

ON THE PREDICTABILITY OF TICK BY TICK EXCHANGE RATES USING THE STRUCTURE OF THE ORDER BOOK

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Outline

- 2 Genetic Algorithm
- 3 Testing procedures
 - Pesaran-Timmermann test
 - Anatolyev-Gerko test
 - White's Reality Check
- 4 Results
- 5 Conclusions

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Genetic Algorithm

- Genetic programming is a computer based optimization procedures which uses the evolutionary principles.
- The genetic algorithm evaluates the fitness of various candidate solutions using the given objective function of the optimization problem and provides as an output solutions that have higher fitness values.
- Operation of crossover and mutation are applied to create new generation based on the genetic information of more fit solution candidates.

Genetic tree consists of:

- Functions

binary algebraic

{+, -, *, /, max, min}

binary order relations

{<, >, ≤, ≥, =}

logical operations

{and, or}

unary functions

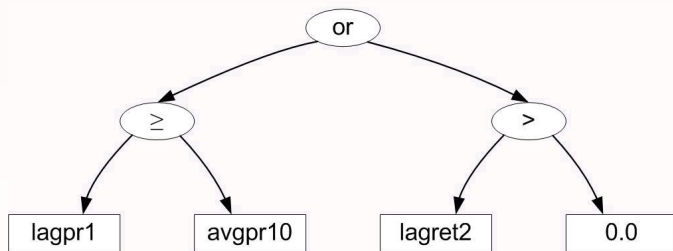
{abs, -}

- Variables

price	bestbid
ret	bestoff
lagpr	bestbidq
lagret	bestoffq
maxpr	quant
minpr	liqbid
avgpr	liqoff
avgret	depthbid
	depthoff
	time

- Real Numbers

Example

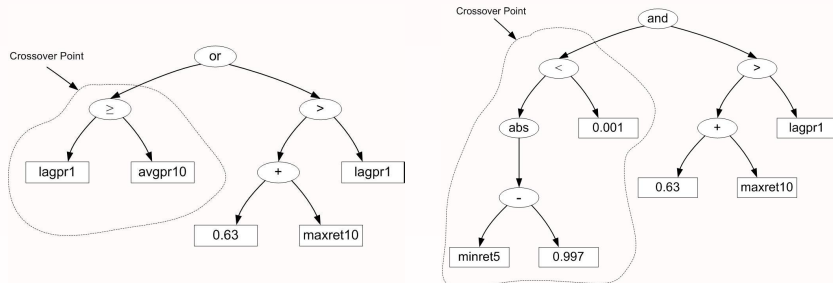


With this trading rule the investor is going to take a long position in USD if $(lagpr1 \geq avgpr10)$ and $(lagret2 > 1.0)$ and short one otherwise.

The evolutionary algorithm

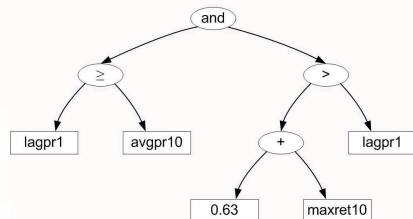
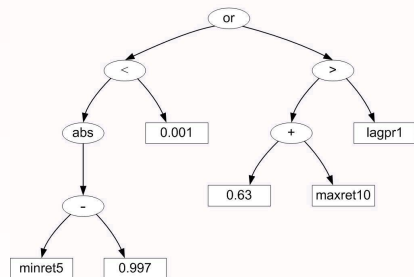
- 1 Create randomly the initial population $P(0)$ and initialize the number of iterations $i = 0$.
- 2 Set $i := i + 1$.
- 3 Evaluate fitness of each tree in the population using the fitness function.
- 4 Generate new population using genetic operations (crossover and mutation)
- 5 Repeat 2) – 5) while $i < N$.

Crossover: select two parents



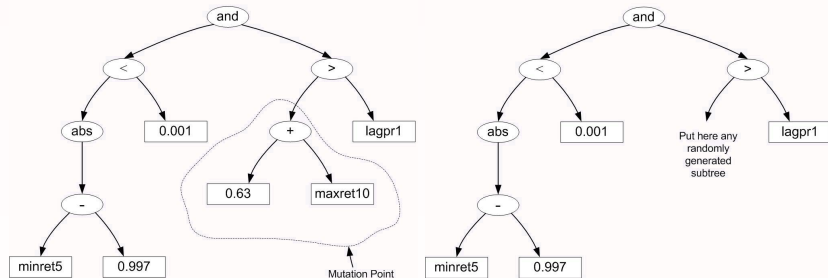
For both of parents we randomly select nodes to crossover

Crossover: get offsprings



Two offsprings after the crossover operation

Mutation: randomly select a node ...



... and substitute it with a random subtree

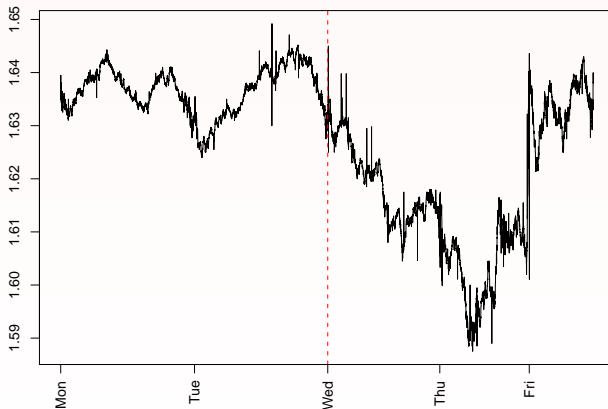
Parameters

- population size – 100 individuals;
- number of evolutionary periods – $N = 50$;
- crossover rate – 40%;
- mutation probability – 0.0 for best 25% and 0.3 for others;

Fitness functions

- DC – percentage of correct predictions of directional changes;
- R – cumulative returns adjusted to the daily basis;
- R^{TC} – cumulative returns under transaction costs;

Data



Tick-by-tick USD/DM exchange rate time series from 5.10.1998 to 9.10.1998

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- Testing procedures

- Pesaran-Timmermann test

Pesaran and Timmermann (1992)

- e_t – realised value of the exchange rate;
- z_t – its forecast;

$$P_{11} = P(z_t < 0, e_t < 0), \quad P_{12} = P(z_t < 0, e_t \geq 0),$$

$$P_{21} = P(z_t \geq 0, e_t < 0), \quad P_{22} = P(z_t \geq 0, e_t \geq 0)$$

$$P_{i0} = P_{i1} + P_{i2}, \quad P_{0i} = P_{1i} + P_{2i}.$$

$$H_0 : \sum_{i=1}^m (\hat{P}_{ii} - \hat{P}_{i0}\hat{P}_{0i}) = 0$$

i.e., realised values and forecasts are independent

- Testing procedures

- Pesaran-Timmermann test

P-T Statistic

$$s_n = \sqrt{n} V_n^{-\frac{1}{2}} S_n \stackrel{a}{\sim} N(0, 1),$$

where

$$S_n = \sum_{i=1}^m (\hat{P}_{ii} - \hat{P}_{i0} \hat{P}_{0i})$$

$$V_n = \left(\frac{\partial f(\mathbf{P})}{\partial \mathbf{P}} \right)'_{\mathbf{P}=\hat{\mathbf{P}}} (\hat{\Psi} - \hat{\mathbf{P}} \hat{\mathbf{P}}') \left(\frac{\partial f(\mathbf{P})}{\partial \mathbf{P}} \right)_{\mathbf{P}=\hat{\mathbf{P}}}$$

$$\hat{\Psi} = \text{diag}(\hat{\mathbf{P}}),$$

$$\left(\frac{\partial f(\mathbf{P})}{\partial \mathbf{P}} \right)_{\mathbf{P}=\hat{\mathbf{P}}} = \begin{cases} 1 - P_{0i} - P_{i0} & \text{for } i = j \\ -P_{j0} - P_{0i} & \text{for } i \neq j \end{cases}$$

└ Testing procedures

└ Anatolyev-Gerko test

Anatolyev and Gerko (2005)

- r_t – observed log-returns of the exchange rate;
- \hat{r}_t – forecasts, depend on past inform. $\mathcal{F}_{t-1} = \{r_{t-1}, r_{t-2}, \dots\}$;

Strategy: take a long position in USD if $\hat{r}_t \geq 0$ and a short position if $\hat{r}_t < 0$;

One-period return from trading the strategy is $R_t = \text{sign}(\hat{r}_t) \cdot r_t$.

$$H_0: E(r_t | \mathcal{F}_{t-1}) = \text{const}$$

i.e., \hat{r}_t and r_t are independent.

- └ Testing procedures
 - └ Anatolyev-Gerko test

A-G Statistic

$E(R_t)$ consistent estimators:

$$A_n = \frac{1}{n} \sum_t R_t$$

$$B_n = \left(\frac{1}{n} \sum_t \text{sign}(\hat{r}_t) \right) \left(\frac{1}{n} \sum_t r_t \right).$$

$$EP = \frac{A_n - B_n}{\sqrt{\hat{V}}} \xrightarrow{d} N(0, 1)$$

$$\hat{V} = \frac{4}{n^2} \hat{p}_r (1 - \hat{p}_r) \sum_t (r_t - \bar{r})^2 \text{ with } \hat{p}_r = \frac{1}{2} \left(1 + \frac{1}{n} \sum_t \text{sign}(\hat{r}_t) \right).$$

- └ Testing procedures
- └ White's Reality Check

White (2000)

- Data snooping involves the re-use of the same sample data in multiple hypothesis testing
- not recognising the impact that prior decisions in the testing sequence have on the significance levels to be used at any stage subsequently down the chain.
- the Reality Check enables us to calculate true empirical p -values of the test statistic based on the null of no predictability beyond the benchmark.

- └ Testing procedures
 - └ White's Reality Check

The Reality Check Procedure

φ_k , ($k = 1, \dots, M$) – performance measure of the k -th trading rule relative to some benchmark;

The aim is to test whether there is a rule within the population that delivers superior performance to the benchmark.

$$H_0: \max_{k=1, \dots, M} \varphi_k \leq 0$$

Test statistic:

$$V = \max_{k=1, \dots, M} \varphi_k.$$

- └ Testing procedures
 - └ White's Reality Check

The Reality Check Procedure: p -values

$\varphi_{k,j}^*$ – performance measure of the k -th trading rule calculated using j -th bootstrap sample.

The empirical distribution based on the bootstrap realizations

$$V_j^* = \max_{k=1, \dots, M} (\varphi_{k,j}^* - \varphi_k), j = 1, \dots, B.$$

White(2000) – distributions of V and V^* are asympt. equivalent.

Comparing V with the quantiles of the empirical distribution of V^* we obtain the Reality Check p -values which are suitable for testing the null hypothesis.

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Conclusions

- We examine the predictability of exchange rates on a transaction level basis using both past transaction prices and the structure of the order book;
- The Pesaran-Timmermann test shows clear ability to predict directional changes in the exchange rate;
- The economic value under transaction costs is shown to be significant out-of-sample using the Anatolyev-Gerkov test;
- Predictability and profitability depend critically on the frequency of trades which is controlled the inertia parameter k ;
- These conclusions are confirmed using White's Reality Check;
- We do not find strong evidence that exploiting the order book structure aids predictability.

- Directional Changes

- Pure Price Information

 $k = 0$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(ret \leq minret10)$ or $(lagpr1 > avgpr5)$	55.35	52.90	5.786	-9.154	4.21	< 0.0001	2.47	0.0068	0.0175
2. $(ret < maxret10)$ and $(lagpr1 > min\{avgpr10, avgpr20\})$	55.20	53.61	7.231	-2.317	5.23	< 0.0001	3.06	0.0011	0.0065
3. $(ret < maxret10)$ and $(lagpr1 > avgpr5)$	55.08	53.67	8.208	-6.090	5.35	< 0.0001	3.49	0.0002	0.0055
4. $(ret \leq minret10)$ or $(lagpr1 \geq avgpr10)$	55.06	53.52	4.677	-6.345	5.12	< 0.0001	2.01	0.0222	0.0075
5. $(price \geq maxpr20)$ or $(maxret5 = lagret1)$ or $(avgret20 \leq maxret5)$ and $(price \geq avgpr20)$	55.00	53.88	-7.663	-12.569	5.62	< 0.0001	-3.49	>0.5000	0.0055
6. $(lagpr1 \geq avgpr10)$ or $(lagpr1 > avgpr5)$	54.98	53.63	2.284	-8.082	5.30	< 0.0001	1.00	0.1587	0.0055
7. $(lagpr1 \geq avgpr20)$ and $(maxret5 = avgret5)$ or $(price \geq avgpr20)$	54.98	53.88	-8.089	-12.856	5.62	< 0.0001	-3.70	>0.5000	0.0055
8. $lagpr1 > \min\{avgpr10, lagpr2\}$	54.96	54.00	2.760	-8.768	5.83	< 0.0001	1.20	0.1151	0.0055
9. $(price \geq avgpr20)$ and $(maxret5 = lagret1)$ or $(price > avgpr20)$	54.94	53.84	-8.126	-12.938	5.56	< 0.0001	-3.71	>0.5000	0.0055
10. $lagpr1 > \min\{avgpr5, avgpr10\}$	54.93	53.58	2.272	-8.154	5.21	< 0.0001	0.99	0.1611	0.0065
11. $price > avgpr20$	54.93	53.84	-8.126	-12.938	5.56	< 0.0001	-3.71	>0.5000	0.0055
12. $(price \geq avgpr20)$ or $(maxret5 = avgret5)$	54.91	53.73	-8.217	-13.278	5.39	< 0.0001	-3.76	>0.5000	0.0055
13. $minpr20 < minpr10 \cdot lagret1$	53.90	52.77	-2.096	-12.556	4.01	< 0.0001	-0.93	>0.5000	0.0024
14. $lagpr1 > \min\{avgpr5, avgpr20\}$	54.89	53.46	2.657	-6.359	5.13	< 0.0001	1.17	0.1210	0.0075
15. $price \geq avgpr20$	54.89	53.83	-8.153	-12.963	5.53	< 0.0001	-3.73	>0.5000	0.0055
16. $lagpr1 > avgpr20$	54.71	53.48	2.910	-2.701	5.03	< 0.0001	1.26	0.1038	0.0075
17. $(lagpr1 > avgpr5)$ or $(lagpr1 = avgpr10)$	54.69	53.40	3.742	-2.701	4.93	< 0.0001	1.61	0.0537	0.0085
18. $(price > avgpr10)$ or $(ret = minret20)$	54.69	54.98	3.742	-9.049	7.28	< 0.0001	-0.17	>0.5000	0.0025
19. $lagpr1 > avgpr5$	54.64	53.34	3.730	-9.108	4.84	< 0.0001	1.61	0.0537	0.0105
20. $(price > avgpr20)$ or $(minpr10 > minpr20)$	54.60	52.42	-9.701	-13.766	3.53	0.0002	-4.51	>0.5000	0.0305
21. Average	55.10	54.11	2.150	-6.123	5.98	< 0.0001	0.93	0.1762	0.0055

Trading rules based on the pure price information only. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Directional Changes

