocks from the model but shocks can easily be reintroduced by making various of the fixed parameters stochastic; in principle this will be required as the model is stochastic. At the beginning of each period 't', the representative agent chooses (a) the commodity bundle necessary for consumption, (b) the total amount of leisure that she would like to enjoy, and (c) the total amount of factor inputs necessary to carry out production. All of these choices are constrained by the fixed amount of time available and the aggregate resource constraint that agents face. During the period 't', the model economy is influenced by various random shocks.

In an open economy goods can be traded but for simplicity it is assumed that these do not enter in the production process but are only exchanged as final goods. The consumption, C_t in the utility function below is composite per capita consumption, made up of agents consumption of domestic goods, C_t^d and their consumption of imported goods, C_t^f. We treat the consumption bundle as the numeraire so that all prices are expressed relative to the general price level, P_t. The composite consumption utility index can be represented as an Armington (1969) CES aggregator of the form

$$C_t = \left[\omega \left(C_t^d \right)^{-\rho} + (1 - \omega) \left(C_t^f \right)^{-\rho} \right]^{\left(\frac{-1}{\rho} \right)} \tag{43}$$

where ω is the weight of home goods in the consumption function, σ, the elasticity of substitution is equal to $\frac{1}{1+\rho}$.

The consumer maximises this composite utility index, given that an amount $\widetilde{C_t}$ has been chosen for total expenditure, with respect to its components, C_t^d and C_t^f subject to $\widetilde{C_t} = p_t^d C_t^d + Q_t C_t^f$. Where p_t^d is the domestic price level relative to the general price level and Q_t is the foreign price level in domestic currency relative to the general price level (the real exchange rate and also the terms of trade). The resulting expression for the home demand for foreign goods[22] is

$$\frac{C_t^f}{C_t} = [(1 - \omega)]^{\sigma} (Q_t)^{-\sigma} \tag{44}$$

We also note that:

$$1 = \omega^{\sigma} \left(p_t^d \right)^{\sigma \rho} + [(1 - \omega)]^{\sigma} Q_t^{\sigma \rho} \tag{45}$$

Hence we can obtain the logarithmic approximation:

$$\log p_t^d = - \left(\frac{1 - \omega}{\omega} \right)^{\sigma} \log (Q_t) + \text{constant} \tag{46}$$

In a stochastic environment a consumer is expected to maximise expected utility subject to the budget constraint. Each agent's preferences are given by

$$U = Max E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(C_t, L_t, \frac{M_t}{P_t} \right) \right], \qquad 0 < \beta < 1 \tag{47}$$

---

[22] we form the Lagrangean $L = \left[\omega \left(C_t^d \right)^{-\rho} + (1 - \omega) \left(C_t^f \right)^{-\rho} \right]^{\left(\frac{-1}{\rho} \right)} + \mu (\widetilde{C_t} - \frac{P_t^d}{P_t} C_t^d - \frac{P_t^f}{P_t} C_t^f)$..Thus $\frac{\partial L}{\partial C_t} = \mu$; also at its maximum with the constraint binding $L = \widetilde{C_t}$ so that $\frac{\partial L}{\partial \widetilde{C_t}} = 1$. Thus $\mu = 1$ - the change in the utility index from a one unit rise in consumption is unity. Substituting this into the first order condition $0 = \frac{\partial L}{\partial C_t^f}$ yields equation (44) . $0 = \frac{\partial L}{\partial C_t^d}$ gives the equivalent equation: $\frac{C_t^d}{C_t} = \omega^{\sigma} (p_t^d)^{-\sigma}$ where $p_t^d = \frac{P_t^d}{P_t}$ Divide (43) through by C_t to obtain $1 = \left[\omega \left(\frac{C_t^d}{C_t} \right)^{-\rho} + (1 - \omega) \left(\frac{C_t^f}{C_t} \right)^{-\rho} \right]^{\left(\frac{-1}{\rho} \right)}$;substituting into this for $\frac{C_t^f}{C_t}$ and $\frac{C_t^d}{C_t}$ from the previous two equations gives us equation (45).

where β is the discount factor, C_t is consumption in period 't', L_t is the amount of leisure time consumed in period 't', $\frac{M_t}{P_t}$ is real money balances and E_0 is the mathematical expectations operator. The essential feature of this structure is that the agent's tastes are assumed to be constant over time.

The objective of this paper is to specify a fully articulated model of an open economy which we propose to calibrate/estimate using data for the UK. We use this model to explain the behaviour of real exchange rate and also evaluate the impact of various demand and supply shocks.

