WBS Summer School

Randomization and Fairness: Axioms and Objections

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Quick survey: With which, if any, of the following statements do you agree?

1. **You won a free ticket to Hawaii. You want to give it to one of your two children. You love them both and both want to go. Flipping a coin between them is a good idea.**

2. If you prefer to live in London over NY, and you prefer to live in NY over Paris, then you prefer to live in London over Paris.

3. You've reached the final stage in the "Maybe You'll be Lucky" TV show. In this stage, a locked box is put in front of you.

Option 1: The host will plip a coin. Heads: A charity will donate $1,000,000 to the school board of Northchester county. Tails: You'll get a key to the box and win whatever is in it.

Option 2: The host will plip a coin. Heads: A charity will donate $1,000,000 to the school board of southchester county. Tails: You'll get a key to the box and win whatever is in it.

As you are indifferent between Northchester and Southchester, you are also indifferent between options 1 and 2.

Basic problem: How to allocate s indivisible units of a certain good among $n > s$ claimants.

Examples:

1. s kidneys, n patients.

2. s dorms, n students.

3. s seats in the last helicopter from Saigon, n embassy workers.

The analysis will be more philosophical / theoretical, less practical.

Call allocations "policies" and denote them $x, y, z \ldots$

Harsanyi (1955): If society can choose one of $\{x_1, \ldots, x_k\}$ policies, then it can choose a lottery

$$(x_1, p_1; \ldots; x_k, p_k)$$

Over these policies.

The policies $x_1, \ldots, x_k$ are given, so society has to choose the probabilities

$$p = (p_1, \ldots, p_k)$$

All probabilities are ≥ 0 and

$$p_1 + \ldots + p_k = 1$$

Analytical benefit: Preferences over social policies can be analyzed using tools from decision theory (that is, the analysis of preferences over lotteries).

For example: Expected utility (EU) theory, where the value of the lottery

$$(x_1, p_1; \ldots; x_k, p_k)$$

is

$$p_1 u(x_1) + \ldots + p_k u(x_k)$$

Observe the combination of objective probabiliyies and subjective utilities.

Harsanyi's model of utilitarianism:

1. **Individuals have expected utility preferences over social lotteries.**

2. **Society has expected utility preferences over social lotteries.**

3. **(Pareto): If no one prefers q to p, and at least one person prefers p to q, then society prefers p to q.**

Theorem: $W(p) = \alpha_1 \mathrm{E}[u_1(p)] + \ldots + \alpha_n \mathrm{E}[u_n(p)]$ $\alpha_1, \ldots, \alpha_n > 0$.

For simple act x:

$$W(x) = \alpha_1 u_1(x) + \ldots + \alpha_n u_n(x)$$

Diamond's criticism: Under Harsanyi, social lotteries don't make society better off, which seems strange.

For this, see Diamond: "Cardinal welfare, individualistic ethics, and interpersonal comparison of utility," JPE 1967.

Suppose we have one unit of an indivisible good (kidney) and we want to give it to one of two individuals, 1 with the utility function u_1 and 2 with the utility function u_2.

Suppose:

1. $u_1(1) = 1,\ u_1(0) = 0$

2. $u_2(1) = 1,\ u_2(0) = 0$

3. $W(u_1, u_2) = u_1 + u_2$**.**

Let a **be the policy that gives the good to person 1 and** b **be the policy that gives the good to person 2. We have**

	Policy a	Policy b
u_1	1	0
u_2	0	1
Sum	1	1

Add now a third policy c which is a coin flip between the two. We get that the utility of person 1 from this policy is

$$u_1(c) = u_1\left(1, \tfrac{1}{2}; 0, \tfrac{1}{2}\right) =$$

$$\tfrac{1}{2}u_1(1) + \tfrac{1}{2}u_1(0) = \tfrac{1}{2}$$

Likewise, $u_2(c) = \frac{1}{2}$.

	Policy a	Policy b	Policy c
u_1	1	0	$\frac{1}{2}$
u_2	0	1	$\frac{1}{2}$
Sum	1	1	1

In other words, if society is indifferent between the two individuals, flipping a coin between them will not make society better off.