- Pure Price Information

$$k = 0.0005$$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $avgr10 > minret20 \cdot maxret20$	56.01	51.28	6.040	3.133	1.07	0.1423	2.57	0.051	0.2345
2. $avgr5 > minret20 \cdot maxret20$	55.57	50.67	7.315	4.193	0.61	0.2709	3.10	0.0010	0.3675
3. $(minret10 \geq ret)$ or $(avgr20 \cdot lagret2 \cdot maxret10 \leq lagpr1)$	56.60	50.87	4.629	-0.473	0.55	0.2912	2.02	0.0217	0.3115
4. $(minret10 \geq lagret2)$ or $(avgr20 \cdot ret \cdot avgr20 \leq lagpr1)$	55.72	54.04	6.728	1.692	3.25	0.0006	2.87	0.0021	0.0065
5. $minret10 \geq \min(lagret2, ret)$	56.16	51.95	6.261	1.308	1.35	0.0885	2.75	0.0030	0.1255
6. $(lagret1 \geq (price - avgr20) \cdot 1743.99)$ or $(maxret10 = avgr20)$	54.99	51.35	9.878	6.894	1.36	0.0869	4.46	<0.0001	0.2215
7. $(price < avgr10)$ or $(price \cdot minret20 < minpr5 \cdot lagret1)$	55.13	50.34	6.081	2.536	0.72	0.2358	2.99	0.0014	0.4355
8. $(avgr10 < avgr5)$ or $(avgr5 > avgr10 \cdot maxret20 \cdot minret5)$	54.69	50.34	0.990	-3.196	0.71	0.2389	0.44	0.3300	0.4355
9. $(minret10 \geq ret)$ or $(lagret1 \geq lagret2)$	54.99	52.29	3.267	-1.362	2.21	0.0136	1.52	0.0643	0.0955
10. $(avgr10 \geq price)$ or $(maxret10 > \max(avgr10, ret))$	54.40	49.80	6.137	2.966	0.43	0.3336	3.38	0.0004	0.5565
11. $(minret20 > avgr20 \cdot minret10)$ or $(lagret1 > lagret2)$	54.55	50.27	-2.792	-6.504	0.63	0.2643	-1.44	>0.5000	0.4455
12. $(minret10 \geq lagret1)$ or $(maxret5 \cdot minret10 > ret)$	55.57	53.90	5.381	0.168	2.95	0.0016	2.32	0.0102	0.0085
13. $avgr5 > avgr10 \cdot maxret20 \cdot minret5$	54.40	52.15	5.909	2.349	1.47	0.0708	2.83	0.0023	0.1055
14. $(minret10 \geq ret)$ and $((lagpr2 - maxpr10) \geq maxret5 - lagret2)$	54.99	51.55	4.727	0.977	0.92	0.1788	2.41	0.0080	0.1785
15. $(minret10 = minret5)$ or $(lagpr1 > lagpr2)$	54.40	50.61	1.714	-1.561	1.10	0.1357	0.87	0.1922	0.3805
16. $(avgr10 \geq price)$ or $(lagret1 > avgr20)$	54.25	50.67	4.189	0.995	1.17	0.1210	2.17	0.0150	0.3675
17. $lagpr2 = minpr5$ or $(ret = minret10)$	54.25	51.68	6.172	1.575	1.07	0.1423	2.96	0.0015	0.1575
18. $(maxret10 > avgr10 \cdot ret)$ and $(minret10 = minret5)$	54.25	51.14	3.813	-0.623	1.06	0.1446	1.67	0.0475	0.2595
19. $(lagret1 > ret \cdot 0.9994)$ and $((lagpr1 > maxpr10 \cdot 0.9994)$ or $(minret5 < 0.9994))$	53.81	53.70	10.212	5.792	0.13	0.4483	-0.51	>0.5000	0.0105
20. $(avgr10 < avgr5)$ or $(avgr10 > lagret2 \cdot maxret20)$	53.67	50.13	-1.159	-5.829	3.54	0.0002	4.96	<0.0001	0.4675
21. Average	58.50	51.95	6.559	3.062	1.28	0.1002	2.49	0.0064	0.1255

Trading rules based on the pure price information only. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Directional Changes

- Pure Price Information

$k = 0.001$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(avgpr10 \geq maxpr10 - ret)$ or $(lagpr1 = avgpr10)$	60.07	55.49	9.390	7.317	3.17	0.0008	4.52	<0.0001	0.0065
2. $avgpr10 \geq maxpr10 - minret20$	59.72	53.98	5.951	4.165	2.26	0.0119	2.57	0.0051	0.0325
3. $(price \leq minpr20)$ and $(ret = minret10)$	59.36	53.43	6.524	5.092	2.05	0.0201	3.71	0.0001	0.0545
4. $(lagpr1 - lagret2 < avgpr10 - avgret5)$ or $(price < avgpr10)$	59.36	50.69	2.015	0.255	0.25	0.4013	0.91	0.1814	0.3005
5. $(avgret20 \geq avgret10)$ or $(price = minpr10)$	59.36	52.34	5.302	3.292	1.44	0.0749	2.36	0.0091	0.1155
6. $(price < avgpr10)$ or $(avgret20 \geq avgret10)$	59.01	52.47	5.979	4.05	1.61	0.0537	2.77	0.0028	0.1025
7. $(lagpr1 \leq avgpr10)$ and $(price \leq avgpr10)$	59.01	50.82	1.619	-0.120	0.33	0.3707	0.73	0.2327	0.2775
8. $(lagpr1 \geq price)$ and $(maxpr20 - minret20 \geq price)$	59.01	52.88	6.385	4.049	1.51	0.0655	2.73	0.0032	0.0785
9. $(price - maxret20 \leq maxpr20)$ and $(price \leq avgpr10)$	58.66	52.88	7.707	5.553	1.50	0.0668	3.30	0.0005	0.0785
10. $(price = minpr20)$ or $(lagpr1 = minpr20)$	58.31	52.47	5.969	4.141	1.27	0.1020	2.59	0.0048	0.1025
11. $(lagpr1 \leq avgpr20)$ or $(avgret20 > avgret10)$	58.30	48.63	0.531	-1.295	-0.64	>0.5000	0.24	0.4052	0.6275
12. $avgpr10 \geq price$	57.95	54.12	8.870	6.395	2.21	0.0136	3.73	0.0001	0.0295
13. $(minpr10 > avgpr10 - avgret20)$ or $(price < avgpr10)$	57.95	53.98	8.826	6.366	2.14	0.0162	3.71	0.0001	0.0325
14. $price \leq minpr20$	57.95	53.98	8.987	7.007	2.12	0.0170	3.93	<0.0001	0.0325
15. $(maxpr5 - avgret5 < avgpr10)$ or $(minret20 = ret)$	56.89	54.95	7.784	5.312	2.65	0.0040	3.36	0.0004	0.0115
16. $(price \leq avgpr10)$ or $(price \leq minpr5)$	57.60	53.98	8.636	6.138	2.14	0.0162	3.63	0.0001	0.0325
17. $(lagpr2 - lagret2 \geq minpr20)$ and $(lagpr1 - avgret5 \geq price)$	57.60	53.16	7.288	4.500	1.69	0.0455	3.09	0.0010	0.0655
18. $price = minpr10$	57.24	54.12	8.214	5.980	2.19	0.0143	3.52	0.0002	0.0295
19. $(minpr5 \geq price)$ or $(avgpr5 \geq lagpr1)$	56.54	53.02	3.749	1.613	1.73	0.0418	1.63	0.0516	0.0735
20. $(lagpr1 < avgpr5)$ or $(avgret20 > avgret5)$	55.83	49.73	0.726	-1.453	-0.09	>0.5000	0.32	0.3745	0.4465
21. Average	58.66	54.12	8.051	5.763	2.19	0.0143	3.42	0.0003	0.0295

Trading rules based on the pure price information only. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Directional Changes

- Pure Price Information

 $k = 0.005$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(\text{minret}20 = \text{ret})$ or $(\text{lagret}1 \leq \text{minret}20)$	84.00	60.38	4.085	3.711	2.21	0.0136	1.76	0.0392	0.0865
2. $(\text{minret}5 - \text{maxret}10 \leq \text{avgret}5)$ or $(\text{minpr}20 = \text{lagpr}1)$	76.00	60.38	4.974	4.604	2.21	0.0136	2.21	0.0136	0.0865
3. $\text{minret}5 \geq \text{ret}$	76.00	72.64	9.634	9.145	5.44	<0.0001	4.19	<0.0001	0.0015
4. $\text{maxpr}5 > \text{max}\{\text{price}, \text{lagpr}2\}$	72.00	66.04	6.027	5.582	3.53	0.0002	2.63	0.0043	0.0185
5. $(\text{lagpr}2 \geq \text{maxpr}20 - \text{minret}10)$ or $(\text{ret} = \text{minret}5)$	72.00	66.04	7.637	7.207	3.53	0.0002	3.35	0.0004	0.00185
6. $(\text{lagpr}1 = \text{minpr}10)$ or $(\text{lagpr}1 = \text{maxpr}5)$	72.00	42.45	-3.324	-3.627	-1.75	>0.5000	-1.53	>0.5000	0.8345
7. $(\text{ret} \leq \text{minret}10)$ or $(\text{avgpr}20 \geq \text{max}\{\text{avgpr}10, \text{lagpr}1\})$	72.00	64.15	6.356	5.981	3.13	0.0009	2.89	0.0019	0.0295
8. $(\text{max}\{\text{price}, \text{avgpr}20\} < \text{lagpr}1)$ or $(\text{maxret}5 = \text{lagret}1)$	72.00	69.81	8.085	7.639	4.70	<0.0001	3.59	0.0002	0.0045
9. $\text{lagpr}1 \geq \text{maxpr}5$	68.00	55.66	2.026	1.843	1.75	0.0401	1.13	0.1292	0.2225
10. $\text{price} < \text{maxpr}5$	68.00	67.92	8.212	7.751	4.01	<0.0001	3.54	0.0002	0.0125
11. $(\text{price} < \text{lagpr}2)$ and $(\text{lagpr}2 < \text{avgpr}5)$	68.00	64.15	4.847	4.491	3.45	0.0003	2.45	0.0071	0.0295
12. $(\text{price} < \text{avgpr}10)$ or $(\text{minpr}10 = \text{lagpr}1)$	68.00	61.32	5.565	5.220	2.47	0.0068	2.59	0.0048	0.0725
13. $(\text{ret} < \text{maxret}20)$ and $(\text{ret} - \text{maxret}10 < \text{lagret}2)$	68.00	67.92	9.070	8.649	4.24	<0.0001	4.03	<0.0001	0.0125
14. $\text{maxpr}5 - \text{minret}10 \leq \text{lagpr}2$	64.00	62.26	6.385	6.031	2.62	0.0044	2.83	0.0023	0.0595
15. $(\text{ret} - \text{maxret}10 < \text{lagret}2)$ and $(\text{lagret}2 < \text{maxret}10)$	64.00	66.04	6.872	6.537	4.08	<0.0001	3.32	0.0005	0.0185
16. $(\text{ret} < \text{lagret}1)$ and $(\text{lagret}2 < \text{maxret}5)$	64.00	66.98	5.460	5.074	3.90	<0.0001	2.57	0.0051	0.0135
17. $(\text{ret} < \text{lagret}1)$ and $(\text{lagpr}2 < \text{maxpr}5)$	64.00	72.64	7.541	7.062	5.33	<0.0001	3.30	0.0005	0.0015
18. $(\text{ret} < \text{lagret}1)$ or $(\text{lagret}1 \leq \text{minret}20)$	64.00	66.98	5.956	5.561	3.85	0.0001	2.75	0.0030	0.0135
19. $(\text{ret} \leq \text{lagret}1)$ or $(\text{lagpr}1 \leq \text{min}\{\text{minpr}10, \text{avgpr}10\})$	64.00	61.32	4.091	3.752	2.50	0.0062	1.81	0.0351	0.0725
20. $\text{price} \leq \text{lagpr}2$	64.00	66.04	7.573	7.143	3.49	0.0002	3.21	0.0007	0.0185
21. Average	76.00	67.92	8.340	7.885	3.97	<0.0001	3.54	0.0002	0.0125

Trading rules based on the pure price information only. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Directional Changes

- Pure Price Information

$$k = 0.01$$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(1.6369 > \text{avgpr}10 \cdot \text{ret})$ and $(\text{lagret}2 \geq \text{avgret}5)$	100.00	56.41	1.536	1.414	0.79	0.2148	0.67	0.2514	0.2575
2. $(1.6369 > \text{avgpr}5 \cdot \text{ret})$ and $(\text{lagret}2 \geq \text{avgret}5)$	100.00	56.41	1.536	1.414	0.79	0.2148	0.67	0.2514	0.2575
3. $(\text{price} \leq \text{maxpr}10 \cdot \text{avgret}10)$ and $(\text{maxret}10 > \text{lagret}1)$	100.00	38.46	-1.775	-1.886	-1.60	>0.5000	-0.98	>0.5000	0.7505
4. $\text{lagret}2 > \text{avgret}5$	83.33	46.15	-2.262	-2.39	-0.48	>0.5000	-1.01	>0.5000	0.5565
5. $\text{maxret}5 < \text{maxret}10$	83.33	56.41	1.498	1.383	0.88	0.1894	0.65	0.2578	0.2575
6. $(\text{avgpr}5 > \text{lagpr}2)$ and $(\text{lagret}1 < \text{maxret}20)$	83.33	61.54	0.585	0.458	1.53	0.0630	0.26	0.3974	0.1525
7. $\text{minret}5 > (0.167 + \text{maxpr}5) \cdot 0.556$	83.33	61.54	2.064	2.003	1.47	0.0708	0.90	0.1841	0.1525
8. $(\text{ret} \leq \text{lagret}1)$ and $(\text{avgret}5 < \text{avgret}10)$	83.33	66.67	3.966	3.862	2.63	0.0043	1.84	0.0329	0.0845
9. $\text{avgpr}5 > \text{lagpr}1$	83.33	58.97	-0.451	-0.610	1.16	0.1230	-0.22	>0.5000	0.1985
10. $\text{maxret}10 > \text{lagret}1$	83.33	35.90	-2.186	-2.284	-2.27	>0.5000	-1.29	>0.5000	0.8065
11. $\text{lagret}2 = \text{maxret}5$	83.33	51.28	0.897	0.817	0.08	0.4681	0.49	0.3121	0.4055
12. $\text{avgpr}10 \leq \text{minpr}5$	83.33	43.59	-1.156	-1.228	-1.43	>0.5000	-0.66	>0.5000	0.6205
13. $(\text{lagpr}2 > \text{lagpr}1)$ and $(\text{avgpr}10 \geq \text{avgpr}5)$	66.67	53.85	-0.404	-0.517	0.47	0.3192	-0.22	>0.5000	0.3055
14. $\text{minret}20 < \text{minret}10$	66.67	53.85	0.372	0.299	0.49	0.3121	0.24	>0.5000	0.3055
15. $\text{lagret}1 < 1$	66.67	51.28	0.388	0.254	0.15	0.4404	0.17	0.4325	0.4055
16. $\text{price} > \text{maxpr}10$	66.67	51.28	-0.055	-0.062	0.00	0.5000	0.00	0.5000	0.4055
17. $\text{minret}10 = \text{avgret}5$	66.67	51.28	-0.055	-0.061	0.00	0.5000	0.00	0.5000	0.4055
18. $\text{avgpr}10 \geq \text{lagpr}1$	66.67	66.67	0.786	0.619	2.20	0.0139	0.35	0.3632	0.0845
19. $\text{lagret}2 = \text{avgret}10$	66.67	51.28	-0.055	-0.062	0.00	0.5000	0.00	0.5000	0.4055
20. sell	66.67	51.28	-0.055	-0.061	0.00	0.5000	0.00	0.5000	0.4055
21. Average	83.33	64.10	2.310	2.208	2.27	0.0116	1.11	0.1335	0.1105

Trading rules based on the pure price information only. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Directional Changes

- All Order Book Information

 $k = 0$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. (bestbid > avgr20) or (bestbid > price) or (bestbid > bestbid2)	58.61	57.30	6.378	-5.628	10.70	< 0.0001	2.61	0.0045	< 0.001
2. (maxpr20 < bestoff) or (bestbid > avgr20) or (bestbid > price)	58.61	57.05	6.307	-3.00	10.32	< 0.0001	2.60	0.0047	< 0.001
3. (bestbid > min(avgr20, price)) or (offcount < 0.4376 + lagr2)	58.07	56.80	6.056	-2.923	10.01	< 0.0001	2.51	0.0060	< 0.001
4. (bestbid > avgr20) or (bestbid > price)	57.93	56.76	6.247	-2.748	9.95	< 0.0001	2.59	0.0048	< 0.001
5. (bestbid2 < bestbid) or (minpr5 < bestbid)	57.84	56.64	4.809	-7.517	9.71	< 0.0001	1.98	0.0239	< 0.001
6. bestbid > min(depthbid1, offcount, minpr5, avgr20)	57.60	56.45	4.770	-5.067	9.45	< 0.0001	1.98	0.0239	< 0.001
7. (maxpr5 > avgr20) and (bestoff > maxpr5)	57.35	56.84	4.571	-3.750	10.36	< 0.0001	1.96	0.0250	< 0.001
8. (bestoff = maxpr5) or (bestoff > maxpr10) or (4.3234 >= bidcount1 - minret20)	57.29	56.93	8.130	-2.763	10.14	< 0.0001	3.31	0.0005	< 0.001
9. (bestoff > avgr20) and (bestoff > maxpr5)	57.10	56.91	4.976	-4.467	10.26	< 0.0001	2.09	0.0183	< 0.001
10. (5 > depthbid) or (bestbid > bestbid2) or (maxpr10 <= max(lagr2, max(bestbid2, bestoff), bestbid))	57.06	55.55	3.871	-6.979	8.14	< 0.0001	1.61	0.0537	< 0.001
11. (bestoff = maxpr5) or (bestoff > maxpr10)	56.97	57.07	8.135	-2.662	10.41	< 0.0001	3.33	0.0004	< 0.001
12. (maxpr10 <= bestoff) or (ret <= minret5)	56.95	56.26	9.653	-3.880	9.20	< 0.0001	3.93	< 0.0001	< 0.001
13. (maxpr10 = lagr1) or (min(maxpr10, depthoff) <= bestoff) or (bidcount - depthbid <= min(time2, 5 + bestbid1) - (9.3 - lagr2))	56.87	57.07	9.737	-0.552	10.32	< 0.0001	3.93	< 0.0001	< 0.001
14. minpr5 < bestbid	56.75	56.07	6.324	-5.207	8.85	< 0.0001	2.59	0.0048	< 0.001
15. (bestbid > minpr20) or (bestoff1 - bidcount2) <= 0.14 + max(liqoff2, $\frac{bidcount2}{4.38}$)	56.56	55.20	2.864	-4.045	8.33	< 0.0001	1.31	0.0951	< 0.001
16. maxpr10 <= bestoff	56.29	56.85	6.261	-2.858	10.33	< 0.0001	2.66	0.0039	< 0.001
17. (bestoff >= maxpr20) or (offcount2 < 1.5)	56.25	55.10	2.188	-4.491	8.46	< 0.0001	1.06	0.1446	< 0.001
18. (bestoff > maxpr20) and (liqoff - max(time1, time2) <= avgr5 - avgr10 - depthoff2)	56.21	55.16	4.367	-2.428	8.57	< 0.0001	2.08	0.0188	< 0.001
19. (bestoff > avgr5) and (bestoff > avgr20)	56.17	55.27	0.676	-6.816	7.70	< 0.0001	0.28	0.3898	< 0.001
20. bestoff > maxpr20	56.15	55.16	4.367	-2.428	8.57	< 0.0001	2.08	0.0188	< 0.001
21. Average	58.76	57.30	5.821	-3.202	10.72	< 0.0001	2.40	0.0082	< 0.001

Trading rules based on the all **order book information**. The fitness function is the percentage of the correct in-sample predictions of the exchange rate **directional changes**. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives a buy signal if more than 9 of best 19 rules give a buy signal and sell otherwise.