9.1 The Representative Household

The model economy is populated by a large number of identical households who make consumption, investment, and labour supply decisions overtime. Each households objective is to choose sequences of consumption and hours of leisure that maximise its expected discounted stream of utility. We assume a time-separable utility function of the form

$$u\left(C_t, 1 - N_t, \frac{M_t}{P_t}\right) = \theta \log C_t + (1 - \theta)(1 - \rho)^{-1}(1 - N_t)^{(1-\rho)} + \log \frac{M_t}{P_t} \tag{48}$$

where $0 < \theta < 1$, and $\rho > 0$ is the leisure substitution parameter.

Individual economic agents view themselves as playing a dynamic stochastic game. Changes in expectations about future events would generally affect current decisions. Each agent in our model is endowed with a fixed amount of time which is spent on leisure L_t and/or work N_t. If H_t, total endowment of time is normalised to unity, then it follows that

$$N_t + L_t = 1 \text{ or } L_t = 1 - N_t \tag{49}$$

Furthermore for convenience in the logarithmic transformations we assume that approximately $L = N$ on average.

The representative agent's budget constraint is

$$\frac{M_t}{P_t} + C_t + \frac{b_{t+1}}{1 + r_t} + \frac{Q_t b^f_{t+1}}{\left(1 + r^f_t\right)} + p_t S^p_t = (1 - \tau) v_t N_t + b_t + Q_t b^f_t + (p_t + d_t) S^p_{t-1} + \frac{M_{t-1}}{P_t} \tag{50}$$

where p_t denotes the real present value of shares, $v_t = \frac{W_t}{P_t}$ is the real consumer wage (w_t, the producer real wage, is the the wage relative to the domestic goods price level; so $v_t = w_t p^d_t$). Labour income is taxed at the rate τ_t, which includes all taxes on households and is assumed to be a stochastic process. b^f_t denotes foreign bonds, b_t domestic bonds, S^p_t demand for domestic shares and $Q_t = \frac{P^f_t}{P_t}$ is the real exchange rate.

In a stochastic environment the representative agent maximizes the expected discounted stream of utility subject to the budget constraint. The first order conditions with respect to C_t, N_t, b_t, b^f_t and S^p_t are:

$$E_0 \beta^t \left(\frac{M_t}{P_t}\right)^{-1} = E_0 \lambda_t - E_0 \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{51}$$

$$E_0 \beta^t \theta C_t^{-1} = E_0 \lambda_t \tag{52}$$

$$E_0 (1 - \theta)(1 - N_t)^{-\rho} = E_0 \lambda_t (1 - \tau_t) v_t \tag{53}$$

$$E_0 \frac{\lambda_t}{1 + r_t} = E_0 \lambda_{t+1} \tag{54}$$

$$E_0 \frac{\lambda_t Q_t}{(1 + r^f_t)} = E_0 \lambda_{t+1} Q_{t+1} \tag{55}$$

$$E_0 \lambda_t p_t = E_0 \lambda_{t+1}(p_{t+1} + d_{t+1}) \tag{56}$$

Substituting equation (54) in (52)and letting t=0 yields :

$$(1 + r_t) = \left(\frac{1}{\beta}\right) E_t \left(\frac{C_t}{C_{t+1}}\right)^{-1} \tag{57}$$

Now substituting (52) and (54) in (53) yields

$$(1 - N_t) = E_t \left\{ \frac{\theta C_t^{-1} (1 - \tau_t) v_t}{(1 - \theta)} \right\} \tag{58}$$

Substituting out for $v_t = w_t p_t^d$ (and noting that $E_t \log v_t = \log v_t + \log p_t^{ue} = \log w_t + \log p_t^d + \log p_t^{ue}$, where the *ue* superscript means 'unexpected') and using (46) equation (58) becomes

$$(1 - N_t) = \left\{ \frac{\theta C_t^{-1} \left[(1 - \tau_t) \ \exp \left(\log w_t - (\frac{1-\omega}{\omega})^\sigma (\log Q_t) + \log p_t^{ue} \right) \right]}{(1 - \theta)} \right\}^{\frac{-1}{\rho}} \tag{59}$$

Substituting (54) in (56) yields

$$p_t = E_t \left(\frac{p_{t+1} + d_{t+1}}{(1 + r_t)} \right) \tag{60}$$

Using the arbitrage condition and by forward substitution the above yields

$$p_t = E_t \sum_{i=1}^{\infty} \frac{d_{t+i}}{\prod_{j=1}^{i} (1 + r_{t+j})} \tag{61}$$

The above equation states that the present value of a share is simply discounted future dividends.