Diamond (and others) claim that this is so obviously wrong, that it makes the whole utilitarian approach useless.

Why do people prefer society to flip a coin?

Fair. In "Iphigeneia in Aulis" by Euripides, Clytemnestra (Agamemnon's wife), tells him when she learns the truth:

> **Would it not have been fair**
> **to say to the Achaians "Men of Argos,**
> **you want to sail to Troy. Draw lots.**
> **Let us see**
> **whose daughter will die." That way**
> **would have had its justice.**

Simple: What other criteria would you like to use?

- **Age: How will you rank 60 years, 25 years, 5 days?**

- **Family: Parents or children?**

  Children need support. But notice the heartbreaking feeling after reading "For sale: Baby shoes. Never worn."

And what about poeple who don't have children?

- **Contribution to society: Surely we are in full agreement regarding the eminent contribution of econ prof's?...**

  Also, what is the meaning of contribution if it is rewarded?

Easier (morally) for the social planner.

Counter arguments (by Harsanyi):

Fair: Lotteries don't create egalitarian allocations. At the end the allocation is 1-0 or 0-1.

Easier: Don't fool yourself. You don't have to follow the coin.

Next aim: To construct a formal (that is, axiomatic) model that will permit strict preferences for randomization.

Central issue: What is the source of preferences for randomization?

Epstein and Segal: "Quadratic social welfare functions," JPE 1994.

The structure is similar to that of Harsanyi:

n individuals

X: A set of k social options

Lotteries over X

Individual preferences over such lotteries.

Social preferences over these lotteries.

Individual preferences are EU

Pareto

Social preferences satisfy EU assumption, except for the independence axiom, which is replaced with:

Mixture Symmetry: $p \sim q$ implies for all $\alpha \in [0, 1]$,

$$(p, \alpha; q, 1 - \alpha) \sim (p, 1 - \alpha; q, \alpha)$$

Justification: If $p \sim q$ and we have a biased $\alpha : 1 - \alpha$ coin, it doesn't matter which side is linked to which outcome.

Preferences for Randomization: $p \sim q$ and for some i, $p \precsim_i q$ imply

$$(p, \tfrac{1}{2}; q, \tfrac{1}{2}) \succ p \sim q$$

Justification: If $p \succ_i q$ but $p \sim q$, then by Pareto there is j such that $q \succ_j p$. Flipping a coin is a natural compromise.

Theorem: The above axioms are satisfied if and only if the social welfare function is given by

$$W(u_1, \ldots, u_n) = Q(u_1, \ldots, u_n) =$$

$$\sum_i a_i u_i^2 + \sum_{i>j} b_{ij} u_i u_j + \sum_i c_i u_i$$

Example: Mean–variance: $\mu - \alpha\sigma^2$.

This demonstrates how ideas from modern decision theory (non EU models) can have a social choice interpretation.

Exactly as Harsanyi did utilitarianism based on EU.

In this approach, the source of preferences for randomization is at the social level.

Individuals are expected utility maximizers and care only for their own welfare.

In particular, they are indifferent to randomizations.

Karni and Safra: "Individual Sense of Justice: A Utility Representation," Econometrica 2002.

Each person has three sets of preferences over lotteries:

$\succeq_S$: Self-interest preferences.

$\succeq_F$: Fairness preferences.

$\succeq$: Actual preferences, revealed by choice.

The source for preferences for randomization is individual sense of justice.

American Geography:

What is the capital of Nebraska?

1. Lincoln

2. Topeka

3. Omaha

4. Des Moines

English History:

Who succeeded King Richard II?

1. **Richard III**
2. **Henry IV**
3. **Edward II**
4. **Oliver Cromwell**

Facts of Life:

What is the weight of a fetus at the end of the first trimester?

1. 25g

2. 50g

3. 100g

4. 200g

A new study reveals abysmal cluelessness about geography, history, and biology among young aspiring medical decision theorists.

___% believe that the capital of Nebraska is _____.

___% believe that _____ succeeded King Richard II.

___% believe that the weight of the fetus at the end of the first trimester is _____.