- Directional Changes

- All Order Book Information

$k = 0.0005$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. (price < bestoff) and (min(3.61 + offcount2, 2 * bestoff1 - lagpr2 + depthbid2) < depthoff)	60.24	52.86	1.103	-3.133	2.55	0.0054	0.47	0.3192	0.0055
2. (price < bestoff) and (depthoff ≥ 14)	59.06	53.13	3.529	-0.722	2.72	0.0033	1.48	0.0694	0.0025
3. (maxret5 4.0661 - quant1 - bestoffq2 : bestbid2 < 2.1426) and ((bestoff1 ≥ maxpr20) or (bestbid > minpr5))	59.06	54.00	3.411	-0.604	2.97	0.0015	1.50	0.0668	0.0015
4. ((time1 ≤ bestoffq2 + max(bestoff2, maxpr10)) or (bestoffq2 ≥ bidcount2) or (lagret1 ≥ bestbidq1)) or (time1 ≥ ret)	58.76	48.22	-0.167	-4.744	-1.56	0.9418	-0.07	>0.5000	0.5915
5. (maxpr10 - lagret1 + bidcount1 ≤ bestoffq2) or (1.8914 * quant ≤ bestoff1) or (minpr5 < bestbid) or (time2 - 4.5848 ≤ bestoffq2)	58.47	52.72	1.635	-2.473	2.39	0.0084	0.70	0.2420	0.0055
6. (avgpr5 ≥ avgpr20 - min(minret10, $\frac{depthoff}{lagpr2}$) - maxret20) or (depthoff ≥ max(bidcount2, maxret20) - lagpr1 - bestbid2)	58.47	49.83	4.098	1.731	0.43	0.0336	2.03	0.0212	0.2445
7. (bestoffq2 ≥ 3) or (quant1 / (max(bestoffq, quant) - 3.2) > 2.99) or (3.1 ≤ bestoffq1)	58.32	50.10	1.198	-3.210	0.20	0.4207	0.51	0.3050	0.1895
8. (price ≤ bestoff1) and (bidcount > bestoff1)	58.03	50.44	6.019	2.332	0.87	0.1922	2.81	0.0025	0.1495
9. (maxpr10 ≤ bestoff1) or (price ≤ bestoff1)	57.44	50.50	6.422	2.776	0.94	0.1736	3.00	0.0013	0.1425
10. (minret10 = ret) or (bestbid > avgpr10)	57.14	51.98	2.921	-1.476	1.75	0.0401	1.24	0.1075	0.0245
11. (liquid1 ≤ lagpr1 - lagpr2) or (9.2395 ≤ max(min(liquid1, 8.10), 7.3) - quant1 - depthoff2 + quant6.77))	57.14	48.49	-4.297	-5.797	-0.79	>0.5000	-2.74	>0.5000	0.5235
12. (min(8.219, bestoff - minpr5, quant ≥ liqoff) and (bestbidq2 < 8.6459) and (5.504 < offcount))	57.00	51.92	1.609	-1.295	2.03	0.0212	0.72	0.2358	0.0255
13. (min(depthoff2, 2.136) ≤ quant2) or (depthbid1 < depthoff2 - bestoff1)	57.00	48.69	-3.464	-5.546	-0.64	>0.5000	-1.62	>0.5000	0.4735
14. (bestoff2 + bidcount2 < offcount1) or (bidcount1 - liquid1 ≥ 6.31 - bestoff2 - min(offcount1, bestoff1) - liqoff)	56.84	49.02	0.099	-1.719	-0.33	0.6293	0.03	0.4880	0.4025
15. (min(bestoff - minpr5, quant) ≥ liqoff) and (bestbidq2 < 8.6459)	56.85	51.92	2.065	-0.799	2.18	0.0148	0.93	0.1762	0.0255
16. (time1 ≥ ret) and (depthoff1 ≥ max(bestbid + offcount, 9.87, offcount + max(bestbid, minpr20 + ret + 0.17))) and (bestbidq2 ≥ ret)	56.84	49.97	1.719	-2.718	0.10	0.4602	0.72	0.2358	0.2135
17. (min(max(bestoffq2, 6.057), quant2) ≥ 2.809) or (4.3 ≤ bestoffq1 + bestoffq2)	56.70	50.24	2.113	-2.269	0.21	0.4168	0.88	0.1894	0.1725
18. price < bestoff	56.55	53.40	5.924	1.426	3.00	0.0013	2.48	0.0066	0.0015
19. (min(bestoffq1, 8.29) > 4.295 - $\frac{\max(\text{bestbid}, \text{bestbidq2})}{\text{bestret}}$) or (offcount > max(max(bestbid, bestbidq2) - bestbid1, 0.485 - depthbid1))	56.41	50.58	-0.403	-4.455	0.52	0.3015	-0.17	>0.5000	0.1315
20. ((offcount > time2) and (bestoff1 > price)) or (max(depthbid2, bestoffq2) < offcount1)	56.26	49.63	-0.944	-4.948	-0.23	>0.5000	-0.40	>0.5000	0.3705
21. Average	64.51	51.31	3.791	0.918	1.88	0.0301	1.91	0.0281	0.0655

Trading rules based on the all order book information. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives a buy signal if more than 9 of best 19 rules give a buy signal and sell otherwise.

- Directional Changes

- All Order Book Information

$k = 0.001$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $((\max(\text{depthoff2} + \text{bestoffq} - \text{quant}, \text{offcount}) - \text{depthbid2} \geq 4.03) \text{ or } (\text{quant1} > \text{avgrp10}) \text{ or } (\text{bestoffq} - \text{quant} \geq 4.03) \text{ or } (\text{liqoff2} \geq \text{avgrt10})) \text{ and } (\text{time2} < \text{depthbid2})$	63.25	45.82	-1.366	-2.964	-2.13	>0.5000	-0.63	>0.5000	0.7505
2. $(\text{bestoffq} = \text{offcount}) \text{ or } (\text{quant2} \geq \text{bestoff}) \text{ or } ((\text{bestbid1} = \text{minpr10}) \text{ or } (\text{bestoffq} = \text{bidcount2})) \text{ and } (\frac{\text{offcount}}{1.34} > \text{bestoff1})$	62.54	44.58	-1.097	-3.165	-2.83	>0.5000	-0.49	>0.5000	0.8735
3. $((\text{time} \leq \text{avgrp5}) \text{ and } (\text{bidcount2} \geq \frac{5.59}{\text{time} \cdot \text{bestbid1}})) \text{ and } (\text{bestbid2} \geq \text{avgrt10}) \text{ or } (\text{offcount} < \text{minpr5}) \text{ or } ((\text{time} < \text{bidcount2}) \text{ and } (\text{quant1} \geq \text{lagpr1}))$	61.13	46.36	-4.426	-6.269	-2.23	>0.5000	-1.96	>0.5000	0.6765
4. $(\text{quant1} \geq \text{avgrp10}) \text{ or } ((4.39 < \text{bestbidq}/1.115) \text{ and } (\text{time2} < \text{bestbid2}))$	60.78	46.50	1.079	-1.206	-1.82	>0.5000	0.46	0.3228	0.6525
5. $(\text{quant} > 1) \text{ or } (\text{offcount1} \geq \text{depthbid})$	60.78	46.63	0.156	-1.958	-1.73	>0.5000	0.06	0.4671	0.8305
6. $(4 < \text{bestbidq}) \text{ or } (\text{quant1} > 2.8) \text{ or } (\text{depthbid1} - \text{quant1} \leq \text{depthoff1}) \text{ or } (4.54 \leq \text{bestbidq} - \text{avgrt5})$	60.78	48.01	-0.016	-0.958	-0.93	>0.5000	-0.02	>0.5000	0.4175
7. $(\text{minpr5} = \text{bestbid1}) \text{ or } (\text{quant} > \text{bestbid})$	60.42	47.33	0.254	-1.803	-1.26	>0.5000	0.11	0.4562	0.5335
8. $(\text{avgrp10} \geq \text{price}) \text{ and } ((\text{time2} < \text{depthbid}) \text{ or } (6 \geq \text{bidcount1}))$	60.42	54.18	9.084	6.632	2.28	0.0113	3.70	0.0001	0.0025
9. $(\text{quant2} \geq \text{avgrp20}) \text{ and } (\min(\text{avgrp20}, \text{price}) < \text{lagpr2})$	60.42	47.87	1.468	-1.133	-1.10	>0.5000	0.52	0.3015	0.4405
10. $(\text{avgrp10} > \max(\text{bestoff1}, \text{time})) \text{ or } (\text{avgrp10} < \text{quant2})$	60.42	46.91	0.276	-2.117	-1.53	>0.5000	0.11	0.4562	0.5965
11. $(\text{depthoff} > 7.26 + \text{depthbid1}) \text{ or } (\text{bestoff} < \text{quant2})$	60.42	47.60	0.544	-0.715	-1.11	>0.5000	0.26	0.3974	0.4875
12. $(\text{avgrp10} > \text{time}) \text{ or } (\text{avgrp10} < \max(\text{quant}, \text{quant2}))$	60.07	45.68	-0.528	-2.816	-2.20	>0.5000	-0.23	>0.5000	0.7655
13. $(\text{time} < \text{bestbid}) \text{ or } (\text{bestoff} < \text{quant2})$	60.07	45.68	-0.528	-2.816	-2.20	>0.5000	-0.23	>0.5000	0.7655
14. $(\text{avgrp10} > \text{time}) \text{ or } (1 < \text{quant})$	60.07	45.68	-0.528	-2.816	-2.20	>0.5000	-0.23	>0.5000	0.7655
15. $(\text{quant2} > 1) \text{ or } (\text{minpr20} > \text{bestbid})$	60.07	46.36	-1.880	-4.136	-1.82	>0.5000	-0.82	>0.5000	0.6765
16. $(\text{quant1} \geq \text{avgrp5}) \text{ or } (\text{lagpr2} > \text{time1}) \text{ or } (5.82 \geq \text{bidcount2})$	60.07	46.23	-1.539	-3.308	-1.94	>0.5000	-0.74	>0.5000	0.7015
17. $(\text{quant1} \geq \text{time} - \frac{0.892}{\text{minpr10}}) \text{ or } (\maxpr5 < \text{quant})$	60.07	45.68	-0.528	-2.816	-2.20	>0.5000	-0.23	>0.5000	0.7655
18. $(\frac{5.76}{\min(2.76, \text{avgrp20}) \cdot \text{bestbid}} > \text{bidcount2}) \text{ or } (\text{quant1} > \text{bestbid1})$	59.36	45.95	-2.906	-5.196	-2.05	>0.5000	-1.26	>0.5000	0.7285
19. $(\text{time2} + \maxpr10 - \frac{\text{offcount10}}{8.89} \leq 8.81 \cdot \text{bestbidq1} + \text{depthbid1}) \text{ and } (\text{avgrp20} < \text{quant})$	59.01	46.23	-0.012	-2.428	-1.97	>0.5000	-0.01	>0.5000	0.7015
20. $(\text{quant} \geq \text{lagret2} + \min(\frac{\text{offcount2}}{\text{offcount} - \text{price}}, \max(\text{lagret1}, \text{time1}))) \text{ and } (7.35 \leq \text{offcount})$	58.66	45.95	-1.624	-3.783	-2.11	>0.5000	-0.69	>0.5000	0.7285
21. Average	59.72	48.36	0.222	-2.161	-1.83	>0.5000	0.09	0.4641	0.6765

Trading rules based on the all order book information. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives a buy signal if more than 9 of best 19 rules give a buy signal and sell otherwise.

- Directional Changes

- All Order Book Information

k = 0.005

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(1 < \text{bestbid1})$ and $(\text{bestoff} > \text{avgret20} + \text{quam2})$	85.19	47.27	0.430	0.156	-0.46	>0.5000	0.24	0.4052	0.5885
2. $(\text{quant} < \text{bestoffq})$ and $(\text{bestbid} < \min(\text{bestbid2}, \text{depthbid1}, \text{bestoffq}))$	85.19	51.82	-0.145	-0.450	0.65	0.2578	-0.06	>0.5000	0.3325
3. $(\text{quam2} < \text{bestoffq})$ and $((\text{liqbid} > \text{bestbid1})$ or $(\text{bestbid1} < \text{bestbid1}))$	85.19	50.00	1.522	1.184	0.21	0.4168	0.73	0.2327	0.4360
4. $(\text{bestbid2} - \text{maxret10}) - (\text{bidcount2} + \text{bestbid}) < \text{offcount}$ and $(\text{bestoffq} > \text{quam1})$	85.19	53.64	2.272	1.959	0.99	0.1611	0.98	0.1635	0.2545
5. $(\text{liqoff2} \leq 0.14 \cdot \text{offcount2} - \min(\text{liqbid2}, \max(\text{bestbid1}, \text{time1}, 4.15)))$ and $(\text{quam2} < \text{bestoffq})$	85.19	53.64	0.709	0.307	1.00	0.1587	0.31	0.3783	0.2545
6. $(\text{avgret5} < \text{bestbid1})$ and $(\max\{-\max\{\text{bestoffq}, \text{avgret5} - \text{liqbid1}\} \cdot \text{quam2}\}, \text{bestoffq} + (\text{maxpr5} - \text{quam}) \cdot \text{quam2}\} > \text{maxpr10})$	85.19	52.73	2.258	1.877	0.79	0.2148	1.02	0.1539	0.2935
7. $((\text{liqoff2} < \text{liqbid2})$ or $(\text{avgpr20} > \text{avgpr10}))$ and $(\max\{\text{avgpr20}, 3.5433 - \min\{\text{bestoffq}, 2.0178\}\} < \text{bestbid1})$	85.19	50.91	1.445	1.127	0.41	0.3409	0.64	0.2611	0.3775
8. $((\text{liqoff2} < \text{liqbid2})$ or $(\text{avgpr20} > \text{avgpr10}))$ and $(\text{maxpr5} < \text{bestbid1})$	81.48	50.91	0.895	0.580	0.41	0.3409	0.40	0.3446	0.3775
9. $(\text{bestbid2} \leq 6)$ and $(\text{quam1} \leq \text{bestoffq} \cdot \text{minret10})$	81.48	49.09	-0.314	-0.672	0.00	0.5000	-0.13	>0.5000	0.4905
10. $(\text{quant} < \text{bestoffq})$ and $(\text{bestbid} \leq \text{bestoffq})$	81.48	52.73	0.934	0.558	0.79	0.2148	0.41	0.3409	0.2935
11. $(\text{bestoffq} > 3.8 - \min\{1.15, 3.5 - \text{quam1}\})$ or $(2 > \text{bidcount2})$	81.48	51.82	0.097	-0.296	0.58	0.2810	0.05	0.4801	0.3325
12. $(\text{depthoff2} \leq \text{bestbid2})$ or $(\text{bestoffq} - \text{avgret10} > 1.35)$ or $(\text{depthoff2} \leq \text{avgpr20})$	81.48	54.55	0.936	0.550	1.16	0.1230	0.40	0.3446	0.2205
13. $(3 \geq \text{bestbidq})$ and $(\text{quam1} < \text{bestoffq})$	81.48	52.73	1.072	0.722	0.79	0.2148	0.47	0.3192	0.2935
14. $(\text{quam2} < \text{bestoffq})$ and $(\text{minret5} < \text{bestoffq2})$	81.48	50.91	0.308	-0.079	0.38	0.3520	0.13	0.4483	0.3775
15. $(\text{avgret10} \leq \text{quam2})$ and $(\text{quam2} < \text{bestoffq})$	81.48	53.64	2.646	2.270	0.99	0.1611	1.11	0.1335	0.2545
16. $(\text{bestoffq} \geq \text{liqoff})$ and $(\text{bestoffq} > \text{quam1} + \text{minret5})$	81.48	50.91	0.430	0.056	0.38	0.3520	0.18	0.4286	0.3775
17. $(\text{bestoffq} > \max\{\text{quam}, 2\})$	77.78	53.64	0.980	0.597	0.98	0.1635	0.42	0.3372	0.2545
18. $\text{bestoffq} > \text{bestbidq2}$	77.78	57.27	1.761	1.375	1.74	0.0409	0.74	0.2296	0.1425
19. $(\text{lagpr2} > \text{price})$ or $(5.69 - 2.866 \cdot \text{bestoffq} > \text{maxret10})$	77.78	55.45	3.405	3.295	2.50	0.0062	3.11	0.0009	0.1925
20. $(\text{maxret10} > \text{depthoff2})$ or $(3 \leq \text{bestoffq})$	77.78	54.55	-0.040	-0.419	1.16	0.1230	-0.01	>0.5000	0.2205
21. Average	85.19	47.27	-0.968	-1.337	-0.38	>0.5000	-0.41	>0.5000	0.5885

Trading rules based on the all **order book information**. The fitness function is the percentage of the correct in-sample predictions of the exchange rate **directional changes**. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives a buy signal if more than 9 of best 19 rules give a buy signal and sell otherwise.