In small open economy models the domestic real interest rate is equal to the world real interest rate, which is taken as given. Further, it is assumed that the economy has basically no effect on the world rate because, being a small part of the world, its affect on the world savings and investment is negligible. These assumptions imply that the real exchange rate for the small open economy is constant. However, we are modelling a medium sized economy. In our set up the economy is small enough to continue with the assumption that world interest rates are exogenous but large enough for the domestic rate to deviate from the world rate. Hence, in our model real exchange rates are constantly varying.

To derive the uncovered interest parity condition equation (54) is substituted into (55)

$$\left(\frac{1 + r_t}{1 + r_t^f} \right) = E_t \frac{Q_{t+1}}{Q_t} \tag{62}$$

In logs this yields

$$r_t = r_t^f + \log E_t \frac{Q_{t+1}}{Q_t} \tag{63}$$

9.2 The Government

The government finances its expenditure per capita, G_t, by collecting taxes on labour income, τ_t. Also, it issues debt, bonds (b_t) each period which pays a return next period.

The government budget constraint is:

$$G_t + b_t = \tau_t v_t N_t + \frac{b_{t+1}}{1 + r_t} \tag{64}$$

where b_t is real bonds.

9.3 The Representative Firm with fixed capital

Firms rent labour from households, who own their shares, and transform them into output according to a production technology and sell consumption goods to households and government. The technology available to the economy is described by a constant-returns-to scale production function where capital is fixed:

$$Y_t = Z_t N_t^{\alpha} \tag{65}$$

where $0 < \alpha < 1$, Y_t is aggregate output per capita and Z_t reflects the state of technology.

In a stochastic environment the firm maximizes present discounted stream, V, of cash flows, subject to the production technology,

$$MaxV = E_t \sum_{i=0}^{T} d^i (Y_{t+i} - w_{t+i} N_{t+i}) \tag{66}$$

Here w_t is the producer real wage. The firm optimally chooses labour so that:

$$N_t = \frac{\alpha Y_t}{w_t} \tag{67}$$

9.4 The Foreign Sector

From equation (2) we can derive the import equation for our economy

$$\log C_t^f = \log IM_t = \sigma \log (1 - \omega) + \log C_t - \sigma \log Q_t \tag{68}$$

Now there exists a corresponding equation for the foreign country which is the export equation for the home economy

$$\log EX_t = \sigma^F \log \left(1 - \omega^F\right) + \log C_t^F + \sigma^F \log Q_t \tag{69}$$

Foreign bonds evolve over time to the balance payments according to the following equation

$$\frac{Q_t b_{t+1}^f}{(1 + r_t^f)} = Q_t b_t^f + p_t^d EX_t - Q_t IM_t \tag{70}$$

9.5   Complete listing:

The above set-up can now be consolidated into a model listing as follows:

Behavioural Equations

(1)   Consumption C_t ; solves for r_t:

$$\begin{aligned}
(1 + r_t) &= \frac{1}{\beta} E_t \left(\frac{C_t}{C_{t+1}} \right)^{-1} \\
\text{or } C_t &= \gamma C_{t+1} - \delta r_t + c_1
\end{aligned}$$

In this linear, representation we use a first order Taylor series expansion around (average) $\overline{C_t}, \overline{E_t C_{t+1}}$, where $\gamma = \overline{\left(\frac{C_t}{E_t C_{t+1}} \right)}$ and $\delta = \gamma \beta \overline{C_t}$ and we would typically assume it to be less than unity on the grounds of growth. By dividing both sides by $\overline{C_t}$, we can approximate this linear expression by

$\log C_t = \gamma E_t \log C_{t+1} - \gamma \beta r_t + c_2$

(2)   Demand for money: $\log \frac{M_t}{P_t} = E_t \log C_{t+1} - \log \theta - \log(r_t + E_t \pi_{t+1})$

(3)   UIP condition:

$$r_t = r_t^F + E_t \log Q_{t+1} - \log Q_t + c_3$$

where r^F is the foreign real interest rate; here we use the property in taking logs that for a lognormal variable x_t, $E_t \log x_{t+1} = \log E_t x_{t+1} - 0.5 \sigma_{\log x}^2$. Thus the constant c_3 contains the variance of $\log C_{t+1}$.

