Understanding and testing for stationarity in finite-sized time-series using surrogate data: Applications to some tornado frequency and financial time-series

Volker Keinhorst,

Project Supervisors: Khurom Kiyani, Sandra Chapman

▶What is stationarity?

On a basic level it means that for a given time series any windows taken from said time series will share certain properties. The two most commonly used types of stationarity are:

▸Strong stationarity:

All moments, and thus the probability distribution function, are identical.

▸Weak stationarity:

Only the first and the second moment (expectation and variance) have to be identical.

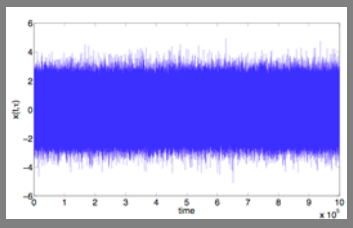
The concept of stationarity is very valuable in the field of time series analysis, as it allows a degree of prediction for the data being analysed. This is the reason why the development of accurate tests for this is important. In the course of the project we have attempted to write a computer routine to test for strong stationarity.

▶What is surrogate data?

The concept of surrogate data was first developed to supply a testing method for nonlinearity in time series. Essentially, given a time series, a new time series will be created that is completely random, but shares as many properties as possible with the original.

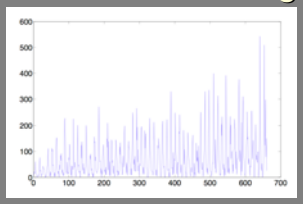
▶A quick preliminary test

Before any elaborate tests should be run to check for strong stationarity, a much simpler technique should be applied, namely eye inspection. If there seems to be any kind of 'development' in the graph, e.g. trends or periodic fluctuations, then the data cannot be stationary:



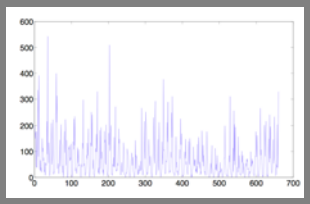
While the time series on the left might be strongly stationary, the one on the right definitely cannot be. Stationarity might be achieved by differencing the data.

▶Three surrogate data algorithms and their results using tornado data



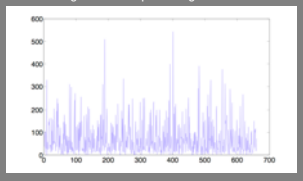
➢The Raw data

Here is the original data used to produce the following three example surrogates.



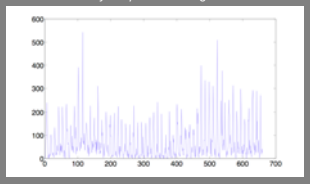
➢The Random Shuffle

Here the data set is just randomly shuffled. Preserves only the *pdf* of the original data.



➢The AAFT algorithm

A more advanced algorithm than above. It rescales a Gaussian white noise set to the original data, Fourier transforms this set, randomises the phases and transforms back. Then the original data is rescaled to the re-transformed set. This preserves the *pdf* and can preserve the power spectrum (although it can suffer from 'power spectrum whitening').



➢The IAAFT algorithm

This algorithm works initially just like the AAFT. But then it rescales the power spectrum of the produced surrogate iteratively to the power spectrum of the original data.

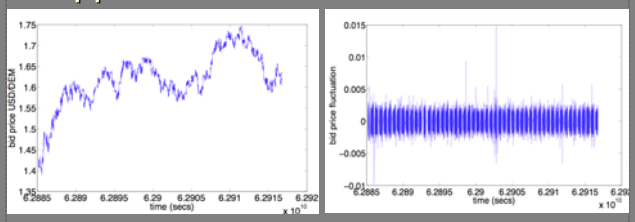
Data obtained from University of Nebraska Lincoln [http://www.hprcc.unl.edu/nebraska/U_S_SEVERE.html]

▶The Routine

The routine consists of the following steps:

- Optional: Difference the data
- Create a certain number of surrogates from the specified data using the IAAFT algorithm
- Take the first 20% of the surrogates and the original data
- Use the Kolmogorov-Smirnov test to compare each surrogate data window to the original data window
- Count how many rejections the test produces
- If there are too many rejections, the data is not strongly stationary

▶Application to financial data



Original data and the differenced form of it. Results were:

- Original data definitely not strongly stationary (determinable by eye).
- Strong stationarity rejected for differenced data by the routine, however need to still check for infinite variance cases.

Data obtained from Olsen Associates [http://www.olsen.ch/]

▶Difficulties and Problems

➢Internal structure

Test only depends on the numbers inside the data window, not on the structure of the data.

➢Problem cases

The test cannot yet fully handle all kinds of non-strong stationarity. A few examples include:

oSpikes

oInfinite variance cases

➢Data window length

All currently available surrogate algorithms will preserve the pdf of the original data. This means one has to use data windows in order to test for strong stationarity, as one would otherwise always obtain a positive result. But how long should this window be?

oToo short -> many false rejections

oToo long -> many false acceptances

Our routine uses 20% of the original data, which seems to be sufficient.

➢Test still in development phase

The test is not yet complete. It can deal with usual cases quite well, but has problems with more exotic situations. Future attempts to correct this include:

oNew distribution comparison test

oNew algorithm for the surrogates

oModifications to deal with problem cases

At the present, the routine works very well at disproving strong stationarity, but still has a high rate of false positive results, which will be corrected in the future.