- Directional Changes

- All Order Book Information

$k = 0.01$

Technical rules	In-sam. DC, %	Out-of-sam. DC, %	Returns, $\theta = 0$	Returns, $\theta = 10^{-4}$	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. (offcount > 6) and (offcount2 ≤ 8)	100.00	50.00	-0.915	-0.935	0.07	0.4721	-3.68	>0.5000	0.4315
2. (max{time - 2.5, maxret20} < $\frac{\text{offcount2} + 8.4}{\text{avgret10}}$) and (offcount < 9)	100.00	50.00	-0.128	-0.141	-1.02	>0.5000	-0.73	>0.5000	0.4315
3. (liqoff2 - offcount1 > -8.9) and (bidcount2 > 7)	100.00	52.38	0.149	0.122	0.10	0.4602	0.61	0.2709	0.3025
4. offcount = offcount1	100.00	50.00	-0.025	-0.212	-0.09	>0.5000	-0.01	>0.5000	0.4315
5. (bestbid > bestbid2) or ((quant2 ≥ lagret1) and (lagpr2 ≥ bestoffq1))	100.00	38.10	-0.806	-0.905	-1.65	>0.5000	-0.43	>0.5000	0.9325
6. (offcount < 9) and (time < time1)	100.00	47.62	-0.312	-0.385	-0.28	>0.5000	-0.40	>0.5000	0.5725
7. time1 > avgpr20 · time	100.00	35.71	-2.670	-2.806	-1.97	>0.5000	-1.20	>0.5000	0.9655
8. (bestbid > bestbid2) or (bestbid > avgpr5)	100.00	28.57	-4.328	-4.475	-3.04	>0.5000	-1.87	>0.5000	>0.999
9. bestoff2 = bestoff	100.00	52.38	0.178	0.104	0.59	0.2776	0.23	0.4090	0.3025
10. ($ \frac{\text{avgpr10} - \text{bestbid1}}{0.3(\text{maxpr2} - \text{bestbid1}) + 3.52} \leq \text{maxpr20}$) or (bestoff2 > bestoffq1)	100.00	47.62	1.264	1.163	-0.22	>0.5000	0.58	0.2810	0.5725
11. (quant < $\frac{\text{depthbid1}}{3g}$) and (avgpr10 < bestbid1)	100.00	50.00	0.072	0.012	0.20	0.4207	0.05	0.4801	0.4315
12. (2.52 ≤ bestoffq) or (max{liqoff2, bestbid1} > maxpr5)	100.00	59.52	1.817	1.702	1.59	0.0559	0.78	0.2177	0.0465
13. bestoffq1 - liqbid1 < bestoffq	100.00	54.76	2.680	2.591	1.13	0.1292	2.33	0.0099	0.1915
14. (bestoff2 ≤ bestoff) or (liqoff2 > avgret10)	100.00	40.48	-1.781	-1.931	-1.22	>0.5000	-0.76	>0.5000	0.8755
15. 8 ≥ offcount2	83.33	45.24	-0.823	-0.869	-0.65	>0.5000	-0.73	>0.5000	0.6885
16. lagret2 > time	83.33	50.00	-0.099	-0.145	0.10	0.4602	-0.04	>0.5000	0.4315
17. (time ≤ avgret10) and (0.23 > liqoff2)	83.33	50.00	-0.099	-0.145	0.10	0.4602	-0.04	>0.5000	0.4315
18. bestoffq1 ≤ minpr10	83.33	45.24	0.846	0.732	-0.55	>0.5000	0.36	0.3594	0.6885
19. minpr5 < min{lagpr1, bestbid}	83.33	57.14	3.741	3.582	0.99	0.1611	1.67	0.0475	0.1065
20. (depthoff2 ≤ 6.54) or (offcount2 < maxpr20 · 5.37)	83.33	45.24	-0.823	-0.869	-0.65	>0.5000	-0.73	>0.5000	0.6885
21. Average	100.00	47.62	-0.985	-1.024	-0.77	>0.5000	-2.27	>0.5000	0.5725

Trading rules based on the all order book information. The fitness function is the percentage of the correct in-sample predictions of the exchange rate directional changes. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives a buy signal if more than 9 of best 19 rules give a buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- Pure Price Information

 $k = 0$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %, $\theta = 0$	O-of-s, Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(ret^2 \cdot \maxret20 < lagret1 \cdot \maxret5)$ or $(ret \cdot \maxret10 < lagret1 \cdot \maxret5)$	6.562	8.689	8.905	51.48	2.15	0.0158	3.67	0.0001	<0.001
2. $(\minret20 < avgret20)$ and $(ret < lagret2)$	6.347	9.092	-10.102	51.42	2.05	0.0202	3.81	0.0001	<0.001
3. $ret < lagret2$	6.347	9.092	-10.102	51.42	2.05	0.0202	3.81	0.0001	<0.001
4. $(lagpr1 > minpr5)$ and $(lagret2 > ret)$	6.321	9.359	-9.966	51.54	2.22	0.0132	2.20	0.0139	<0.001
5. $(ret < lagret2)$ or $(\maxret20 = lagret2)$	6.314	9.186	-10.036	51.52	2.19	0.0143	3.85	0.0001	<0.001
6. $(lagret2 = \maxret10)$ or $(lagret1 > ret)$	6.222	9.631	-10.688	50.88	1.28	0.1003	4.08	<0.0001	<0.001
7. $(ret < lagret1)$ or $(ret < lagret2)$	6.158	9.935	-9.190	51.63	2.44	0.0073	4.32	<0.0001	<0.001
8. $(avgret5 > ret)$ or $(lagret2 \geq ret)$	5.996	8.547	-9.362	51.40	2.06	0.0197	3.66	0.0001	0.0015
9. $(lagpr1 \cdot lagret1 > price)$ and $(price \leq \maxpr5)$	5.861	8.806	-12.084	51.12	1.61	0.0537	3.70	0.0001	0.0015
10. $ret \leq lagret1$	5.861	8.806	-12.084	51.12	1.61	0.0537	3.70	0.0001	0.0015
11. $(avgret5 > ret)$ or $(avgpr10 = avgpr5)$	5.861	7.153	-11.074	51.10	1.58	0.0571	3.03	0.0012	0.0115
12. $ret \leq avgret5$	5.771	7.467	-10.814	50.96	1.39	0.0823	3.15	0.0008	0.0065
13. $lagpr1 \cdot avgret5 > price$	5.771	7.465	-10.814	50.96	1.39	0.0823	3.15	0.0008	0.0065
14. $(lagpr1 > minpr5)$ and $(\maxret10 > ret)$	5.658	9.868	-5.755	51.67	2.75	0.0030	4.75	<0.0001	<0.001
15. $\maxret20 \cdot ret < \maxret10$	5.416	9.460	-5.930	50.62	0.93	0.1762	4.10	<0.0001	<0.001
16. $(\maxret20 = lagret2)$ or $(ret < avgret10)$ or $(avgpr20 = minpr10)$	5.343	8.593	-9.228	50.92	1.33	0.0918	3.61	0.0002	0.0015
17. $(price = avgpr20)$ or $(avgret5 = avgret10)$ or $(price \leq lagpr1)$	5.169	8.534	8.925	50.54	0.78	0.2177	3.59	0.0002	0.0015
18. $(\minret5 = ret)$ and $(price < lagpr2)$	5.113	6.198	-5.470	49.94	-0.06	>0.5000	3.21	0.0007	0.0385
19. $price \leq lagpr1$	5.054	8.494	-8.934	50.50	0.72	0.2358	3.57	0.0002	0.0025
20. $(\maxpr10 < price)$ or $(ret = \minret5)$	5.021	5.422	-8.751	50.81	1.34	0.0901	2.60	0.0047	0.0795
21. Average	5.654	8.327	-10.267	51.00	1.44	0.0749	3.51	0.0002	0.0025

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- Pure Price Information

$$k = 0.0005$$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %; $\theta = 0$	O-of-s. Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(\max_{pr10} \cdot \min_{ret20} < \text{avgpr5})$ and $(\max_{ret20} \cdot \min_{ret20} \leq \text{lagret2})$	4.569	6.233	2.477	52.62	1.89	0.0294	2.74	0.0031	0.0065
2. $(\max_{pr5} > \min_{pr10})$ and $(\text{lagret1} > \max_{ret20} \cdot \min_{ret20})$	4.017	6.407	2.092	51.62	1.33	0.0918	2.73	0.0032	0.0065
3. $(\text{ret} \leq \min_{ret10})$ or $(\text{lagret1} \leq \min_{ret10})$ or $(\text{avgret5} = \min_{ret20})$	3.447	4.934	0.224	52.15	1.52	0.0643	2.19	0.0143	0.0155
4. $(\text{ret} = \min_{ret10})$ or $(\text{lagret1} = \min\{\text{avgret20}, \min_{ret10}\})$	3.415	4.938	0.227	52.15	1.52	0.0643	2.20	0.0139	0.0155
5. $(\text{ret} \leq \min_{ret10})$ or $(\text{lagret1} \leq \min_{ret20})$	3.372	4.076	-0.731	50.94	0.51	0.3050	1.87	0.0307	0.0205
6. $(\text{price} < \max_{pr5})$ or $(\max_{ret20} > \max_{ret10})$	3.325	5.083	1.415	50.54	0.86	0.1949	2.44	0.0073	0.0135
7. $(\text{ret} \cdot \max_{ret20} < \text{lagret1})$ and $(\text{price} = \min_{pr10})$	3.304	3.335	0.884	51.55	0.90	0.1841	3.39	0.0003	0.0305
8. $\text{price} \cdot \min_{ret20} \leq \text{avgpr20}$	3.273	5.098	2.757	50.61	0.97	0.1660	2.51	0.0060	0.0135
9. $(\text{lagpr1} = \min_{pr20})$ or $(\text{ret} = \min_{ret10})$	3.263	3.289	-1.315	50.67	0.32	0.3745	1.49	0.0681	0.0305
10. $(\text{ret} < \max_{ret10})$ or $(\max_{pr5} > \min\{\text{price}, \max_{pr10}\})$	3.184	4.895	1.937	49.66	0.33	0.3707	2.79	0.0026	0.0155
11. $(\text{price} < \max_{pr10})$ or $(\text{ret} < \max_{ret10})$	2.104	4.231	1.930	49.60	0.38	0.3520	2.68	0.0037	0.0205
12. $\max\{\max_{pr5}, \text{price} - \text{avgret20} + \min_{ret10}\} > \text{price}$	3.155	7.652	1.930	51.35	1.19	0.1770	3.27	0.0005	0.0015
13. $\text{price} < \max_{pr5}$	3.124	7.333	2.761	51.35	1.19	0.1770	3.13	0.0009	0.0025
14. $(\text{ret} \leq \min_{ret10})$ and $(\text{avgret5} \cdot \text{lagret2} > \text{ret})$	3.098	6.425	1.885	52.02	1.35	0.0885	3.14	0.0008	0.0065
15. $((\text{lagret2} \leq \max_{ret20})$ and $(\text{avgret5} = \max_{ret5}))$ or $((\text{price} \leq \text{avgret5} \cdot \text{avgpr20})$ and $(\min_{pr10} = \min_{pr20}))$	3.000	11.058	7.669	51.14	0.77	0.2206	4.64	<0.0001	<0.001
16. $\text{price} + \min_{pr20} - \text{avgpr20} < \text{lagpr2}$	2.932	5.003	2.534	51.28	2.18	0.0146	3.09	0.0010	0.0145
17. $(\text{ret} < \max_{ret10})$ or $(\text{price} < \max\{\text{lagpr2}, \max\{\text{avgpr5}, \min_{pr20}\}\})$	2.926	4.488	1.245	50.07	0.67	0.2514	2.43	0.0075	0.0175
18. $\min\{\text{ret}, \text{avgret5}\} \leq \min_{ret10}$	2.883	6.127	1.492	51.62	1.01	0.1562	2.92	0.0018	0.0075
19. $(\text{ret} \leq \min_{ret10})$ or $(\text{avgret5} = \min_{ret20})$	2.858	6.141	1.532	51.62	1.01	0.1562	2.95	0.0018	0.0075
20. $\text{ret} \leq \min_{ret10}$	2.821	6.172	1.575	51.68	1.07	0.1423	2.96	0.0015	0.0065
21. Average	3.460	6.676	1.833	51.75	1.31	0.0951	2.84	0.0023	0.0065

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- Pure Price Information

$$k = 0.001$$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., $\theta = 0$	O-o-f-s, Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(\max_{pr10} - \text{avgret5} + \maxret5 \leq \max_{pr20})$ or $(\max_{pr10} - \text{avgret5} + \minret10 \leq \text{avgpr20})$	3.781	6.049	4.484	51.92	0.98	0.1635	2.61	0.0045	0.0035
2. $(\min_{pr10} = \text{lagpr1})$ or $(\minret20 \geq \text{ret})$	3.574	4.581	2.633	52.88	1.51	0.0655	2.29	0.0110	0.0115
3. $(\maxret10 - \text{avgpr20} \geq \maxpr5)$ or $(\text{avgpr5} = \text{lagpr2})$ or $(\text{avgpr5} = \text{avgpr10})$	3.567	1.473	-0.077	51.10	0.70	0.2420	0.64	0.2611	0.0865
4. $\text{avgpr20}^2 \cdot \text{lagpr2} > \minpr5 \cdot \maxpr5^2$	3.465	6.909	5.191	53.16	1.66	0.0485	2.97	0.0015	<0.001
5. $(\text{price} = \minpr20)$ or $(\min\{\text{lagpr1}, \text{avgpr5}\} = \minpr20)$	3.452	6.013	4.170	52.61	1.35	0.0885	2.62	0.0044	0.0035
6. $(\text{avgpr10} = \text{avgpr5})$ or $(\min\{\text{price}, \text{lagpr1}\} = \minpr20)$	3.400	5.339	3.494	51.92	0.98	0.1635	2.32	0.0102	0.0065
7. $(\min\{\text{price}, \text{lagpr1}\} = \text{avgpr10})$ or $(\min\{\text{price}, \text{lagpr1}\} = \minpr20)$	3.287	6.012	4.170	52.61	1.35	0.0885	2.61	0.0045	0.0035
8. $(\minret20 = \text{lagret1})$ or $(\text{price} \cdot \maxret5 \leq \text{avgpr10})$	3.282	2.001	0.061	51.10	0.54	0.2946	0.88	0.1894	0.0645
9. $(\text{lagret1} = \minret20)$ or $(\text{ret} = \minret10)$	3.264	4.346	2.164	53.16	1.65	0.0495	2.01	0.0222	0.0125
10. $(\text{avgret10} < \text{avgret20})$ or $(\text{price} = \minpr20)$	3.262	5.390	3.379	52.34	1.41	0.0793	2.39	0.0084	0.0065
11. $(\text{lagret1} = \minret10)$ or $(\text{ret} = \minret20)$	3.252	6.350	4.043	55.63	3.08	0.0010	2.93	0.0017	0.0025
12. $(\text{lagpr2} = \text{avgpr5})$ or $(\minret10 \geq \text{ret})$	3.204	6.251	3.911	54.26	2.32	0.0102	2.93	0.0017	0.0025
13. $(\text{avgret10} < \text{avgret20})$ or $(\min\{\text{avgret10}, \text{lagret1}^2 \cdot \text{ret}\} \leq \text{avgret20})$	3.029	1.287	2.570	51.10	0.80	0.2119	2.08	0.0188	0.0995
14. $(\text{lagret1} = \minret10)$ or $(\text{ret} = \minret10)$	3.006	5.397	2.973	54.26	2.26	0.0119	2.39	0.0084	0.0035
15. $\minret10 = \min\{\text{ret}, \text{lagret1}\}$	3.006	5.397	2.973	54.26	2.26	0.0119	2.39	0.0084	0.0035
16. $(\minpr10 = \minpr20)$ and $(\text{ret} \leq \min\{\minret10, \frac{\text{avgret5}}{\text{lagpr1}}\})$	2.890	5.874	4.235	52.06	1.05	0.1469	3.20	0.0007	0.0035
17. $(\text{ret} \leq \minret10)$ and $(\minpr10 = \minpr5)$	2.864	6.172	4.383	53.43	1.94	0.0262	3.24	0.0006	0.0035
18. $(\minret10 \geq \text{ret})$ and $(\text{price} \leq \text{avgpr10})$	2.839	6.602	4.694	53.98	2.27	0.0116	3.33	0.0004	0.0015
19. $(\text{avgret10} \leq \text{avgret20})$ or $(\text{ret} \leq \minret10)$	2.791	3.068	0.960	50.55	0.44	0.3300	1.38	0.0838	0.0305
20. $\text{ret} \leq \minret10$	2.645	6.387	4.225	53.30	1.78	0.0375	3.13	0.0009	0.0025
21. Average	3.555	4.150	2.190	52.75	1.42	0.0778	1.88	0.0301	0.0165