(4)   Production function Y_t:

$$\begin{aligned}
Y_t &= Z_t (N_t)^{\alpha} \qquad\qquad \text{or} \\
\log Y_t &= \alpha \log N_t + \log Z_t
\end{aligned}$$

(5)   Demand for labour :

$$\begin{aligned}
N_t &= \left(\frac{\alpha Y_t}{w_t} \right) \qquad\qquad \text{or} \\
\log N_t &= \log \alpha + \log Y_t - \log w_t
\end{aligned}$$

(6)   The producer wage is derived by equating demand for labour, N_t, to the supply of labour given by the consumer's first order conditions:

$$(1 - N_t) = \left\{ \frac{\theta C_t^{-1} \left[(1 - \tau_t) \ \exp \left(\log w_t - (\frac{1-\omega}{\omega})^\sigma (\log Q_t) + \log p_t^{ue} \right) \right]}{(1 - \theta)} \right\}^{\frac{-1}{\rho}} \quad \text{or}$$

$$\log(1 - N_t) = -\log N_t = c_4 + \frac{1}{\rho} \left[\log C_t - \log(1 - \tau_t) - \log w_t - \log p_t^{ue} + (\frac{1-\omega}{\omega})^\sigma \log Q_t \right]$$

where Q_t is the real exchange rate, $(1 - \omega)^\sigma$ is the weight of domestic prices in the CPI index.

(7)   Imports IM_t:

$$\log IM_t = \sigma \log (1 - \omega) + \log C_t - \sigma \log Q_t$$

(8)   Exports EX_t:

$$\log EX_t = \sigma^F \log (1 - \omega^F) + \log C_t^F + \sigma^F \log Q_t$$

Budget constraints, market-clearing and transversality conditions:

(9)   Market-clearing condition for goods:

$$Y_t = C_t + G_t + EX_t - IM_t$$

and we assume the government expenditure share is an exogenous process.

The remainder of the model can be ignored for our purposes since we are now in a position to derive the various equations of the New Classical model- viz IS, LM, and Phillips Curve. For this purpose we will ignore the constants.

The IS curve:

To derive the IS curve note that loglinearising the market-clearing condition directly yields

$\log Y_t = c \log C_t + g_t + x(\log C_t^F - \log C_t) + x\sigma^* \log Q_t$

where c is the share of consumption in GDP, x is the share of trade in GDP and $.\sigma^* = \sigma + \sigma^F$. Hence substituting for $\log C_t$ from above yields:

$$\log Y_t = \frac{-(c - x)\gamma\beta}{1 - \gamma B^{-1}} r_t + g_t + x C_t^F - x\sigma^* \log Q_t$$

and multiplying through by $1 - \gamma B^{-1}$ gives::

$$\log Y_t = -\gamma\beta r_t + \gamma E_t Y_{t+1} + (1 - \gamma B^{-1})g_t + x(1 - \gamma B^{-1}) \log C_t^F - x\sigma^*(1 - \gamma B^{-1}) \log Q_t$$

This is our IS curve.

The LM curve:

To obtain the LM curve we take (2) above and noting that $R_t = r_t + E_t \pi_{t+1}$ obtain:
$\log M_t - \log P_t = E_t \log C_{t+1} - \log(R_t)$

Using the loglinearised market-clearing condition above yields

$E_t \log C_{t+1} = \frac{1}{c-x}(E_t \log Y_{t+1} - E_t g_{t+1} - x E_t \log C_{t+1}^F - x\sigma^* E_t \log Q_{t+1})$

We now linearise $\log R_t$ around $\overline{R}$ and substitute from the above to obtain:

$\log(\frac{M_t}{P_t}) = \psi_1 E_t \log Y_{t+1} - \psi_2 R_t + \epsilon_t$

where $\psi_1 = \frac{1}{c-x}$ and $\psi_2 = \overline{R}^{-1}$
Here the error term $\epsilon_t = \frac{1}{c+x}(-E_t g_{t+1} - x E_t \log C_{t+1}^F - x\sigma^* E_t \log Q_{t+1})$

The New Classical Phillips Curve:

We obtain the New Classical Phillips Curve. We solve the 3 equations production function (4), demand for labour (5), and supply of labour (6), first for expected (equilibrium) values, assuming $\log C_t$ is also at its expected value. Then we solve the same three equations for the effect of unexpected prices, $\log P_t^{ue}$ (we assume consumption is smoothed to stay at its expected value). From the production function we have $\log Y_t^{ue} = \alpha \log N_t^{ue}$ Hence from the demand for labour we have $\log N_t^{ue} = -\frac{1}{1-\alpha}\log w_t^{ue}$. Finally from the labour supply equation we have $\log N_t^{ue} = \frac{1}{\rho_2}(\log w_t^{ue} + \log P_t^{ue})$. So it follows that $\log w_t^{ue}$. Finally from the labour supply equation we have $\log w_t^{ue} = \frac{-\rho_2(1-\alpha)}{\rho_2+1-\alpha}(\log P_t^{ue})$ so that $\log Y_t^{ue} = \frac{\alpha\rho_2}{\rho_2+1-\alpha}(\log P_t^{ue})$. This is of course the 'surprise' Phillips Curve:

$\log Y_t = \log Y^* + \psi(\log P_t - E_t\log P_t)$ where it has been assumed that households do not have contemporaneous knowledge of the general price level.

The New Keynesian model:

We may take over all the equations derived above except the surprise Phillips Curve. We now assume that households all have monopoly power in their particular product and set the price in Calvo contracts but with lagged indexation. The resulting pricing equation has been derived carefully in Le and Minford (2006) for price-setting firms with fixed capital and variable labour as

$$\pi_t = \zeta(\log Y_t - \log Y^*) + \nu E_t\pi_{t+1} + (1-\nu)\pi_{t-1}$$

A similar form was derived by Christiano et al (2005). According to Le and Minford $\nu = \frac{\beta}{1+\beta}$ and should therefore be just below one half. However this formulation assumes that every household indexes prices up by last period's price; different assumptions on indexation will clearly change the formulation.

The specification effect of Fixed versus floating exchange rates:

Under fixed exchange rates the uncovered interest parity equation ties real interest rates to the relevant foreign real interest rate while the money supply becomes endogenous and money demand redundant. The real exchange rate is determined by inflation and the relevant exogenous foreign inflation. Thus under fixed rates interest rates are substituted out in favour of exogenous foreign interest rates while the real exchange rate is substituted out by the log of prices and of exogenous foreign prices.

Thus $r_t = r_t^F + E_t\log Q_{t+1} - \log Q_t$; and $\log Q_t = \log P_t^F - \log P_t$. So the IS curve becomes:

$\log Y_t = -\gamma\beta r_t^F + \gamma E_t\log Y_{t+1} + (1-\gamma B^{-1})g_t + x(1-\gamma B^{-1})\log C_t^F + [\gamma\beta(1-B^{-1}) + x\sigma^*(1-\gamma B^{-1})]\log Q_t$

where B^{-1} is the forward operator instructing one to lead the variable, holding the date of the expectation constant.

Since γ is less than unity the last term on the RHS becomes $[\gamma\beta + x\sigma^*](1-\gamma^* B^{-1})(\log P_t^F - \log P_t)$, where $\gamma < \gamma^* = \frac{\gamma\beta + \gamma x\sigma^*}{\gamma\beta + x\sigma^*} < 1$

Thus our IS curve under fixed rates becomes:

$\log Y_t = \gamma E_t\log Y_{t+1} - \phi(1-\gamma^* B^{-1})(\log P_t) + v_{FXt}$

where the error term contains the remaining terms above; $v_{FXt} = -\gamma\beta r_t^F + (1-\gamma B^{-1})g_t + x(1-\gamma B^{-1})\log C_t^F + [\gamma\beta + x\sigma^*](1-\gamma^* B^{-1})\log P_t^F$

and $\phi = \gamma\beta + x\sigma^*$

Under floating exchange rates uncovered interest parity allows the home interest rate (set by home policy) to determine the real exchange rate (and hence also its expected future value) given exogenous foreign interest rates. Thus we can substitute out the real exchange rate in terms of home and exogenous foreign interest rates. using UIP again we have $(1-B^{-1})\log Q_t = -(r_t - r_t^F)$. Noting that γ is close to unity we may approximate $(1-\gamma B^{-1})\log Q_t$ as $-(r_t - r_t^F)$. and remove $(1-B^{-1})\log Q_t$ from the IS curve in favour of $r_t - r_t^F$, yielding an IS curve under floating rates of:

$\log Y_t = \gamma E_t\log Y_{t+1} - \phi r_t + v_{FLt}$

where again $\phi = \gamma\beta + x\sigma^*$

where the error term again contains the remaining terms in this case

$v_{FLt} = (1-\gamma B^{-1})g_t + x(1-\gamma B^{-1})\log C_t^F + x\sigma^* r_t^F$

9.5.1 Summary of the IS/LM/Phillips Curve reduction and its solution method

Fixed Exchange Rate model:

(IS) $y_t = \gamma E_t y_{t+1} - \phi(1 - \gamma^* B^{-1})(\log P_t] + v_{FXt}$

where y_t is log detrended output and $\phi = \gamma\beta + x\sigma^*$

Under Fixed rates the LM curve is redundant. The model is completed by a Phillips Curve, either New Keynesian
$$\pi_t = \zeta(y_t - y^*) + \nu E_t \pi_{t+1} + (1 - \nu)\pi_{t-1} + u_{Kt}$$

or New Classical

$$y_t = \delta \log P_t^{ue} + u_{Ct}$$

Floating Exchange rate model:

(IS) $y_t = \gamma E_t y_{t+1} - \phi r_t + v_{FLt}$

Now again there is either a New Keynesian or New Classical Phillips curve as above.