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- Pure Price Information

$$k = 0.005$$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %, $\theta = 0$	O-of-s, Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(\text{minret}10 \cdot 1.637268 \geq \text{avgpr5})$ or $(\text{minret}20 + \text{maxret}20 < 2 \cdot \text{lagret}2)$	3.292	0.644	0.550	53.77	1.49	0.0681	0.53	0.2981	0.1555
2. $(\text{min}(\text{minret}20, \text{ret}) = \text{lagret}1)$ or $(\text{minret}20 = \text{ret})$	2.918	4.085	3.711	60.38	2.21	0.0136	1.76	0.0392	0.0145
3. $(\text{minpr}10 \geq \text{lagpr}1)$ or $(\text{avgret}20 \leq \text{avgret}5)$ and $(\text{lagpr}1 \geq \text{price})$	2.635	-1.315	-1.656	44.34	-1.20	>0.5000	-0.58	>0.5000	0.3005
4. $(\text{avgret}20 \leq \text{avgret}5)$ and $(\text{lagret}1 \geq \text{avgret}20)$	2.293	-2.298	-2.591	47.17	-0.61	>0.5000	-1.07	>0.5000	0.3985
5. $\max(\frac{\text{maxpr}10 - \text{lagpr}2}{\text{avgpr5}}, \text{avgpr5}) > \text{lagpr}2$	2.070	2.784	2.415	60.38	2.33	0.0099	1.37	0.0853	0.0325
6. $\text{ret} \cdot \text{maxret}20 < \text{lagret}1$	1.994	7.952	7.543	68.87	4.64	<0.0001	3.96	<0.0001	0.0015
7. $\text{ret} \leq \text{minret}5$	1.992	9.634	9.145	72.64	5.44	<0.0001	4.19	<0.0001	0.0015
8. $(\text{maxpr}5 \geq \text{maxpr}20)$ and $(\text{maxpr}5 > \text{price})$	1.964	3.718	3.497	56.60	1.73	0.0418	2.08	0.0188	0.0175
9. $(\text{lagret}2 > \text{minret}5)$ and $(\text{price} \geq \frac{\text{maxpr}20}{\text{avgpr5}})$	1.934	-5.758	-6.059	35.85	-3.06	>0.5000	-2.60	>0.5000	0.7185
10. $(\text{lagpr}1 < \text{avgpr}20)$ or $(\text{max}(\text{avgpr}20, \text{minpr}5) \geq \text{price})$	1.813	4.914	4.628	59.43	2.07	0.0192	2.27	0.0116	0.0085
11. $\text{price} \leq \text{max}(\text{avgpr}20, \text{lagpr}2)$	1.813	7.812	7.367	66.98	3.72	0.0001	3.32	0.0005	0.0015
12. $(\text{price} \leq \text{max}(\text{lagpr}1, \text{lagpr}2))$ or $(\text{maxpr}5 \leq \text{lagpr}1)$	1.813	8.690	8.198	69.81	4.48	<0.0001	3.71	0.0001	0.0015
13. $\text{maxpr}5 > \text{price}$	1.813	8.212	7.751	67.92	4.01	<0.0001	3.54	0.0002	0.0015
14. $\text{avgret}20 \leq \text{avgret}5$	1.635	-0.105	-0.418	48.11	-0.39	>0.5000	-0.05	>0.5000	0.1935
15. $(\text{price} \leq \text{minpr}5)$ or $(\text{lagpr}1 \leq \text{avgpr}20)$	1.497	7.905	4.604	59.43	2.04	0.0207	2.20	0.0139	0.0085
16. $(\text{minpr}10 = \text{minpr}20)$ and $(\text{price} \leq \text{minpr}20 \cdot \text{maxret}20)$	1.491	7.862	7.410	67.92	3.96	<0.0001	3.32	0.0005	0.0015
17. $\text{min}(\text{price}, \text{lagpr}1) \leq \text{avgpr}20$	1.491	4.914	4.628	59.43	2.07	0.0192	2.27	0.0116	0.0085
18. $\frac{\text{avgpr}10}{\text{lagpr}2 - \text{price} \cdot \text{lagpr}2} > 0.6707$	1.472	6.690	6.229	67.92	3.96	<0.0001	2.84	0.0023	0.0025
19. $(\text{lagret}2 = \text{maxret}5)$ or $(\text{lagpr}2 > \text{price})$	1.472	7.983	7.537	67.92	3.96	<0.0001	3.37	0.0004	0.0015
20. $\text{price} < \text{lagpr}2$	1.472	8.148	7.602	67.92	3.95	<0.0001	3.39	0.0003	0.0015
21. Average	1.813	8.690	8.198	69.81	4.48	<0.0001	3.71	0.0001	0.0015

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- Pure Price Information

 $k = 0.01$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %, $\theta = 0$	O-of-s, Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $avgpr20 < 1.6345$	1.707	-1.062	-1.095	48.72	-0.05	>0.5000	-1.31	>0.5000	0.3635
2. $(minpr20 \geq price)$ or $(avgpr5 \geq avgpr10)$	1.707	3.503	3.399	56.41	1.21	0.1131	3.16	0.0008	0.0145
3. $(maxpr5 < 1.6422)$ and $(lagpr1 < maxpr5)$	1.645	-2.457	-2.548	41.03	-1.18	>0.5000	-1.79	>0.5000	0.5675
4. $(lagret2 \geq 1)$ and $(lagret2 - avgret20 \geq avgret10)$	1.645	1.827	1.705	53.85	0.55	0.2912	0.86	0.1949	0.0575
5. $maxpr20 \cdot minret10 \geq avgpr10 \cdot maxret20$	1.645	0.318	0.238	53.85	0.49	0.3121	0.26	0.3974	0.1625
6. $avgret20 \leq \min(lagret1, lagret2)$	1.085	-1.234	-1.346	51.28	0.09	0.4641	-0.60	>0.5000	0.3895
7. $maxpr10 \leq 1.6394$	1.085	1.166	1.106	58.97	1.24	0.1075	0.83	0.2033	0.0925
8. $\min(avgret20, maxret5) \leq lagret2$	1.078	-2.909	-3.026	41.03	-1.13	>0.5000	-1.31	>0.5000	0.6025
9. $lagret2 > avgret20$	1.078	-2.909	-3.026	41.03	-1.13	>0.5000	-1.31	>0.5000	0.6025
10. $(minret20 < minret5)$ and $(avgpr20 \leq lagpr2)$	1.023	-3.189	-3.305	33.33	-2.62	>0.5000	-1.68	>0.5000	0.6445
11. $(maxpr20 > lagpr1)$ and $(avgret20 > 1)$	1.023	0.071	-0.083	38.46	-1.52	>0.5000	0.04	0.4840	0.1955
12. $minret20 - lagpr1 > lagpr2$	1.023	-0.958	-0.991	48.72	-0.56	>0.5000	-3.31	>0.5000	0.3505
13. $minpr20 < minpr5$	1.023	-2.846	-2.955	33.33	-2.34	>0.5000	-1.39	>0.5000	0.5975
14. $(lagret2 < lagret1)$ and $(lagret2 < ret)$	1.023	-0.146	-1.506	46.15	-1.03	>0.5000	-5.25	>0.5000	0.2475
15. $lagret1 < maxret5$	1.016	-2.186	-2.284	35.90	-2.27	>0.5000	-1.29	>0.5000	0.5325
16. $lagpr1 < maxpr5$	1.016	-1.279	-1.351	41.03	-1.52	>0.5000	-1.00	>0.5000	0.3965
17. $ret > minret5$	1.016	-5.612	-5.781	23.08	-4.01	>0.5000	-2.77	>0.5000	0.8705
18. $2.6339 > price + maxret20$	0.832	7.942	7.777	79.49	5.04	>0.0001	5.27	>0.0001	<0.001
19. $price = minpr20$	0.832	8.108	7.899	82.05	5.25	>0.0001	3.56	0.0002	<0.001
20. $minpr20 > price - avgret5$	0.770	5.530	5.383	74.36	3.82	0.0001	2.37	0.0089	0.0045
21. Average	1.645	1.447	1.359	51.28	0.20	0.4207	0.65	0.2578	0.0745

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- All Order Book Information

 $k = 0$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %, $\theta = 0$	O-of-s, Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(lagret2 > ret)$ and $(minpr5 \leq bestbid)$	8.404	12.256	-0.476	53.76	5.75	<0.0001	5.18	<0.0001	0.0035
2. $(12.13 \cdot price \geq time1)$ and $(price \leq bestoff)$	7.984	10.097	-2.631	53.26	5.43	<0.0001	4.76	<0.0001	0.0035
3. $bestbid > \min\{avgpr20, price, bidcount1\}$	7.675	5.540	-3.446	56.76	9.96	<0.0001	2.30	0.0107	0.0205
4. $(\max\{avgpr20, price\} \leq bestoff)$ or $(bestbidq = maxret5)$	7.534	7.326	-2.231	57.43	11.06	<0.0001	3.02	0.0013	0.0105
5. $(bidcount2 \geq time1)$ and $(price \leq bestoff)$	7.436	7.571	-6.112	53.16	4.98	<0.0001	3.39	0.0003	0.0085
6. $(bestoff1 = maxpr20)$ or $(ret \leq lagret2)$	7.272	9.669	-8.193	51.78	2.63	0.0043	3.98	<0.0001	0.0045
7. $(ret < lagret2)$ or $(\max\{\min\{\frac{1.91 - 7.2 \cdot maxpr20}{avgcount10 - 0.097}, avgret10\}, ret\} < lagret1)$	6.851	11.624	-8.112	51.57	2.34	0.0096	4.78	<0.0001	0.0035
8. $(ret \leq lagret2)$ or $(ret < lagret1)$	6.773	10.730	-8.044	51.59	2.41	0.0080	4.50	<0.0001	0.0035
9. $price \leq bestoff$	6.768	9.490	4.102	54.89	13.15	<0.0001	7.61	<0.0001	0.0045
10. $bestbid \geq price$	6.526	9.258	-4.932	51.05	1.61	0.05367	3.84	0.0001	0.0045
11. $(avgret10 > depthbid2)$ or $(\text{offcount2} > \text{offcount})$ or $(time \leq 2.08 \cdot lagret2)$ and $(ret \leq \min\{bestbid2, avgret10\})$	6.465	1.618	-10.178	52.14	3.66	0.0001	0.83	0.2033	0.1375
12. $(9.31 - \frac{8.78}{time} \leq 8.79)$ and $(price \leq bestoff1)$	6.357	0.117	-13.384	51.80	2.63	0.0043	0.05	0.4801	0.2505
13. $ret \leq lagret2$	6.229	10.213	-9.154	51.14	1.65	0.0495	4.10	<0.0001	0.0035
14. $ret \leq avgret5$	5.771	8.439	-10.006	50.97	1.40	0.0808	3.41	0.0003	0.0075
15. $(\min\{depthoff, ret\} = \minret5)$ or $(depthbid1 \leq 2.4)$	5.453	5.948	-8.345	50.91	1.48	0.0694	2.73	0.0032	0.0185
16. $(\minpr5 = price)$ or $(depthbid2 \leq 5.01)$ or $(bestbid = price)$ and $(bestoff1 + 7.61 > \min\{time, depthbid\})$	5.296	2.101	-8.594	49.32	-1.14	>0.5000	1.00	0.1587	0.1175
17. $(maxpr5 \leq bestoff)$ or $(lagret1 = maxret20)$	5.233	7.786	-3.993	57.18	10.49	<0.0001	3.16	0.0008	0.0075
18. $(bestoffq1 > ret)$ and $(bestoff \geq lagpr2)$ and $(offcount2 \geq 1.51 \cdot (0.16 + bestbidq1))$	5.036	5.803	-10.145	53.24	4.75	<0.0001	2.40	0.0082	0.0195
19. $(time1 \leq (\frac{6.54 \cdot price}{\minpr10 - 4.71 - bestoff1} + 7.61) \cdot (bestoffq1 - 5.55))$ or $(price \leq bestoff1)$	5.017	7.777	-1.426	51.45	2.84	0.0023	4.29	<0.0001	0.0075
20. $(depthbid1 \leq 4.83 - ret)$ or $(\minret10 = ret)$	5.011	8.972	-1.041	50.86	1.64	0.0505	4.85	<0.0001	0.0095
21. Average	9.165	11.830	-8.481	53.37	4.95	<0.0001	4.79	<0.0001	0.0035

Trading rules based on the all order book information. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- All Order Book Information