Finally there are differing regimes:

Either Incomes Policy:

(IP) $\pi_t = (1 - \chi)\pi_{t-1} + c_t$

Notice that in this regime the Incomes Policy equation bypasses the rest of the model, χ representing the 'toughness' of controls; needless to say such a control regime cannot be expected to last because it could not indefinitely override market forces in the rest of the model; the effect of these is seen in the error term c_t. Nor does it of course, as the regime changes by the end of the decade.
Or Monetary Targeting

(MT) $\Delta m_t = m + \mu_t$

with LM Curve

(LM) $m_t - p_t = \psi_1 E_t y_{t+1} - \psi_2 R_t + \epsilon_t$

where $m_t = \log M_t, p_t = \log P_t$.
Or Inflation Targeting

(IT) $R_t = \varkappa(\pi_t - \pi^*) + i_t$

Notice here no LM curve is required.

10 Solution method:

Incomes Policy: The solution is straightforwardly obtained by substituting the c_t process.

10.1 New Classical Models:

(a) Fixed rates   Here the model is:

$y_t = \gamma E_t y_{t+1} - \phi(1 - \gamma^* B^{-1})(\log P_t) + v_{FXt}$
and $y_t = \delta \log P_t^{ue} + u_{Ct}$

We assume here the case where $E_t x_t = E(x_t \mid \Phi_{-1})$ and for simplicity we set the error term in the IS curve to zero. Let $u_t = \kappa u_{t-1} + \epsilon_t$ or $u_t = \frac{\epsilon_t}{1 - \kappa L}$ where L is the lag operator; $p_t = \log P_t$. The solution is most easily obtained by taking expectations at t of the model and then adding on the unexpected elements of each variable. Taking E_t of the model yields:

$(1 - \gamma^* B^{-1})E_t p_t = \phi^{-1}(\gamma B^{-1} - 1)E_t u_t$

The forward recursion on the error term then yields:

$E_t p_t = \frac{\phi^{-1}\kappa(\gamma\kappa - 1)}{1 - \gamma^*\kappa} u_{t-1}$

The unexpected component of prices is:

$p_t^{ue} = \frac{-1}{\delta + \phi}\epsilon_t$ so that:

$p_t = \frac{-1}{\delta + \phi}\epsilon_t - \frac{\phi^{-1}\kappa(1 - \gamma\kappa)}{1 - \gamma^*\kappa} u_{t-1}$

Multiplying through by $(1 - \kappa L)$ and differencing the equation gives us the inflation solution:

$\pi_t = \kappa\pi_{t-1} - \varphi_1\epsilon_t + [\varphi_1(1 + \kappa) - \varphi_2]\epsilon_{t-1} + (\varphi_2 - \kappa\varphi_1)\epsilon_{t-2}$

where $\varphi_1 = (\delta + \phi)^{-1}$ and $\varphi_2 = \frac{\phi^{-1}\kappa(1 - \gamma\kappa)}{1 - \gamma^*\kappa}$.

The solution in this simplified case where we have only one error is ARMA(1,2). Notice that the sum of the two moving average terms has the value φ_1 which is equal and opposite to the impact effect of the error. Hence the Impulse Response Function has this element working to offset the autoregressive element κ, reducing persistence. In period 2 the response is $\frac{\varphi_2}{\varphi_1} - 1$; in period 3 it is $-(\frac{\varphi_2}{\varphi_1})(1 - \kappa)$; from then on it decays at κ. With our calibration here φ_1 is of order 1 while φ_2 is of order 2κ. Thus in period 2 there is some echo of the impulse but by period 3 it has turned modestly negative

b) Floating rates under Monetary Targeting: Here we have:

$y_t = \gamma E_t y_{t+1} - \phi r_t + v_{FLt}$

$y_t = \delta p_t^{ue} + u_t$

and the LM curve

$m_t - p_t = \psi_1 E_t y_{t+1} - \psi_2 R_t + e_t$

with money supply process

$\Delta m_t = \mu_t$

where for simplicity again we will suppress all errors other than the productivity process $u_t = \frac{\epsilon_t}{1 - \kappa L}$ and the money supply error process $\mu_t = \frac{\eta_t}{1 - \kappa_1 L}$ which is now added.

The solution proceeds in the same way, except that now the real interest rate is determined in the money market ($R_t = r_t + E_t p_{t+1} - E_t p_t$). We obtain:

$$\pi_t = \kappa\pi_{t-1} - \varphi_1\epsilon_t + [\varphi_1(1+\kappa) - \varphi_2]\epsilon_{t-1} + [\varphi_2 - \kappa\varphi_1]\epsilon_{t-2} + (1-\kappa L)\frac{[\varphi_3\eta_t + \{\varphi_4 - \varphi_3(1+\kappa_1)\}\eta_{t-1} + \kappa_1\varphi_3\eta_{t-2}]}{1-\kappa_1 L}$$

where $\varphi_1 = \frac{\psi_2}{\phi+\delta\psi_2}; \psi = \frac{\psi_2}{1-\psi_2}; \varphi_2 = \frac{\phi\kappa(1-\kappa)+\kappa(\psi_1/\psi_2)}{1-\kappa\psi}; \varphi_3 = \frac{\phi}{\phi+\delta\psi_2}; \varphi_4 = \frac{1}{1-\psi}.$

Here we find the same pattern as under fixed rates for the IRF of the productivity shock; but now we have in addition the effect of the money shock. If we assume that approximately $\kappa = \kappa_1$, then this adds an MA(2) where since $\varphi_4 > 1 > \varphi_3$ the lagged errors reinforce the effect of lagged inflation. This will mean that this money IRF will have persistence greater than κ; thus in period 1 the value is $\frac{\varphi_4}{\varphi_3} - 1$; in period 2 $\kappa\frac{\varphi_4}{\varphi_3}$; from then decaying at the rate κ. Given that monetary volatility in the UK was high this term is likely to dominate the overall IRF.

c) Floating rates under Inflation Targeting:

Here we have:
$$y_t = \gamma E_t y_{t+1} - \phi r_t + v_{FLt}$$

$$y_t = \delta p_t^{ue} + u_t$$

$$r_t = -E_t\pi_{t+1} + \psi\pi_t + i_t$$

Again we suppress all errors other than the productivity process $u_t = \frac{\epsilon_t}{1-\kappa L}$ and the monetary error process $i_t = \frac{\eta_t}{1-\kappa_1 L}$. The solution is now:

$$\pi_t = \kappa\pi_{t-1} - \varphi_1\epsilon_t + [\varphi_1\kappa - \varphi_2]\epsilon_{t-1} - \frac{(1-\kappa L)}{1-\kappa_1 L}[\varphi_3\eta_t + (\varphi_4 - \kappa_1\varphi_3)\eta_{t-1}]$$

where $\varphi_1 = \frac{1}{\delta+\phi\psi}; \varphi_2 = \frac{\kappa(1-\kappa)}{\phi\psi(1-\kappa/\psi)}; \varphi_3 = \frac{\phi}{\delta+\phi\psi}; \varphi_4 = \frac{\kappa_1}{\psi(1-\kappa/\psi)}$

Let us assume, as occurs in our models, that the inflation response of interest rates, ψ, is fairly large; then both φ_2 and φ_4 will tend to be small. It can then be seen that the MA(1) term in both errors will tend to cancel out the lagged inflation effect, eliminating persistence (assuming as before that $\kappa_1 = \kappa$ approximately).

10.2   New Keynesian Models:

(a) Fixed rates   Here we approximate γ^* as unity so that the model can be written conveniently as:

$$y_t = \gamma E_t y_{t+1} + \phi(E_t\pi_{t+1}] + v_{FXt} \quad \text{(IS)}$$
$$\text{and } \pi_t = \zeta(y_t - y^*) + \nu E_t\pi_{t+1} + (1-\nu)\pi_{t-1} + u_{Kt} \quad \text{(Phillips)}$$

We assume here the case where $E_t x_t = E(x_t \mid \Phi_t) = x_t$ (full current information) as is normal in New Keynesian models; again for simplicity we set the error term in the IS curve to zero. $u_t = \frac{\epsilon_t}{1-\kappa L}$ where L is the lag operator.

The solution is obtained directly in terms of forward and backward operators as:

$$(1 - \nu B^{-1} - [1-\nu]L)\pi_t = \frac{\zeta\phi B^{-1}\pi_t}{1-\gamma B^{-1}} + u_t$$

which we can rewrite as:

$$\{\nu\gamma B^{-2} - [\gamma + \nu + (1-\nu)\gamma\nu + \zeta\phi]B^{-1} + [1 + (1-\nu)(\gamma+\nu)] - (1-\nu)L\}\pi_t = (1-\gamma\kappa)u_t$$