 $k = 0.001$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %, $\theta = 0$	O-of-s, Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $((\text{depthoff}2 > \max(8.62, \text{bidcount1}) - \text{minret}5) \text{ and } (\text{quant} > \text{liqid}) \text{ and } (\text{price} \leq \text{bestbid}1))$ or $((\text{depthoff}2 > \max(8.62, \text{bidcount1}) + 0.46) \text{ and } (16.19 + \text{liqid}2 > \text{offcount}))$	4.843	2.008	0.476	50.62	0.67	0.2514	0.87	0.1922	0.0725
2. $(\text{lagret}1 \leq \text{minpr}20 \cdot \text{quant} + \text{minret}10 - 2.2713) \text{ and } (3.89 < \text{time}1 + \text{minret}20))$ or $(9 < \text{bestbid}4)$	4.561	-2.606	-2.162	49.25	-0.58	>0.5000	-0.04	>0.5000	0.4845
3. $(\text{bestoff} > \text{price})$ and $(\text{liqid} + \text{minret}10 \leq \text{quant}2)$	4.076	4.967	3.143	53.09	2.21	0.0136	2.32	0.0102	0.0155
4. $(1.96 < \text{time}1 \cdot \max(-\text{bestbid}2, \text{quant}1 - \text{avgr}10) \cdot \max(-\text{bestbid}2, \text{quant}1 - \text{avgr}10))$ and $(\text{offcount}1 > \text{bestbid}2 + \min(\text{bestbid}4, (\text{bestbid}2 + \text{bestbid}2 \cdot 5.32) \cdot \frac{\text{time}1}{\text{quant}1}))$	4.056	-1.231	-3.490	48.97	-0.64	>0.5000	-0.53	>0.5000	0.3385
5. $(\text{bestoff} > \text{price} \cdot \text{quant}1 \geq \frac{\min(\text{depthbid}1, 5.416)}{(\text{quant}1 - \text{maxret}10) \cdot \text{quant}2})$ or $(\text{bestoff}2 \leq \min(\max(\text{bestoff}7, 7.59), \frac{\max(8.15, \text{lagpr}1) \cdot \text{offcount}2 \cdot \text{quant}2}{\text{time}1}))$	4.026	-5.129	-7.415	45.68	-2.38	>0.5000	-2.22	>0.5000	0.7345
6. $(\text{avgr}20 > \text{avgr}10)$ and $(\max(\text{bestoff}, \text{bestoff}2) < \text{bestoff}2)$ or $((\text{quant}1 > 1) \text{ and } (\text{price} \leq \text{lagpr}2))$	3.943	3.439	1.384	51.03	0.48	0.3156	1.49	0.0681	0.0535
7. $(\{(1.843 \leq \text{time}1) \text{ or } (\text{bestoff}2 < \text{quant})\} \text{ and } (\text{quant} > 1))$ or $(\text{depthbid}1 \leq 4.85)$	3.929	-3.185	-5.443	47.33	-1.47	>0.5000	-1.37	>0.5000	0.5415
8. $(5.97 \cdot \text{bestoff}2 \cdot \text{bestbid}1 < \text{depthoff} \cdot \text{bestbid}1 - 80.14)$ or $(\text{avgr}10 > \text{avgr}10 - \text{price})$	3.865	8.542	6.428	53.50	2.15	0.0158	3.63	0.0001	0.0035
9. $(\text{quant}1 > \text{maxret}10 + \text{ret})$ or $((\text{quant}1 > 1.23) \text{ and } (\text{lagpr}1 < \text{time}1))$	3.607	0.648	-1.705	47.74	-1.16	>0.5000	0.27	0.3936	0.1585
10. $((1.15 < \text{quant}1) \text{ or } (\text{avgr}10 - \text{liqid}2 \geq \text{time}))$ and $(\text{time}1 \geq 4.64 - \text{bestoff})$	3.566	-0.269	-2.485	49.11	-0.64	>0.5000	-0.11	>0.5000	0.2245
11. $(\text{quant}1 \geq 1.35)$ and $(3.04 \leq \max(\text{time}, \text{time}1))$	3.549	0.597	-1.861	48.15	-0.96	>0.5000	0.25	0.4013	0.1595
12. $(1 < \text{quant}2)$ and $((\text{time} \geq 4.24) \text{ or } (\text{time} > \text{price}))$	3.435	0.812	-1.890	47.87	-1.09	>0.5000	0.34	0.3669	0.1435
13. $(\text{avgr}20 \geq \text{ret})$ or $(\text{avgr}20 \geq \text{avgr}10)$	3.417	5.550	3.591	51.71	1.31	0.0951	2.57	0.0051	0.0125
14. $\text{bestbid}1 < \min(\frac{\text{time}1}{1.11}, \text{quant})$	3.379	-0.159	-2.299	47.87	-1.31	>0.5000	-0.06	>0.5000	0.2155
15. $(\text{quant}1 > \text{bestoff})$ or $(2.98 - \text{bestbid}1 - \text{bestoff} \geq (\min(\text{quant}1, 0.34) + \text{minret}10) \cdot 8.345 - \frac{\text{avgr}10}{\text{lagpr}1} - \text{bestoff})$	3.351	-0.375	-2.679	49.52	-0.29	>0.5000	-0.16	>0.5000	0.2335
16. $(6.85 < \text{depthoff}1 - \text{bidcount})$ or $(\text{quant}2 - 0.82 \geq \text{avgr}10)$	3.300	1.407	0.861	48.70	-0.31	>0.5000	1.05	0.1469	0.1025
17. $(9.6 < \max(\text{depthoff}1 - \text{ret}, \text{bidcount}1))$ and $(\text{avgr}10 > \text{price})$	3.210	4.955	2.624	53.50	1.93	0.0268	2.04	0.0207	0.0155
18. $(\frac{\max(\text{quant}1, \text{bestbid}4)}{\text{bestbid}2 \cdot \text{maxret}5} > \frac{\text{bidcount}}{\text{bestoff}})$ or $(\frac{\max(\text{quant}1, \text{bestbid}4)}{\text{bestbid}2 \cdot \text{maxret}5} > \frac{2.23 \cdot \text{avgr}10 \cdot \max(\text{quant}1 - 6.09, \text{bestbid}2)}{\text{bestoff}} > \text{bidcount})$	3.124	0.251	-1.056	48.42	-0.65	>0.5000	0.11	0.4562	0.1785
19. $(\max(\text{bestoff}, 7.8, \text{depthoff}) \geq \max(\text{depthbid}2, 8.28))$ or $(1.27 \geq \text{time})$	3.126	-0.692	-1.615	47.74	-1.01	>0.5000	-0.32	>0.5000	0.2735
20. $(\text{bestbid}2 > \text{price} + \text{liqoff}1)$ or $(3.66 > \frac{\text{avgr}10 - \text{bidcount}}{\text{min}(\text{price}, \text{liqid}1 \cdot 3)} - \text{bestoff})$	3.016	8.351	6.383	52.54	1.34	0.0901	3.94	<0.0001	0.0035
21. Average	4.668	0.810	-1.586	48.42	-0.78	>0.5000	0.34	0.3669	0.1435

Trading rules based on the all order book information. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- All Order Book Information

$$k = 0.005$$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %, $\theta = 0$	O-of-s. Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(\text{bestoffq1} \geq \text{lagret2})$ and $(\text{bestoffq1} \geq \text{ret})$ and $(\text{bestoffq} - \text{ret}) \geq \text{ret}$	4.198	4.774	4.418	59.09	2.20	0.0139	1.99	0.0233	0.0345
2. $(\text{bestoffq} > \text{bestbidq2})$ or $(\text{bestbidq2} \geq \frac{\text{bestoffq} - \text{bestbidq2}}{\min(\text{max}\{\text{bidcount}, \text{quant2}\}, \text{max}\{\text{bidcount}, \text{quant2}\} - 4.89)})$	3.971	3.876	3.530	58.18	1.98	0.0239	1.63	0.0516	0.0535
3. $(\text{offcount1} - \text{time} < \text{bestbidq2})$ or $(\text{bestbidq2} < \text{bestoffq})$	3.639	1.675	1.432	54.55	1.40	0.0808	0.79	0.2148	0.1415
4. $(\text{time} - \text{bestoffq1} - \text{avgret20} - 7.88) > \text{bestbidq2}$ and $(\text{bestoffq} > \text{maxret20})$ and $(\text{time} \leq \text{maxret20})$ or $(\text{depthoff} > \text{liqoff})$	3.596	2.606	2.279	59.09	2.20	0.0139	1.12	0.1314	0.0945
5. $(\text{liqoff1} \leq 0.24)$ and $(\min(\text{avgret20}, \text{quant2}) = \text{bestbidq2})$ or $(\min(\text{ret5} > \text{bestoffq}))$ or $(\text{bestoffq} \geq \text{max}\{\text{quant}, \text{bestbidq}, \text{bestbid}\})$	3.546	3.738	3.379	56.36	1.54	0.0618	1.55	0.0606	0.0585
6. $(\text{quant1} < \min(8.14, \text{bestoffq}))$ or $(2.9 < \text{bestbid} - 1.7)$ and $(3.08 \geq \text{bestbidq})$	3.546	1.072	0.722	52.73	0.79	0.2148	0.47	0.3192	0.1775
7. $(\text{quant1} < \text{bestoffq})$ and $(\text{avgret10} + \text{bestbidq}) - \text{maxpr10} \leq 8.27$	3.546	1.221	0.858	52.73	0.78	0.2177	0.53	0.2981	0.1705
8. $(\text{bestbidq2} - \text{minret20} < 5.04)$ and $(\text{quant2} < \text{bestoffq})$	3.545	-0.362	-0.703	49.09	0.00	0.5000	-0.15	>0.5000	0.3155
9. $(\text{max}\{\text{quant2}, \text{price}\} < \min(\text{bestoffq}, 3.38) - \text{avgret5})$ or $(\text{max}\{\text{bestbid1}, \text{quant2}\} < \min(\text{bestoffq}, 8.7, \text{time1}))$	3.537	0.519	0.144	51.82	0.58	0.2810	0.23	0.4090	0.2335
10. $(\text{bestoffq} \geq \text{bestbidq2})$ and $(4.63 - \text{bestoffq} + \text{quant} > 8.7)$	3.531	1.754	1.395	57.27	1.55	0.0606	0.75	0.2266	0.1375
11. $(5.41 \leq \text{bestbidq1} - \text{bestoffq})$ and $(\text{bestbidq1} \leq 3.08 + \text{maxpr5})$	3.518	1.318	1.062	50.00	0.23	0.4090	0.71	0.2389	0.1605
12. $(\text{bestbidq1} > \text{bestbid} - 6.96)$ or $(\text{bestoffq} \geq \text{max}\{\frac{\text{bestbidq} - \text{bestbid}}{\text{bestoffq}}, \text{maxpr10}\} - \text{avgpr20})$	3.517	1.805	1.432	56.36	1.56	0.0594	0.77	0.2206	0.1355
13. $(-5.94 - \text{bestbid1} > -0.069 - \text{bestbidq1})$ or $(\text{avgret10} < \text{avgret20})$ or $(3 \leq \text{bestoffq})$	3.301	-0.139	-0.464	50.91	0.41	0.3409	-0.07	>0.5000	0.2945
14. $(4.09 \leq \text{bestoffq})$ and $(\min(\text{minret5}, \text{liqoff}) > \text{time2})$ or $(\text{bestoffq} \geq \text{max}\{\text{bestoff}, \text{bestbidq}, \text{quant2}\})$	3.225	1.640	1.289	54.55	1.15	0.1251	0.69	0.2451	0.1425
15. $(9.87 < \text{bestbidq1})$ or $(\text{bestoff} \geq \frac{\text{bestoff1} + \text{bestbidq1}}{\min(\text{bestbidq1}, \text{bestoff2} + \frac{\text{bestoff1}}{\text{bestoff2}})})$	3.212	1.782	1.403	56.36	1.54	0.0618	0.75	0.2266	0.1365
16. $(5.9 \geq \text{max}\{\text{bestoffq1}, \frac{\text{bestoff2}}{\text{bestoff1}}\})$ and $(\text{bestoff1} < \text{bestoffq})$ or $(\text{quant} > 2 - \text{price})$	3.212	1.839	1.520	56.36	1.39	0.0823	0.80	0.2119	0.1325
17. $(\text{bestoff} > \min(\text{bestbidq2}, \text{depthbid})$ and $(\text{bestoff} > 1))$	3.209	1.829	1.476	57.27	1.74	0.0409	0.78	0.2177	0.1335
18. $(\min\{3.22, \text{quant2}\} < \text{max}\{\text{quant2}, \text{bestoffq}\})$	3.200	0.727	0.318	51.82	0.38	0.3520	0.30	0.3821	0.2085
19. $\text{quant2} \geq \text{bestoffq} - (\text{ret} - 2.41) + 4.94$	3.175	1.575	1.183	56.36	1.35	0.0885	0.66	0.2546	0.1475
20. $(\text{bestbid} = \text{bestoffq1})$ and $(\text{liqoff} + \text{bestoff1} > \text{bestoffq2})$	3.164	-2.100	-2.257	46.36	-0.84	>0.5000	-1.28	>0.5000	0.5115
21. Average	3.868	1.664	1.299	54.55	1.15	0.1251	0.70	0.2420	0.1415

Trading rules based on the all order book information. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give buy signal and sell otherwise.

- Cumulative Returns: No Transaction Costs

- All Order Book Information

 $k = 0.01$

Technical rules	In-sam., Ret., %	Out-of-sam., Ret., %, $\theta = 0$	O-of-s, Ret., $\theta = 10^{-4}$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. (offcount ≤ 9.98) and (time2 - maxret20 < maxpr10)	1.744	-3.335	-3.406	42.86	-1.38	>0.5000	-7.98	>0.5000	0.6635
2. (bestoffq1 < avgrpr5) or (9.7 \geq (bestoff - (7.95 - offcount2) (6.58 - minpr10)))	1.779	-0.561	-0.673	42.86	-0.88	>0.5000	-0.24	>0.5000	0.4175
3. (offcount < 9.88) and ((1 < quant) or (quant < avgrt10))	1.744	2.983	2.893	54.76	0.57	0.2843	1.76	0.0392	
4. (time1 \geq quant) and (time1 \geq bestbid) and (maxret20 \leq depthoff2)	1.744	-0.243	-0.389	45.24	-0.24	>0.5000	-0.12	>0.5000	0.3895
5. (7.33 > offcount - max(bestoff, liqoff2)) and (time2 \leq 9.46)	1.744	-0.966	-1.078	50.00	0.15	0.4404	-1.03	>0.5000	0.4635
6. (quant1 > 2) or (lagpr1 > bestoff2)	1.744	0.248	0.108	50.00	-0.17	>0.5000	0.12	0.4522	0.3345
7. ($-\frac{\text{depthoff1}}{\max(4.45, \text{maxret20}, \text{offcount})} \leq 8.23 - \text{offcount}$) and (4.7 > time2)	1.744	-4.098	-4.181	40.48	-1.74	>0.5000	-9.82	>0.5000	0.7245
8. ($(\frac{\max(\text{depthoff1}, \text{liqoff2})}{7.84} > \text{bestbid1})$ and (avgrpr10 \leq time1)) or (avgrpr10 = time1)	1.744	-1.856	-1.980	45.24	-0.44	>0.5000	-0.79	>0.5000	0.5465
9. ((0.47 \leq liqbid2 + (bidcount1 - bidcount2) * (1.7 - quant2)) and (6.755 * avgrpr20 \geq 1.46)) or (bestbid1 = lagret2)	1.744	1.819	1.683	54.76	0.92	0.1788	0.80	0.2119	0.1885
10. (bestoffq1 \leq minpr20) or (bestbidq1 $\leq -\frac{5.98}{8.85 - \text{offcount1}}$)	1.744	0.477	0.377	47.62	-0.20	>0.5000	0.20	0.4207	0.3135
11. ($\frac{\text{avgrpr10}}{\text{bestbid1}} \leq \text{bestoffq}$) and (depthoff1 \geq 8.17 * ret - avgrt5)	1.744	2.553	2.430	61.90	1.61	0.0537	1.38	0.0838	0.1295
12. (time1 \geq price) and (maxpr5 \geq bestoff2)	1.744	-1.767	-1.900	38.10	-1.23	>0.5000	-0.75	>0.5000	0.5375
13. offcount2 \geq offcount	1.744	1.431	1.282	52.38	0.32	0.3745	0.60	0.2743	0.2235
14. (offcount2 > maxret5) and (lagpr2 < time1)	1.744	-0.057	-0.190	45.24	-0.22	>0.5000	-0.04	>0.5000	0.3645
15. (6.36 > minpr5) and (depthoff < depthoff2)	1.744	1.827	1.664	50.00	0.00	0.5000	0.77	0.2206	0.1875
16. (8.37 \geq max(offcount1, 3.33)) and (3 < depthbid)	1.744	-3.815	0.96114	38.10	-1.74	>0.5000	-3.62	>0.5000	0.7015
17. (bestbid1 < time1) and (time1 < depthoff) and (lagpr1 < time1)	1.744	-1.430	-1.568	45.24	-0.36	>0.5000	-0.67	>0.5000	0.5115
18. (quant2 > 2.39) or (avgrt5 \geq max(time, minret5))	1.744	0.381	0.281	50.00	-0.22	>0.5000	0.16	0.4364	0.3245
19. (4 < depthoff) and (8.38 > liqbid1 + min(offcount1, 8.66))	1.744	-2.496	-2.593	45.24	-0.88	>0.5000	-2.46	>0.5000	0.6105
20. (0.71 * time1 > time) or (maxret5 > max(minret20, bestbidq1))	1.744	-2.625	-2.793	35.71	-1.90	>0.5000	-1.14	>0.5000	0.6195
21. Average	1.744	-0.447	-0.586	50.00	-0.06	>0.5000	-0.19	>0.5000	0.4115

Trading rules based on the all order book information. The fitness function is the cumulative return for 3 days trading. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 9 of best 19 rules give signal and sell otherwise.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ Pure Price Information

$k = 0$

Technical rules	In-sam., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 0$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $0.032718 < \text{lagpr1} \cdot (\text{minret20} + 14.407 + \text{maxpr5} - \frac{0.4314}{\text{minpr5}}) \cdot (\text{avgpr20} - \text{minpr10})$	1.761	3.930	6.291	50.48	0.97	0.1660	3.54	0.0002	0.2425
2. $0.02524 < \text{lagpr1} \cdot (\text{lagret1} + 14.437 + \text{minpr10} - \text{avgpr20}) \cdot (\text{avgpr20} - \text{minpr10})$	1.642	2.760	5.333	49.69	-0.48	>0.5000	2.66	0.0039	0.2725
3. $\text{minret20} < 0.9992$	1.511	3.134	4.066	50.52	0.97	0.1660	2.20	0.0139	0.2605
4. $\text{maxret10}^2 < \frac{\text{lagpr2} \cdot \text{avgpr10}}{8.7857 - 6.391 \cdot \text{minret5} - \text{avgpr10}}$	1.247	2.825	3.292	51.15	1.91	0.0281	1.67	0.0475	0.2695
5. $3.0792 \leq \frac{2.188}{\text{minret10}} - \frac{\text{minret5}}{8.8874 + \text{maxret20}}$	0.992	-0.430	0.216	50.17	0.50	0.3085	0.19	0.4247	0.4015
6. $\text{price} - \text{maxret20} < 0.62988$	0.971	3.724	4.321	51.35	2.32	0.0102	2.28	0.0113	0.2445
7. $\text{price} < 1.6410$	0.935	1.918	2.033	50.23	1.55	0.0606	4.54	<0.0001	0.3275
8. $\text{price} \leq 1.6315$	0.629	7.127	7.807	51.17	2.15	0.0158	4.37	<0.0001	0.3405
9. $\text{price} < 1.6346$	0.615	5.822	6.268	50.69	1.53	0.0630	4.29	<0.0001	0.2065
10. sell	0.095	-0.062	-0.055	49.96	0.00	0.5000	0.00	0.5000	0.3865
11. Average	1.500	2.850	3.531	50.87	1.50	0.0668	1.89	0.0294	0.2695