We can factor the left hand side to obtain:

$$k(1 - \alpha_1 B^{-1})(1 - \alpha_2 B^{-1}){:}(1 - \alpha_3 L)\pi_t = (1-\gamma\kappa)u_t$$

where the roots α_i must all be stable. Note that as $\zeta\phi$ tends to zero these roots are (forward) γ and ν and (backward) $(1 - \nu)$. Hence the two backward roots that determine the IRF of the supply shock η_t are κ and $(1 - \nu)$; there is no offset from moving average terms. (Moving average terms will be introduced as we introduce other errors; but each of the IRFs of these errors will have the same form considered individually. The MA terms come from their cross-effects which will not alter this basic IRF pattern). The impact effect of η_t is determined by the forward roots so that the overall solution with this one shock is:

$$\pi_t = \left[\frac{1 - \gamma\kappa}{k(1 - \alpha_1\kappa)(1 - \alpha_2\kappa)}\right] \frac{\epsilon_t}{(1 - \alpha_3 L)(1 - \kappa L)}$$

We can see therefore that under Fixed rates there will be substantial persistence imparted by the backward indexation term in the New Keynesian Phillips Curve as well as the autoregressiveness of the supply shock. Only the terms $\zeta\phi$ can affect the size of the backward root α_3; however it turns out that this tends to raise the value of α_3 in our calibration here where α_3 is barely short of unity.

b) Floating rates under Monetary Targeting: Here we have:

$$y_t = \gamma E_t y_{t+1} - \phi r_t + v_{FLt}$$

$$\pi_t = \zeta y_t + \nu E_t \pi_{t+1} + (1 - \nu)\pi_{t-1} + u_{Kt}$$

and the LM curve

$$m_t - p_t = \psi_1 E_t y_{t+1} - \psi_2 R_t + e_t$$

with money supply process

$$\Delta m_t = \mu_t$$

where for simplicity again we will suppress all errors other than the productivity process $u_t = \frac{\epsilon_t}{1 - \kappa L}$ and the money supply error process $\mu_t = \frac{\eta_t}{1 - \kappa_1 L}$ which is now added.

The solution proceeds by using the forward and backward operators as above, except that now the real interest rate is determined in the money market ($R_t = r_t + E_t p_{t+1} - E_t p_t$). We obtain:

$$\left((1 - L)\{1 - (\gamma - \frac{\phi\psi_1}{\psi_2})B^{-1}\}(1 - \nu B^{-1} - [1 - \nu]L) - \left[\zeta\phi B^{-1} - \zeta\phi(1 + \frac{1}{\psi_2})\right]\right)\pi_t$$
$$= (1 - L)\{1 - (\gamma - \frac{\phi\psi_1}{\psi_2})B^{-1}\}u_t + \frac{\zeta\phi}{\psi_2}\mu_t$$

The LHS of this factorises as in the earlier New Keynesian model into a fourth order difference equation with two forward and two backward roots. As ϕ tends to zero, the backward roots tend to unity and $(1 - \nu)$, the forward to γ and ν. However because the supply shock is differenced, the MA process in it cancels out the strong persistence coming from these two backward roots and also from its own autoregressiveness. However this is not true of the monetary growth shock μ_t whose effect will be highly persistent from these two backward roots plus its autoregressiveness.

The only way to bring down this persistence would be for the backward roots to be diminished in size by the effect of the terms in ϕ. However, at least in our calibration here they are increased.

c) Floating rates under Inflation Targeting: Here we have:
$$y_t = \gamma E_t y_{t+1} - \phi r_t + v_{FLt}$$

$$\pi_t = \zeta y_t + \nu E_t \pi_{t+1} + (1 - \nu)\pi_{t-1} + u_{Kt}$$

$$r_t = -E_t \pi_{t+1} + \psi\pi_t + i_t$$

Again we suppress all errors other than the productivity process $u_t = \frac{\epsilon_t}{1 - \kappa L}$ and the monetary error process $i_t = \frac{\eta_t}{1 - \kappa_1 L}$. We now obtain, following the same operator methods:

$$\left(\{1 - \gamma B^{-1}\}(1 - \nu B^{-1} - [1 - \nu]L) - \zeta\phi B^{-1} + \zeta\phi\psi \right) \pi_t$$
$$= \{1 - \gamma B^{-1}\}u_t - \zeta\phi i_t$$

Here again the LHS factorises into two forward roots and one backward root (which is again tends to $(1 - \nu)$ as ϕ tends to zero). Hence the IRFs of both the supply and the monetary shock have their persistence determined by this root and by their own individual autoregressiveness. The only way that this persistence can be reduced is by the terms in ϕ which reflect the strength of the response to higher inflation and its pass-through via the Phillips Curve and the IS curve. In our calibration here we have embodied a high long-run inflation response of interest rates (thus long-run $\psi = 10$). The backward root is approximately halved by this. Hence the Inflation Targeting regime under the New Keynesian model does reduce persistence substantially, though not enough to match the data.