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 4 of best 9 rules give buy signal and sell otherwise.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ Pure Price Information

$$k = 0.0005$$

Technical rules	In-sam., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 0$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $\text{minpr20} < \frac{4.289}{\text{minpr10}} - \text{maxret20}$	1.244	4.632	4.873	51.48	1.99	0.0233	2.63	0.0043	0.0095
2. $\text{minret10} + \text{minret20} > \text{maxpr20} + 0.3638$	0.982	0.159	0.393	49.39	0.06	0.4761	0.22	0.4129	0.2805
3. $\text{maxpr5} \leq \text{avgpr20} \cdot \text{maxret20}$	0.559	-0.006	2.088	49.26	-0.33	>0.5000	0.96	0.1685	0.4055
4. $\text{maxret20} > \text{ret} - \text{avgret5}$	0.514	1.764	3.661	50.61	1.72	0.0427	2.71	0.0034	0.0935
5. $\text{maxret5} - \text{avgret10} \leq \text{avgret5}$	0.493	-0.547	0.125	50.74	-0.38	>0.5000	0.20	0.4207	0.4925
6. $\text{ret} < \text{maxret20}$	0.180	3.155	6.133	50.61	1.67	0.0475	3.66	0.0001	0.0315
7. $\text{maxret5} = \text{avgret5}$	0.167	-0.417	-0.277	50.74	-1.22	>0.5000	-0.61	>0.5000	0.4645
8. $\text{avgpr10} = \text{price}$	0.166	-0.014	0.020	51.14	1.42	0.0778	0.36	0.3594	0.4065
9. $(\text{minret20} > 1.04) \text{ or } (\text{tagpr2} = \text{avgpr10})$	0.155	-0.118	-0.015	51.08	0.43	0.3336	0.09	0.4641	0.4215
10. $\text{avgret20} \leq \text{minret10}$	0.154	0.183	0.241	51.01	0.04	0.4840	1.31	0.0951	0.2755
11. Average	0.638	0.039	2.818	50.87	0.56	0.2877	1.23	0.1093	0.3055

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 4 of best 9 rules give buy signal and sell otherwise.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ Pure Price Information

$$k = 0.005$$

Technical rules	In-sam., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 0$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(\text{lagpr1} > \text{avgpr20} - \text{minret10})$ and $(\text{minpr20} > \text{price} \cdot \text{minret10})$	5.847	4.427	5.808	61.32	2.45	0.0071	2.69	0.0036	0.0745
2. $(\text{minret20} = \text{ret})$ or $(\text{lagret1} - \text{minret5} + \text{minret20} = \text{minret10})$	5.562	3.346	3.726	58.49	1.78	0.0375	1.61	0.0537	0.0985
3. $(\text{ret} = \text{minret20})$ or $(\text{avgret10} < \text{avgret20})$	5.287	2.843	3.179	58.49	1.80	0.0359	1.37	0.0853	0.1075
4. $(\text{maxpr5} = \text{lagpr1})$ or $(\text{minret10} = \max\{\frac{\text{minpr20}}{\text{lagpr2}}, \text{minret5} \cdot \text{minret5}\})$	5.133	0.608	0.869	54.72	1.02	0.1539	0.38	0.3520	0.1545
5. $\frac{6.55}{\text{minpr20}} \geq 0.719 + \text{price} + \text{minpr10}$	5.129	2.436	2.477	52.83	1.39	0.0823	3.70	0.0001	0.1145
6. $(\text{minpr5} = \text{lagpr2})$ and $(\text{minpr20} = \text{lagpr2})$ or $(\text{minret20} = \text{minret5})$	4.361	2.289	2.617	53.77	0.79	0.2148	1.14	0.1271	0.1055
7. $\text{ret} = \text{minret20}$	4.324	7.518	7.912	67.92	4.41	<0.0001	4.04	<0.0001	0.0355
8. $(\text{avgret10} < \text{avgret20})$ or $(\text{avgret20} < \text{avgret5})$	3.943	-0.763	-0.624	50.00	0.00	0.5000	-0.56	>0.5000	0.2285
9. $(\text{avgret5} \geq \text{lagret2})$ and $(\text{avgret5} \geq \text{ret})$	3.785	4.148	4.475	58.49	1.96	0.0250	2.08	0.0188	0.0805
10. $(1 \leq \text{avgret10})$ and $(\text{avgret20} \leq \text{lagret1})$ and $(\text{minpr5} \leq \text{lagret2} \cdot \text{lagpr1})$	3.664	-3.887	-3.662	45.28	-1.10	>0.5000	-1.93	>0.5000	0.3145
11. Average	6.252	3.743	4.111	60.38	2.19	0.0143	1.77	0.0384	0.0895

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 4 of best 9 rules give buy signal and sell otherwise.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ Pure Price Information

$k = 0.01$

Technical rules	In-sam., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 0$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $\frac{\minret20 - \text{avgpr}5}{\minret5 - \text{avgpr}5} - \text{avgpr}10 > 0.3888 \geq \minpr5$	1.667	-0.292	-0.239	48.72	-0.11	>0.5000	-0.20	>0.5000	0.3495
2. $3.2678 \geq \text{avgpr}20 + \minpr5$	1.667	1.048	1.082	56.41	1.88	0.0301	1.19	0.1170	0.1515
3. $(\text{avgpr}10 < \text{avgpr}5)$ or $(\text{lagpr}1 < \text{avgpr}5)$	1.667	-0.780	-0.734	51.28	0.56	0.2877	-1.12	>0.5000	0.4225
4. $\text{avgpr}5 > \text{avgpr}10$	1.045	-1.313	-1.148	38.46	-1.47	>0.5000	-0.51	>0.5000	0.4775
5. $(\text{lagret}2 \geq \text{avgret}20)$ and $(\text{avgret}5 \leq \text{lagret}1)$	1.045	1.107	1.222	58.97	1.24	0.1075	0.61	0.2709	0.1495
6. $(\text{avgpr}5 > \text{lagret}1 \cdot \minpr20)$ or $(\text{avgret}10 < \text{lagret}2)$	1.038	-0.495	-0.449	46.15	-0.65	>0.5000	-0.39	>0.5000	0.3815
7. $\text{avgpr}20 \leq \minpr5$	0.993	-2.961	-2.851	33.33	-2.62	>0.5000	-1.61	>0.5000	0.6505
8. $(\text{ret} > \text{avgret}10)$ and $(\minret10 > \minret20)$	0.993	-0.577	-0.518	48.72	-0.42	>0.5000	-0.29	>0.5000	0.3955
9. $(\text{ret} > \minret5)$ and $(\minpr5 > \minpr20)$	0.993	-0.518	-3.352	30.77	-2.99	>0.5000	-1.78	>0.5000	0.3835
10. $(\text{lagpr}1 > \text{lagpr}2)$ and $(\text{ret} > \text{avgret}10)$	0.993	-1.506	-1.460	46.15	-1.03	>0.5000	-5.29	>0.5000	0.4975
11. Average	1.045	-1.495	-1.383	43.59	-0.80	>0.5000	-0.69	>0.5000	0.4965

Trading rules based on the pure price information only. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 4 of best 9 rules give buy signal and sell otherwise.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ Pure Price Information

GA based no-transaction region

Technical rules		In-sample $\theta = 0.0001$	Out-of-s $\theta = 0.0001$
1a.	$\frac{1.6324}{\minret5} > \max\{\text{price}, \text{avgpr10}\}$	1.836	3.542
1b.	true		
2a.	$2.6361 \geq \minret10 + \maxpr20$	1.401	3.544
2b.	true		
3a.	$3.2724 > \maxpr10 + \minpr10$	1.198	2.939
3b.	true		
4a.	false	1.148	2.565
4b.	$5.4633 \leq \frac{\maxret10 - \maxret20}{1.18225 - \text{avgret20} - \minret5}$		
5a.	false	0.892	1.562
5b.	$1.639 < \text{price}$		
6a.	false	0.753	2.802
6b.	$ \text{price} - \maxpr10 \leq 0.0036$		
7a.	$(1.00524 \geq \text{ret}) \text{and} (1.00524 < \maxret20)$	0.689	1.782
7b.	true		
8a.	$(\minpr20 + 7.7928 - \text{avgret5} + \minret20 \leq \frac{9.77 \cdot \text{avgret5}}{\text{avgret20} - \text{avgpr10} (\minpr20 - \text{avgret20})})$	0.664	1.332
8b.	true		
9a.	$\maxpr5 \geq \minret20 \cdot 1.648865$	0.652	2.041
9b.	true		
10a.	false	0.401	0.559
10b.	$\frac{9.83103}{\minpr20} + \text{avgret5} < 7.101135$		

Trading rules based on the pure price information. Buy if a) rule is true, Sell if a) is false and b) is true, No Trade if both a) and b) are false. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ All Order Book Information

$$k = 0.0005$$

Technical rules	In-sam., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 0$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $((\text{minpr5} < \text{bestoff2}) \text{ or } (0.4 \geq \min\{\text{bestoffq}, \text{offcount2} + \text{liqoff1}\} + \text{liqoff1}) \text{ or } (\frac{1}{4.16 - \min\{\text{depthbid1}, \frac{\text{bestoff1} - 0.3}{\text{price}}\}} > 1)) \text{ and } (13 < \text{depthhoff}))$	1.522	-1.044	-0.229	48.89	-0.17	>0.5000	-0.25	>0.5000	0.4625
2. $(\text{depthbid1} < \text{depthhoff} + \text{bestbidq2} - 2.192) \text{ or } (\text{depthbid1} < 2.19)$	1.375	-4.988	-3.719	48.62	-0.81	>0.5000	-1.64	>0.5000	0.8285
3. $(9.76 < \text{offcount}) \text{ or } (\text{bestoffq2} \cdot \text{bidcount1} < 2.9 \cdot \text{price})$	1.146	-2.666	-2.054	49.50	0.68	0.2483	-2.08	>0.5000	0.6415
4. $(\text{quant1} > \text{offcount1}) \text{ or } (\text{offcount} > 9)$	1.033	0.256	0.839	49.56	0.72	0.2358	0.75	0.2266	0.2905
5. $(2 < \text{bestoffq2}) \text{ and } (\text{bestoff2} = \text{maxpr20})$	1.002	-0.149	0.565	51.18	0.44	0.3300	0.62	0.2676	0.3465
6. $(\text{depthbid1} < \max\{7.48 + \text{liqbid1}, \text{avgpr20}\}) \text{ or } (9.74 - \text{avgret5} \geq \text{maxpr20} \cdot 5.3)$	0.994	0.048	0.055	48.89	0.0	0.5000	0.0	0.5000	0.3225
7. $(2.13 \leq \max\{\text{liqbid1}, \text{minret10}\} \cdot \max\{\text{bestoffq2}, 1.26\}) \text{ and } (\text{bestoff2} = \text{bestbid})$	0.957	-0.951	-0.454	50.50	-0.15	>0.5000	0.49	0.3121	0.4475
8. $(\text{bestoffq} \cdot \text{maxret5} \cdot \text{bestoff2} > 9.3) \text{ or } (\text{depthbid1} \cdot \text{maxret5} < \min\{9.3, \frac{\text{bestoff1}}{\text{bestoff2}}\})$	0.897	-3.064	-0.417	50.03	-0.81	>0.5000	-0.73	>0.5000	0.6795
9. $9.44 \cdot \text{ret} \leq \text{offcount}$	0.852	-0.761	-0.263	49.63	0.76	0.2236	-0.29	>0.5000	0.4115
10. $\text{depthhoff} > 2.35 \cdot \max\{\text{bestbidq}, 8.27\}$	0.827	-0.898	-0.292	48.89	-0.21	>0.5000	-0.25	>0.5000	0.4395
11. Average	2.342	0.152	0.842	49.02	-0.04	>0.5000	0.69	0.2451	0.2995

Trading rules based on the all order book information. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 4 of best 9 rules give buy signal and sell otherwise.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ All Order Book Information

$$k = 0.005$$

Technical rules	In-sam., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 0$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(\text{bestbidq2} \leq 5)$ and $(\max\{(\text{bestbidq1} - 6.7) \cdot (\text{liqbid} + 3.6) , \text{minpr5}\} \geq \min\{\text{liqbid1}, 3.2\})$ and $(\text{bestoffq} > \min\{\text{quant2}, 5.16\})$	4.137	1.026	1.350	50.00	0.22	0.4129	0.65	0.2578	0.1895
2. $(\text{minpr20} \geq \frac{3.81}{\text{avgrt10} - \text{minpr10}})$ and $(\text{depthbid} \leq \text{bestoffq} \cdot \max\{\min\{\text{depthoff}, \text{depthbid1}\} - \max\{\text{bidcount}, 2.12\}, \text{bestoffq}\})$	3.162	2.570	2.965	55.45	1.36	0.0869	1.26	0.1038	0.0895
3. $(\text{bestoffq} > \text{lagret2})$ and $((8.6 = \text{liqbid}) \text{ or } (\text{bestbidq1} > \text{lagret2} \cdot 1.99))$	3.060	1.141	1.520	52.73	0.80	0.2119	0.68	0.2483	0.1615
4. $(\text{bestoffq} - \text{liqoff} > 1.62)$ and $(\text{minpr10} = \text{minpr5})$	2.841	1.961	2.308	57.27	1.74	0.0409	0.96	0.1685	0.1315
5. $(\min\{8.63, \text{bestoffq1}\} > \text{price})$ and $(4.72 > \text{bestoffq1})$ or $(\min\{\text{bestoffq}, \text{avgrt20}\} > \text{price})$	2.835	3.982	4.266	59.09	2.16	0.0154	1.83	0.0336	0.0505
6. $((\text{bestbidq} \cdot \text{bestoff2} - \text{bestbidq} - \text{lagpr2}) \cdot 6.97 \leq \text{bestoff1} \cdot \text{bestoffq} \cdot \text{bestoff2})$ and $(\text{bestoffq2} \leq \text{bestoff2} + 3)$	2.546	0.175	0.924	54.55	1.25	0.1056	0.39	0.3483	0.2675
7. $(\text{bestbidq} \leq 3)$ and $(\text{price} \leq \text{lagpr2})$	2.528	5.140	5.519	60.91	2.58	0.0049	2.29	0.0110	0.2335
8. $(\frac{3.23}{\text{depthbid}} + \text{bestoffq2} < 4.05)$ or $(\text{avgrt10} \leq \text{avgrt20})$	2.284	3.367	3.539	55.45	1.74	0.0409	2.19	0.0143	0.0585
9. $(\text{offcount1} \cdot \text{bestoffq2} \leq \text{depthoff})$ or $(\text{offcount1} \leq \text{time})$	2.268	-0.321	-0.101	48.18	-0.46	>0.5000	-0.07	>0.5000	0.3175
10. $\text{avgrt20} \geq \min\{\text{ret}, \text{avgrt10}\}$	2.196	5.017	5.367	49.93	2.60	0.0047	2.52	0.0059	0.0315
11. Average	4.362	5.368	5.672	60.00	2.35	0.0094	2.36	0.0091	0.0275

Trading rules based on the all order book information. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 4 of best 9 rules give buy signal and sell otherwise.

- Cumulative Returns: Transaction Costs $\theta = 0.0001$

- All Order Book Information

$k = 0.01$

Technical rules	In-sam., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 10^{-4}$	Out-of-s., Ret., $\theta = 0$	DC, %	P-T stat.	P-T p-value	A-G stat.	A-G p-value	White's p-value
1. $(6.157 + \text{minret}10 < \text{depthbid1})$ and $(\text{time1} \geq \text{bestoff}2)$	1.703	-1.353	-1.514	47.62	0.00	0.5000	-0.55	>0.5000	0.5295
2. $(\text{offcount}2 > 1)$ and $(\text{offcount}2 < 11)$	1.703	-1.309	-1.237	42.86	-0.99	>0.5000	-1.11	>0.5000	0.5255
3. $(\text{liqoff} < 4.415)$ and $(\text{depthoff} < \text{depthoff}2)$	1.703	1.664	1.827	50.00	0.0	0.5000	0.77	0.2206	0.2215
4. $(7.6 \geq \text{maxpr}20 + \frac{\text{time}2}{1.72})$ and $(7 \leq \text{offcount}2)$	1.703	-0.492	-0.472	47.62	-1.86	>0.5000	-3.01	>0.5000	0.4505
5. $(\text{offcount} > 3)$ and $(\text{time1} > \text{bestoff}1)$	1.703	-1.463	-1.331	45.24	-0.29	>0.5000	-0.64	>0.5000	0.5415
6. $(\text{offcount}1 < 9)$ and $(\text{time}2 < \text{minret}10 \cdot 9.3176)$	1.703	-2.414	-2.316	47.62	-0.28	>0.5000	-5.96	>0.5000	0.6465
7. $\text{avgpr}5 \cdot \text{time} \leq \text{time}1$	1.651	-2.806	-2.670	35.71	-1.97	>0.5000	-1.20	>0.5000	0.6835
8. $(20.52 \cdot \text{time}2 > \text{maxret}20 + \text{bestbid}1)$ and $(\text{depthbid} \geq \text{depthoff}1)$	1.651	-2.079	-1.968	38.10	-1.61	>0.5000	-0.92	>0.5000	0.6105
9. $(\text{quant}2 \leq \text{avgret}10)$ or $(\text{time}1 \geq 5.31)$	1.092	1.684	1.806	50.00	0.32	0.3745	0.75	0.2266	0.2205
10. $(\text{lagpr}2 < \text{maxpr}5)$ and $(6 \leq \text{depthbid}1)$	1.087	-1.249	-1.143	42.86	-0.88	>0.5000	-0.76	>0.5000	0.5205
11. Average	1.703	-0.886	-0.774	47.62	-0.13	>0.5000	-0.38	>0.5000	0.4905

Trading rules based on the all order book information. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday. The average rule gives buy signal if more than 4 of best 9 rules give buy signal and sell otherwise.

└ Cumulative Returns: Transaction Costs $\theta = 0.0001$

└ All Order Book Information

GA based no-transaction region

Technical rules	In-sample $\theta = 0.0001$	Out-of-s $\theta = 0.0001$
1a. $(\max\{6.166, 2.3 \cdot (\text{liqbid1} - 14.564 + \text{avgpr5})\maxpr10, 18.41 - \text{bidcount}\} \leq 8.438) \text{and} (\maxpr10 \leq \text{liqbid1} - \text{minpr20} \cdot 8 + \text{avgpr5})$		
1b. true	1.988	1.203
2a. $(77.827 \cdot \text{liqoff1} \cdot \text{bestoff2} \cdot \text{liqbid} \geq \text{minpr20}^2 \max\{4.712, \text{time}\} \cdot \max\{\text{avgpr10}, \text{time}\}) \text{and} (2.64 \geq \text{depthbid1})$		
2b. $\frac{10}{1.6437 - \text{price}} < \text{minret5}$	1.009	0.000
3a. false		
3b. $0.99723 \leq \text{minret5}^2$	0.901	2.809
4a. $(\text{avgpr5} - \max\{\text{liqoff1}, \text{minpr20}\} = \text{liqbid} - \text{avgret5}) \text{or} (3.918 < \text{liqbid2})$		
4b. $(\text{price} \cdot \text{maxret10} \geq \text{bestbid}) \text{or} (\text{price} \cdot \text{maxret20} \geq \text{bestbid} \cdot \text{maxret10})$	0.803	2.881
5a. false		
5b. $(1.63289 \leq \text{minpr20}) \text{or} (\max\{\min\{\text{depthbid1}, \text{time2} - \text{maxpr10} \}, 3.48\} = \text{depthoff} - \text{bestbidq1})$	0.721	0.123
6a. $(\frac{\text{bestoffq1}}{\text{time2}} + \text{avgpr10} \leq 4.1) \text{and} (\frac{0.4965}{\text{bestoffq1}} + \text{avgpr10} \geq \text{depthbid1})$		
6b. true	0.538	-0.840
7a. $1.5 > \text{depthbid1}$		
7b. $\text{minpr5} - \text{maxret20} \geq 0.627$	0.466	-0.366
8a. $(\max\{\text{bidcount} - \frac{\text{bestoff2}}{\text{avgret20} \cdot \text{depthbid2}} < 5.1\}) \text{and} (\max\{\text{bidcount} - \frac{\text{lagpr2}}{\text{bidcount1}} \cdot \text{depthoff}\} \geq \frac{6.77}{\text{bidcount} - \text{bestoffq} \cdot 0.637})$		
8b. $(1.1 \leq \text{offcount}) \text{or} (\text{liqbid} > 1.766)$	0.453	-0.757
9a. $(\text{bestbid1} > \min\{\text{bestoffq1}, \text{bestbidq2}\}) \text{and} (\text{depthbid1} \leq 2.7)$		
9b. true	0.453	-0.181
10a. $(5.0 \geq \text{depthbid2}) \text{or} (\text{bestbid1} > 34 + (\text{lagpr1} - \text{time2}) \cdot 0.1)$		
10b. $(\max\{\text{depthbid}, \text{depthoff} - 6.5\} \geq 2.7) \text{or} (\text{bidcount1} \leq \text{maxret20})$	0.402	1.786

Trading rules based on the all order book information. Buy if a) rule is true, Sell if a) is false and b) is true, No Trade if both a) and b) are false. The fitness function is the cumulative return for 3 days trading under proportional transaction costs, $\theta = 0.0001$. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday.

- └ Average Rules
- └ Pure Price Information

Average Rules: Pure Price Info

Fitness function: directional changes

k	In-sample, Dir. change., %	Out-of-sample, Dir.Ch., %	Daily return, %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	55.10	54.11	2.150	-6.123	5.98	<0.0001	0.93	0.1762	0.0055
0.0005	58.50	51.95	6.559	3.062	1.28	0.1002	2.49	0.0064	0.1255
0.001	58.66	54.12	8.051	5.763	2.19	0.0143	3.42	0.0003	0.0295
0.005	76.00	67.92	8.340	7.885	3.97	<0.0001	3.54	0.0002	0.0125
0.01	83.33	64.10	2.310	2.208	2.27	0.0116	1.11	0.1335	0.1105

Fitness function: cumulative returns, no-transaction costs

k	In-sample, Daily ret., %, $\theta = 0$	Out-of-sample, Daily ret., %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	5.654	8.327	-10.267	51.00	1.44	0.0749	3.51	0.0002	0.0025
0.0005	3.460	6.676	1.833	51.75	1.31	0.0951	2.84	0.0023	0.0065
0.001	3.555	4.150	2.190	52.75	1.42	0.0778	1.88	0.0301	0.0165
0.005	1.813	8.690	8.198	69.81	4.48	<0.0001	3.71	0.0001	0.0015
0.01	1.645	1.447	1.359	51.28	0.20	0.4207	0.65	0.2578	0.0745

Fitness function: cumulative returns, $\theta = 0.0001$

k	In-sample, Daily ret., %, $\theta = 10^{-4}$	Out-of-sample, Daily ret., %, $\theta = 10^{-4}$	Daily return, %, $\theta = 0$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	1.500	2.850	3.531	50.87	1.50	0.0668	1.89	0.0294	0.2695
0.0005	0.638	0.039	2.818	50.87	0.56	0.2877	1.23	0.1093	0.3055
0.001	2.614	3.594	5.828	52.61	1.34	0.0901	2.68	0.0037	0.1445
0.005	6.252	3.743	4.111	60.38	2.19	0.0143	1.77	0.0384	0.0895
0.01	1.045	-1.495	-1.383	43.59	-0.80	>0.5000	-0.69	>0.5000	0.4965

Average of best 20 trading rules for different fitness functions based on the pure price information. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday.

Average Rules

All Order Book Information

Average Rules: All Order Book Info

Fitness function: **directional changes**

k	In-sample, Dir. change, %	Out-of-sample, Dir.Ch., %	Daily return, %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	58.78	57.30	5.821	-3.202	10.72	<0.0001	2.40	0.0082	<0.001
0.0005	64.51	51.31	3.791	0.918	1.88	0.0301	1.91	0.0281	0.0655
0.001	59.72	46.36	0.222	-2.161	-1.83	>0.5000	0.09	0.4641	0.6765
0.005	85.19	47.27	-0.968	-1.337	-0.38	>0.5000	-0.41	>0.5000	0.5885
0.01	100.00	47.62	-0.985	-1.024	-0.77	>0.5000	-2.27	>0.5000	0.5725

Fitness function: **cumulative returns, no-transaction costs**

k	In-sample, Daily ret., %, $\theta = 0$	Out-of-sample, Daily ret., %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	9.165	11.830	-8.481	53.37	4.95	<0.0001	4.79	<0.0001	0.0035
0.0005	5.518	1.173	-2.606	49.76	-0.40	>0.5000	0.51	0.3050	0.0795
0.001	4.668	0.810	-1.586	48.42	-0.78	>0.5000	0.34	0.3669	0.1435
0.005	3.868	1.664	1.299	54.55	1.15	0.1251	0.70	0.2420	0.1415
0.01	1.744	-0.447	-0.586	50.00	-0.06	>0.5000	-0.19	>0.5000	0.4115

Fitness function: **cumulative returns, $\theta = 0.0001$**

k	In-sample, Daily ret., %, $\theta = 10^{-4}$	Out-of-sample, Daily ret., %, $\theta = 10^{-4}$	Daily return, %, $\theta = 0$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	0.255	-0.840	-0.754	49.99	-0.38	>0.5000	-4.95	>0.5000	0.3255
0.0005	2.342	0.152	0.842	49.02	-0.04	>0.5000	0.69	0.2451	0.2995
0.001	4.328	-1.913	-0.058	49.11	-0.71	>0.5000	-0.02	>0.5000	0.4935
0.005	4.362	5.368	5.672	60.00	2.35	0.0094	2.36	0.0091	0.0275
0.01	1.703	-0.886	-0.774	47.62	-0.13	>0.5000	-0.38	>0.5000	0.4905

Average of best 20 trading rules for different fitness functions based on the all order book information. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday.

- Maximum Rules

- Pure Price Information

Maximum Rules: Pure Price Info

Fitness function: directional changes

k	In-sample, Dir. change, %	Out-of-sample, Dir.Ch., %	Daily return, %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	55.35	52.90	5.786	-9.154	4.21	<0.0001	2.47	0.0068	0.0175
0.0005	56.01	51.28	6.040	3.133	1.07	0.1423	2.57	0.0051	0.2345
0.001	60.07	55.49	9.390	7.317	3.17	0.0008	4.52	<0.0001	0.0065
0.005	84.00	60.38	4.085	3.711	2.21	0.0136	1.76	0.0392	0.0865
0.01	100.00	56.41	1.536	1.414	0.79	0.2148	0.67	0.2514	0.2575

Fitness function: cumulative returns, no-transaction costs

k	In-sample, Daily ret., %, $\theta = 0$	Out-of-sample, Daily ret., %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	6.562	8.689	8.905	51.48	2.15	0.0158	3.67	0.0001	<0.001
0.0005	4.569	6.233	2.477	52.62	1.89	0.0294	2.74	0.0031	0.0065
0.001	3.781	6.049	4.484	51.92	0.98	0.1635	2.61	0.0045	0.0035
0.005	3.292	0.644	0.550	53.77	1.49	0.0681	0.53	0.2981	0.1555
0.01	1.707	-1.062	-1.095	48.72	-0.05	>0.5000	-1.31	>0.5000	0.3635

Fitness function: cumulative returns, $\theta = 0.0001$

k	In-sample, Daily ret., %, $\theta = 10^{-4}$	Out-of-sample, Daily ret., %, $\theta = 10^{-4}$	Daily return, %, $\theta = 0$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	1.671	3.930	6.291	50.48	0.97	0.1660	3.54	0.0002	0.2425
0.0005	1.244	4.632	4.873	51.48	1.99	0.0233	2.63	0.0043	0.0095
0.001	4.319	3.676	5.012	53.30	2.05	0.0201	3.17	0.0008	0.1435
0.005	5.847	4.427	5.808	61.32	2.45	0.0071	2.69	0.0036	0.0745
0.01	1.667	-0.292	-0.239	48.72	-0.11	>0.5000	-0.20	>0.5000	0.3495

Average of best 20 trading rules for different fitness functions based on the pure price information. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday.

- Maximum Rules

- All Order Book Information

Maximum Rules: All Order Book Info

Fitness function: directional changes

k	In-sample, Dir. change, %	Out-of-sample, Dir.Ch., %	Daily return, %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	58.61	57.30	6.378	-5.628	10.70	<0.0001	2.61	0.0045	<0.001
0.0005	60.24	52.86	1.103	-3.133	2.55	0.0054	0.47	0.3192	0.0055
0.001	63.25	45.82	-1.366	-2.964	-2.13	>0.5000	-0.63	0.4641	0.7505
0.005	85.19	47.27	0.430	0.156	-0.46	>0.5000	0.24	0.4052	0.5885
0.01	100.00	50.00	-0.915	-0.935	0.07	0.4721	-3.68	>0.5000	0.4315

Fitness function: cumulative returns, no-transaction costs

k	In-sample, Daily ret., %, $\theta = 0$	Out-of-sample, Daily ret., %, $\theta = 0$	Daily return, %, $\theta = 10^{-4}$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	8.404	12.256	-0.476	53.76	5.75	<0.0001	5.18	<0.0001	0.0035
0.0005	4.945	-1.098	-5.057	48.89	-1.08	>0.5000	-0.47	>0.5000	0.2825
0.001	4.843	2.008	0.476	50.62	0.67	0.2514	0.87	0.1922	0.0725
0.005	4.198	4.774	4.418	59.09	2.20	0.0139	1.99	0.0233	0.0345
0.01	1.744	-3.335	-3.406	42.86	-1.38	>0.5000	-7.98	>0.5000	0.6635

Fitness function: cumulative returns, $\theta = 0.0001$

k	In-sample, Daily ret., %, $\theta = 10^{-4}$	Out-of-sample, Daily ret., %, $\theta = 10^{-4}$	Daily return, %, $\theta = 0$	Dir. change, %	P-T statistic	P-T p-value	A-G statistic	A-G p-value	White's p-value
0	0.385	-0.919	-0.767	49.97	-0.53	>0.5000	-3.04	>0.5000	0.3265
0.0005	1.522	-1.044	-0.299	48.89	-0.17	>0.5000	-0.25	>0.5000	0.4625
0.001	3.130	-1.111	1.391	51.71	1.08	0.1401	0.59	0.2776	0.4155
0.005	4.137	1.026	1.350	50.00	0.22	0.4129	0.65	0.2578	0.1895
0.01	1.703	-1.353	-1.514	47.62	0.00	0.5000	-0.55	>0.5000	0.5295

Average of best 20 trading rules for different fitness functions based on the all order book information. In-sample period: Monday and Tuesday, out-of-sample: Wednesday – Friday.

Probabilities based on bootstrap method

Pure Price Information

k	$P\{DC \geq 50\%\}$	Ω_{DC}	$P\{r \geq 0\}$	$P\{r \geq r_{BH}\}$	Ω_{EV}	$P\{r_{TC} \geq 0\}$	$P\{r_{TC} \geq r_{BH}\}$	Ω_{TC}
0	0.943	83.384	0.956	0.974	152.380	0.534	0.607	1.327
0.0005	0.744	4.508	0.939	0.946	36.118	0.571	0.620	1.871
0.001	0.799	7.689	0.915	0.931	43.846	0.798	0.811	9.265
0.005	0.831	14.035	0.893	0.911	26.625	0.869	0.870	15.960
0.01	0.813	6.461	0.753	0.789	6.087	0.731	0.760	5.655

All Order-Book Information

k	$P\{DC \geq 50\%\}$	Ω_{DC}	$P\{r \geq 0\}$	$P\{r \geq r_{BH}\}$	Ω_{EV}	$P\{r_{TC} \geq 0\}$	$P\{r_{TC} \geq r_{BH}\}$	Ω_{TC}
0	0.975	261.221	0.912	0.914	39.415	0.681	0.693	3.882
0.0005	0.825	9.203	0.872	0.881	17.734	0.640	0.683	2.620
0.001	0.787	84.811	0.870	0.873	14.355	0.677	0.723	4.086
0.005	0.815	6.837	0.834	0.850	12.539	0.775	0.768	5.823
0.01	0.759	3.553	0.731	0.765	4.535	0.674	0.715	3.231

Probabilities of non-losing money and Ω statistics for bootstrapped data.

Performances of some standard rules

Trading rules	Performance measure	$k = 0$	$k = 0.0005$	$k = 0.001$	$k = 0.005$	$k = 0.01$
MA Oscillator	DC	52.21	50.87	50.41	50.94	43.59
	R	-0.088	-0.720	0.003	-0.032	0.067
	R^{tc}	-3.472	-3.223	-1.512	-0.345	-0.087
Variable length MA Oscillator	DC	50.04	48.99	49.18	50.00	48.72
	R	0.055	0.055	0.055	0.055	0.055
	R^{tc}	0.048	0.048	0.048	0.048	0.048
Filter Rule	DC	49.67	50.27	50.00	46.23	41.03
	R	-2.908	-2.908	-2.908	-2.908	-2.908
	R^{tc}	-2.953	-2.953	-2.953	-2.953	-2.953
Breakout Rule	DC	49.96	51.01	50.82	50.00	51.28
	R	-0.055	-0.055	-0.055	-0.055	-0.055
	R^{tc}	-0.062	-0.062	-0.062	-0.062	-0.062
Bollinger Band	DC	51.56	52.89	51.79	50.00	48.72
	R	1.309	1.795	1.122	0.055	0.055
	R^{tc}	1.275	1.761	1.102	0.048	0.048
Aroon Oscillator	DC	48.58	50.00	49.45	46.23	43.59
	R	-1.757	-1.327	-1.327	-1.298	-1.306
	R^{tc}	-1.797	-1.353	-1.353	-1.324	-1.333